Chapter 1: Electric Charges and Fields

Electrostatics is a branch of physics which deals with the study of electric charge at rest

Or

It is a branch of physics which deals with the study of force, electric field and potential due to electric charge at rest

What is an electric charge?

Electric charge is a basic property of a body due to which the body attracts or repels another body.

Charge is a scalar quantity.S.I. unit of charge is coulomb (C)

Note: Electric charge is best understood by what it does and not by what it is

Types of charges: There are two types

1) Positive charge, 2) Negative charge

By convention:

The charge on proton is taken as positive charge.
The charge on electron is taken as negative charge.

Note: Old convention

1. Charge on glass rod is called positive charge, Charge on plastic rod is called negative charge
2. The charges were named as positive and negative by an American scientist Benzamin Franckline.

Neutral body or uncharged body: The body is said to be neutral if it has no charge. **Ex**: Atom

Charged body or electrified body: The body is said to be charged if it has net charge **Ex**: Ion

Polarity of charge:

The property which differentiate the two types of charges is called polarity of charge.

Note:

1. When two bodies are rubbed together, then electrons are transferred from one body to another body. The body which looses an electron becomes positively charged body and the body which gains an electrons becomes negatively charged body

2. Addition of electrons gives negatively charged body and removal of electrons gives positively charged body.

3. When the substance listed in column (1) are rubbed with the substances listed in column (2). Then the substances in column (1) acquire the positive charge and the substance in column(2) acquires negative charge.

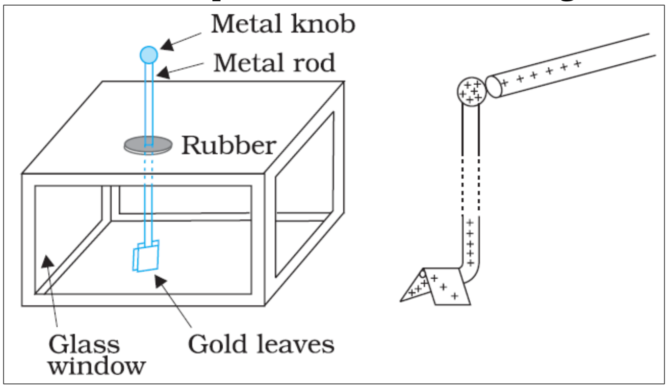
Column (1) (+ve charge)	Column (2) (–ve charge)
Glass rod	Silk cloth
Cat skin or fur	Ebonite rod
Woollen cloth	Amber rod
Woollen cloth	Plastic
Woollen cloth	rubber

4. Thales of Miletus was the first scientist who discovered the fact that an amber rubbed with silk or wool attracts the light objects. It was around 600 B.C.
5. Mass of a body is always is positive, whereas charge can be either positive or negative.

Electroscope: It is a device used to detect the charge on a body.

Gold leaf electroscope:

It is one type of electroscope used to detect charge on a body.



Construction:

It is one type of electroscope used to detect the charge on a body. It consists of a vertical metal rod kept on a box.A metal knob is present at the upper end and gold leaves are attached at bottom end of a metal rod.When a charged body touches the metal knob, the charge flows to the gold leaves and gold leaves diverge (repelled).The degree of divergence of leaves depends on charge on a body.

Conductor: The substance which allows the charges to pass through it easily is called conductor

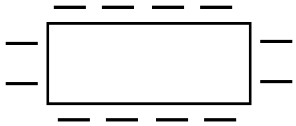
Ex: All metals (Cu, Al, Ag, Fe etc…), thehumanbody, animal body and earth.

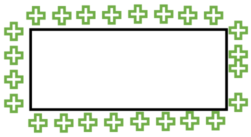
Insulator: The substance which doesnot allow the charges to pass through it is called insulator.

Ex: Wood, plastic, Diamond, Glass

Note (1): When a conductor is charged then charges are distributed over the entire surface of the conductor

Ex:

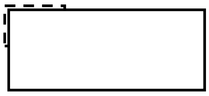




Negatively charged conductor Positively charged conductor

Note(2): If some charge is put on an insulator, it stay at the same place

Ex:



Negatively charged Insulator Positively charged Insulator

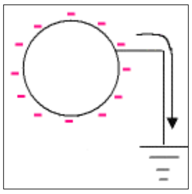
That is the charge doesn't get distributed over the surface of insulator.

Electrification:The process of charging a body is called electrification.

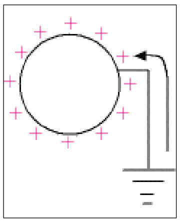
Earthing or Grounding:

The process of sharing the charges with the earth is called earthing or grounding.

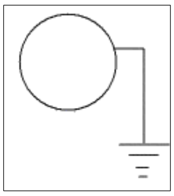
Ex 1): When a negatively charged body is connected to earth, then the electrons flow from body to earth till the body becomes neutral.



Ex 2): When a positively charged body is connected to earth, then the electron flow from earth to body till the body becomes neutral.



Ex 3): When a neutral body is connected to earth, then there is no flow of charge from body to earth.



Earthing is necessary for house, why?

The electricity from the mains is supplied to our houses using a 3 core wiring called live wire, neutral wire and earth wire.

The live and neutral wires carry the current from the power station. One end of the earth wire is connected to a thick metal plate which is buried in the earth and the other end of earth wire is connected to metallic bodies of electrical appliances (refrigerator, T.V, etc).

When live wire touches the metallic body then charges flow to the earth through earth wire.Therefore, there is no damages to the electrical appliances.

Note:

1. Usually the live wire is red in colour neutral wire is black in colour and the earth wire is green in colour.

Methods of charging a body

There are three methods to charge a body

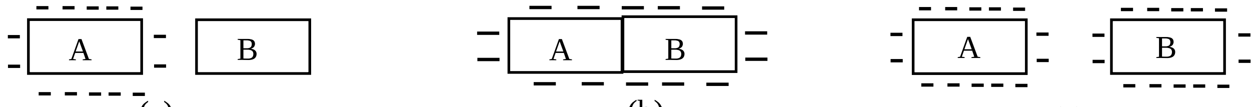
a) Charging by friction
b) Charging by conduction
c) Charging by induction

Charging by friction:

When a glass rod is rubbed with silk cloth then glass rod loses the electron and silk cloth gains electron therefore glass rod becomes positively charged body and the silkcloth becomesnegatively charged body. This process is called charging by friction

Note: Normally insulator can be charged by friction method. In this method opposite charges are developed.

Charging by conduction:



(a) (b) (c)

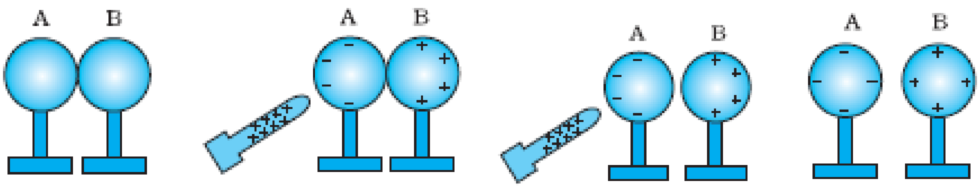
Let A and B are the two conductors A is negatively charged and B is neutral when A and B are brought in contact, free electrons flow from A to B therefore conductor B gets negatively charged.This process is called charging by conduction.

Note: Charging by conduction is suitable for conductors. In this method same charges are developed.

Charging by Induction:

When a charged body is brought near the neutral body then opposite charges are developed on a near surface of a neutral body. This process is called charging by Induction

Describe how two spheres can oppositely charged by induction method



(a) (b) (c) (e) (e)

The two metal spheres can be oppositely charged by following step:

Step 1: Bring two metal spheres A and B in contact mounted on insulating stand as shown in fig (a).

Step 2: Bring a positively charged glass rod near the sphere A. Then the free electrons in A and B are attracted towards glass rod. As a result excess negative charges and positive charges are developed on A and B as shown in fig (b).

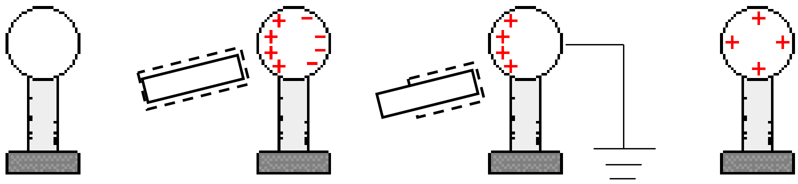
Step 3: Separate the spheres A and B by a small distance while the glass rod is held near 'A' as shown in fig (c).

Step 4: Remove the glassrod, the negative charges on A and positive charges on 'B' are uniformly distributed as shown in fig (d).

∴By induction method the two metal spheres acquired equal and opposite charges

Note: Charging by induction is suitable for conductors. In this method opposite charges are developed.

Note:
Describe how canyou charge a single metal sphere positively without touching it (i.e. by induction)



(a) (b) (c) (d)

The metallic sphere can be positively charged by following steps

Step 1: Consider a neutral metal sphere kept on insulating stand as shown in fig (a).
Step 2: Bring a negatively charged plastic rod near the sphere. Then free electrons move away from the rod due to repulsion. As a result excess positively and negatively charges are developed (induced) on the sphere as shown in fig (b)
Step 3:negatively charged part of a sphere is connected to earth withoutremoving plasticrod, then electrons flow into earth as shown in fig (c).
Step 4: Remove the plasticrod and earthing, now positive charges are distributed over the surface of sphere as shown in fig (d).
i.e. Metallic sphere is positively charged by induction method

Note:
1. Similar steps are involved on charging a metal sphere negatively by bringing a positively charged glass rod.
2. Why charged body attracts light objects?
   When a charged body is brought near the light objects like piece of paper, the charged body induces opposite charges on near surface of a light objects. As a result charged body attracts light objects.

What are Point charges?

Charges whose sizes are very small compared to the distance between them are called point charges.

Basic unit of charge or elementary charge or fundamental charge

The charge on an electron or proton is called basic unit of charge or elementary charge

It is denoted by the letter 'e'

The value of elementary charge is $e=1.602192 \times 10^{-19}C$

Note:

1) Charge on electron= $-e = -1.602 \times 10^{-19}C$.
2) Charge on proton = $+e = +1.602 \times 10^{-19}C$
   ∴Magnitude of charge on electron and proton is equal
3) Charge on neutron = 0
4) The least possible amount of charge that can exist independently in the nature is called elementary charge.

Basic properties of electric charge:

1. The electric charge is conserved.
2. The electric charge is quantized.
3. The charge is additive.
4. Like charges repel each other and unlike charges attract each other.
5. Charge is scalar, its SI unit is coulomb.

Explain Additivity of charge:

Charges can be added algebraically, i.e., if a system contains positive charges $+q_1, +q_2, +q_3$ and negative charges $-q_4, -q_5, -q_6$.

Then, total charges of a system or net charge $= q_1 + q_2 + q_3 - q_4 - q_5 - q_6$.

Ex: If a system contains the charges +4C, +2C, –3C and –1C

Then, net charge = 4C+2C–3C–1C=+2C

Explain Conservation of charge:

Conservation of charge states that "The total charge of an isolated system remains always constant", i.e. total charge of the isolated system can neither be created nor be destroyed but there is a transfer of electrons from one body to another body.Therefore charge is conserved.

Ex: In unstable nucleus neutron is converted into proton and electron.

i.e, neutron = proton + electron

A/C to law of conservation of charge,

Charge on neutron = charge on proton + charge on electron

$0 = +e - e$

$0 = 0$

Explain Quantization of charge:

The charge on a body is always an integral multiple of charge on electron.

- Quantization of charge means charge on a body can be increased or decreased in steps of 'e'

- Charge on a body is given by (Expression for quantization of charge)

  $q = n\,e$

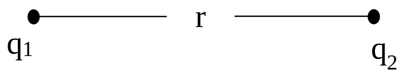
  Where, n= number of electrons added or removed from the body
  Where, q= charge on a body
  e=elementary charge

Note:

1. Quantization of charge was experimentally demonstrated by R.A Millikan through his oil-drop experiment.
2. Quantization of charge was first suggested by the experimental laws of electrolysis discovered by Faraday.
3. The force of attraction or repulsion between two charges at restis called electrostatic force.

State and explain coulomb's law:

It states that "the force between two point charges is directly proportional to the product of magnitudes of two chargesand inversely proportional to square of the distance between them. The force is acting along the line joining two charges."



Let, q_1 and q_2 = two point charges, r = distance between them,
F = force between q_1 and q_2

$$F \,\alpha\, q_1 q_2 \quad \text{and} \quad F \,\alpha\, \frac{1}{r^2}$$

$$F \,\alpha\, \frac{q_1 q_2}{r^2}$$

$$F = K\frac{q_1 q_2}{r^2}$$

Where, K = proportionality constant

but, $K = \dfrac{1}{4\pi\varepsilon_0}$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

This is the expression for electrostatic force or columb's law

Note 1: ε_0 is called permittivity of free space

S.I unit is $C^2\,N^{-1}\,m^{-2}$

$\varepsilon_0 = 8.854 \times 10^{-12} C^2\,N^{-1}\,m^{-2}$

Note2: To find the value of $\dfrac{1}{4\pi\varepsilon_0}$, $\dfrac{1}{4\pi\varepsilon_0} = \dfrac{1}{4\times3.14\times8.854\times10^{-12}}$,

$\dfrac{1}{4\pi\varepsilon_0} = 0.00899\times10^{12}$, $\dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$, $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2/c^2$

Note3: Coulomb used a Torsion balance for measuring the force between two charged metallic spheres.
Note4: Torsion balance is a sensitive device to measure a force.
Note5: Cavandis also used Torsion balance to measure the very feeble gravitational force.

Define one columb using coulomb's law:
According to coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$ if $q_1=q_2=1C$ and $r=1m$

Then $$F = \frac{1}{4\pi\varepsilon_0}\frac{1\times1}{1^2}$$

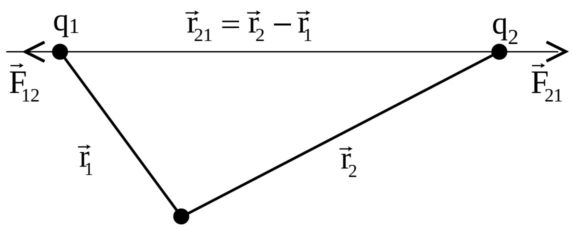
$$F = \frac{1}{4\pi\varepsilon_0}$$

$F=9\times10^9$ N

Definition: *One columb is the charge that when placed at a distance of 1m from another identical charge kept in vacuum experiences a force of repulsion of 9×10⁹N.*

Note:
Define one coulomb of charge in terms of current:
One coulomb is that charge flowing through a wire in one second if the current is one ampere.

Columb's law in vector form:



$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

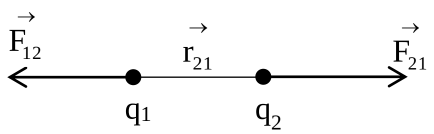
Where,
$\vec{F}_{21}$ = Force on q_2 by q_1
$\vec{r}_{21}$ = position vector.
$\hat{r}_{21}$ = unit vector which gives direction of force.

Note: 1) The force on q_2 and q_1 = force on q_1 by q_2

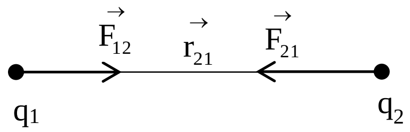
$$\vec{F}_{21} = -\vec{F}_{12}$$

Thus, Coulomb's law obeys Newton's III law.

Note: 2) Pictorial representation of force of repulsion between two like charges ($q_1 q_2>0$)



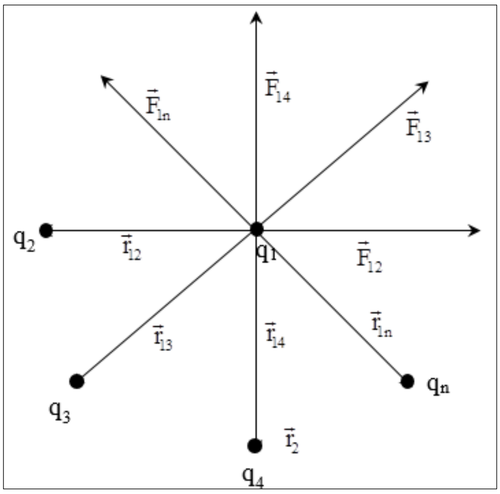
Note3) Pictorial representation of force of attraction between two unlike charges ($q_1 q_2 < 0$).



Principle of superposition of electrostatic forces :

It states that "Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to other charges taken one at a time. The individual forces are unaffected due to the presence of other charges.

Application of Principle of superposition to find the force between system of charges:



Consider a system of n stationary charges.

Let force on q_1 by q_2 is

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

Force on q_1 by q_3 is

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13}$$

Similarly, force on q_1 by q_n is

$$\vec{F}_{1n} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_n}{r_{1n}^2} \hat{r}_{1n}$$

According to principle of superposition, net force on q_1 is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \ldots\ldots\ldots \vec{F}_{1n}$$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} \ldots\ldots\ldots\ldots + \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_n}{r_{1n}^2} \hat{r}_{1n}$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0} \left[\frac{q_2}{r_{12}^2} \hat{r}_{12} + \frac{q_3}{r_{13}^2} \hat{r}_{13} \ldots\ldots\ldots\ldots + \frac{q_n}{r_{1n}^2} \hat{r}_{1n} \right]$$

$$\vec{F}_1 = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{1i}^2} \hat{r}_{1i}$$

Electric field or electric field intensity or strength:

Electric field at a point is the force per unit positive charge.

$$\text{i.e. Electric field} = \frac{\text{Force}}{\text{unit positive charge}}$$

$$E = \frac{F}{q_0}$$

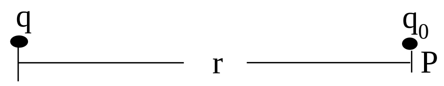
S.I. unit is newton/coulomb(N/C)or volt / meter (v/m),
Electric field is a vector quantity.

Source charge: The charge which produces the electric field is called source charge.

Test charge: The charge that detects the effect of source charge is called test charge.

Note: UPC=Unit positive charge. In SI, UPC = 1 coulomb.

Mention the expression for electric field due to a point charge:



Let, q=source charge, q_0=test charge (upc), P=any point

According to coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{qq_0}{r^2}$$

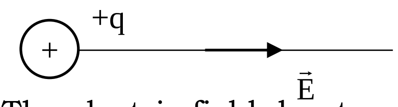
$$\frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$$

But $$\frac{F}{q_0} = E = \text{Elecric field}$$

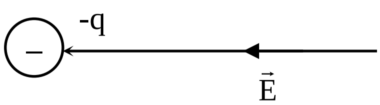
$$\therefore \quad \boxed{E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}}$$

This is the expression for electric field at a point p due to a charge.

Note:

1. Electric field in vector form: $\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}\hat{r}}$
2. 

The electric field due to positive charge is directed radially outward.

3. 

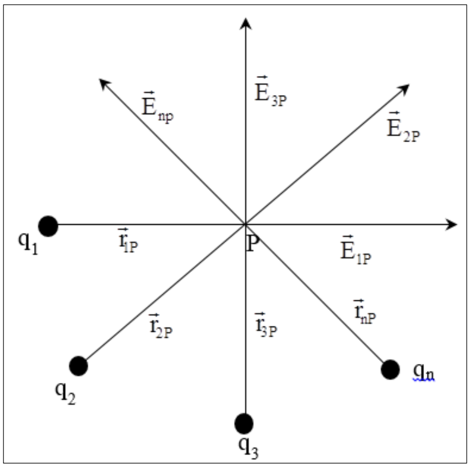
The electric field due to negative charge is directed radially inward.

Principle of superposition of electric field:

It's state that "The net electric field at a point is the vector sum of all the electric fields at that point due to number of charges."

Application of principle of superposition to find Electric field due to system of charges:

Consider a system of 'n' charges arranged as shown.



Electric field at a point P by q_1 charge is

$$\vec{E}_{1P} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1}{r_{1P}^2}\hat{r}_{1P}$$

Electric field at a point P by q_2 charge is

$$\vec{E}_{2P} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_2}{r_{2P}^2}\hat{r}_{2P}$$

Similarly

$$\vec{E}_{3P} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_3}{r_{3P}^2}\hat{r}_{3P}$$

$$\vec{E}_{nP} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_n}{r_{nP}^2}\hat{r}_{np}$$

According to principle of superposition net electric field at P

$$\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P} + \vec{E}_{3P} + \dots\dots\dots + \vec{E}_{nP}$$

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\varepsilon_0} \times \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \dots\dots\dots + \frac{1}{4\pi\varepsilon_0} \times \frac{q_n}{r_{nP}^2}\hat{r}_{nP}$$

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \dots\dots\dots + \frac{q_n}{r_{nP}^2}\hat{r}_{nP}\right]$$

$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

Physical significance of electric field:

Electric field is a vector quantity. Electric field was first introduced by Faraday. Electric field transport the energy. It is a characteristic of system of charges and it does not depend on test charge. It deals with theelectromagnetic phenomena.

Electricfield lines or Electric lines of force:

It is a curved path drawn such that tangent at any point on it gives the direction of net electric field at that point.

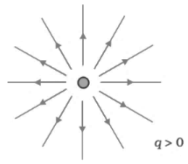
Or

It is an imaginary path along which unit positive charge moves.

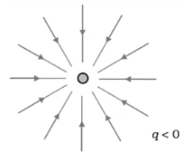
The concepts of electric field lines was developed by Faraday.

Note:

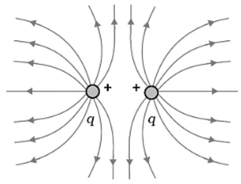
1. Electric field lines due to a single positive charge are directed radially outward.



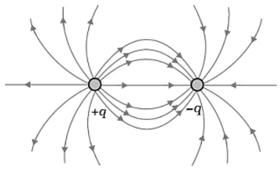
2. Electric field lines due to a single negative charge are directed radially onward.



3. The electric field lines around two opposite charges are as shown in figure.



4. The electric field lines around two same charges are as shown.



Properties of electric field lines:

1. The electric field lines start from the positive charge and ends at negative charge.
2. The electric field lines donot intersect each other.
3. The electric field lines donot form a closed loop.
4. The electric field lines are normal to the surface of charged conductor.

Note: If electric field lines intersect, the field at the point of intersection will not have a unique direction. That's why two electric field lines never intersect.

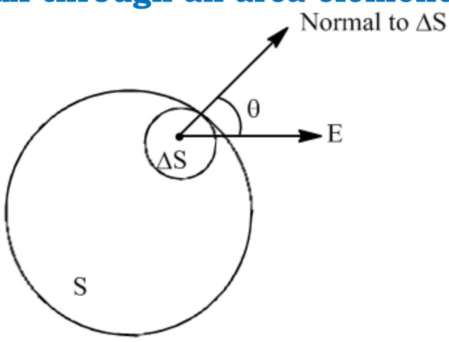
Electric flux:

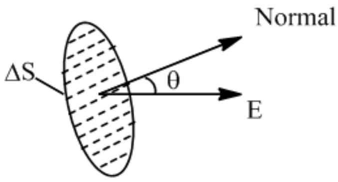
The total number of electric field lines passing through the surface area held perpendicular to the direction of field lines is called electric flux.

S.I unit is $NC^{-1}\ m^2$.

Electric flux is a scalar quantity.

Electric flux through an area element:





Consider a surface 'S'.

Let, ΔS = small area element of surface 'S'

E = Electric field

θ = Angle between 'E' and normal to ΔS

Δϕ = Electric flux through ΔS.

The projection of area 'ΔS' normal to E is given by ΔScosθ

The electric flux through the area element ΔS is given by

$$\Delta\phi = E\ \Delta S\ \cos\theta$$

$$\Delta\phi = \vec{E} \bullet \overrightarrow{\Delta S}$$

Therefore, electric flux through a surface is defined as the dot product of electric field and area element.

Note 1) Electric flux through small area ΔS is given by

$$\Delta\phi = E\Delta S\cos\theta$$

$$\Delta\phi = \vec{E} \bullet \overrightarrow{\Delta S}$$

Note 2) Total electric flux through entire surface is given

$$\phi = \sum \vec{E} \cdot \overrightarrow{\Delta S} \quad \text{or} \quad \phi = \oint \vec{E} \cdot \overrightarrow{\Delta S}$$

Note 3) We have Δϕ = E ΔS cosθ

$$\therefore E = \frac{\Delta\phi}{\Delta S\cos\theta}$$

i.e. Electric field is the electric flux per unit normal area.

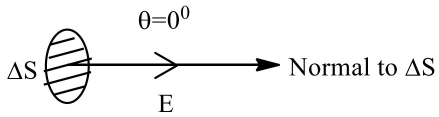
If $\theta = 0^0$, $\cos 0^0 = 1$, then $E = \frac{\Delta\phi}{\Delta S}$

Note 4) If $\theta = 0^0$, $\cos 0^0 = 1$

$$\Delta\phi = E\Delta S\cos 0^0$$

$$\Delta\phi = E\Delta S$$

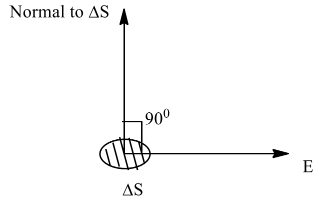
i.e. flux is maximum



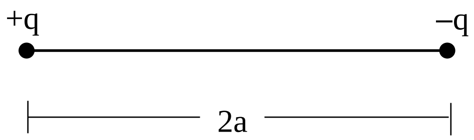
Note 5) If $\theta = 90^0$, $\cos 90^0 = 0$

Then, $\Delta\phi = E\Delta S \times 0 = 0$

i.e. flux is minimum.



Electric dipole:A set of two equal and opposite point charges separated by a small distance is called electric dipole.



+q and –q = point charges

2a = distance between +q and –q = dipole length

Note: Net charge of an electric dipole is zero

Electric dipole moment $(\vec{P})$:

It is the product of magnitude of one of the charges and the distance between two charges of dipole.

Dipolemoment = charge × distance

$$P = q \times 2a$$

S.I unit is coulomb × metre (Cm)

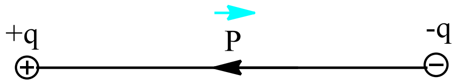
Dipole moment is a vector quantity

Note (1) Dipole moment in vector form

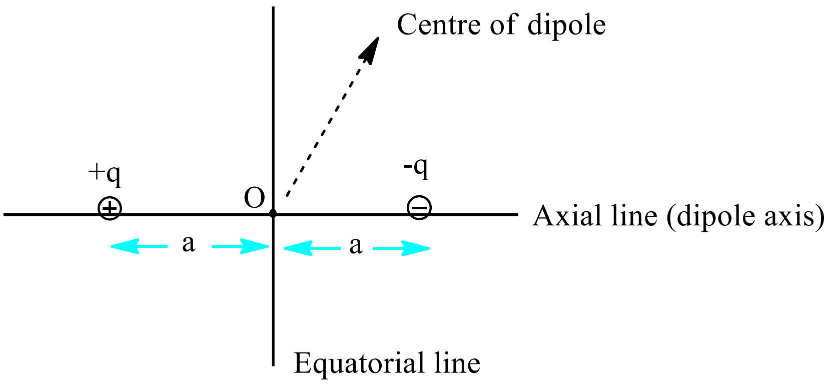
$$\vec{p} = q \times 2a\ \hat{P}$$

Where $\hat{P}$ = unit vector which gives direction of dipole moment.

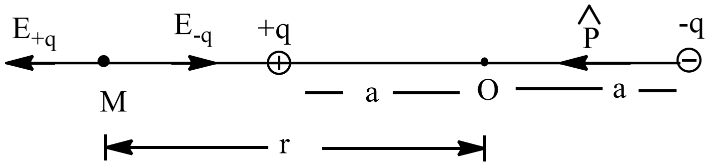
Note (2) The direction of dipole moment is directed from negative charge to positive charge along the dipole axis



Note (3)



Obtain an expression for the electric field at a point along the axis of dipole



Consider a point M on the axis of electric dipole.

Let O = mid point of dipole

+q and –q = two point charges, q = magnitude of point charge

2a = dipole length, r = distance between O and M

$\vec{P}$ = dipole moment acting from –q to +q, $\hat{P}$ = unit vector of $\vec{P}$

but P = q × 2a ------------> (1)

Electric field at M due to +q charge is

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r-a)^2}\hat{P} \qquad \text{away from } +q \qquad \text{------------- (2)}$$

Electric field at M due to –q charge is

$$\vec{E}_{-q} = -\frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r+a)^2}\hat{P} \qquad \text{towards } -q$$

(–ve sign indicates that $\vec{E}_{-q}$ is opposite to $\vec{P}$)

According to the principle of superposition the total electric field at M is

$$\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r-a)^2}\hat{P} - \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r+a)^2}\hat{P}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{P}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]\hat{P}$$

$$\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} = \frac{(r+a)^2-(r-a)^2}{(r-a)^2\,(r+a)^2}$$

$$= \frac{r^2+a^2+2ar-r^2-a^2+2ar}{(r-a)^2\,(r+a)^2}$$

$$\frac{4ar}{(r^2-a^2)^2}$$

If, $\quad r>>a$

then $(r^2-a^2)^2 = (r^2)^2 = r^4$

$$\therefore \vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{r^4}\right]\hat{P}$$

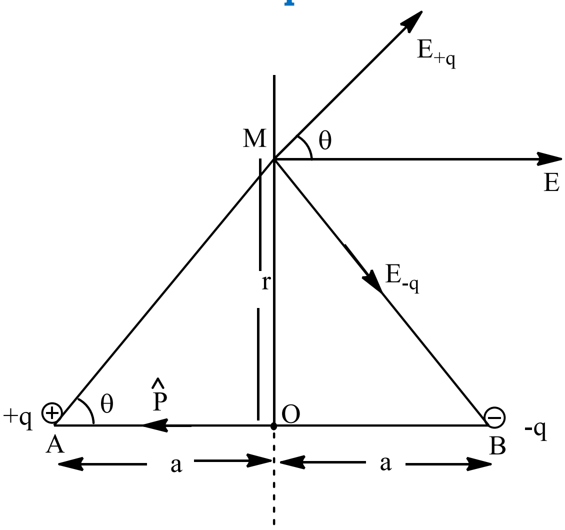
$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{4a}{r^3}\right)\hat{P}$$

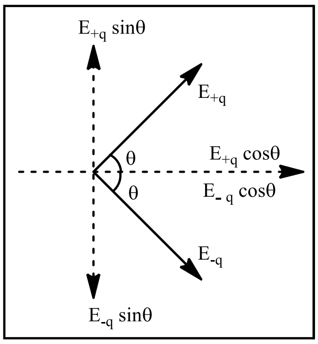
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\times 2a\times q}{r^3}\right]\hat{P}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2P}{r^3}\right]\hat{P}$$

This is the expression for electric field at a point on the axis of dipole.

Obtain an expression for the electric field at a point on the equatorial plane of electric dipole.





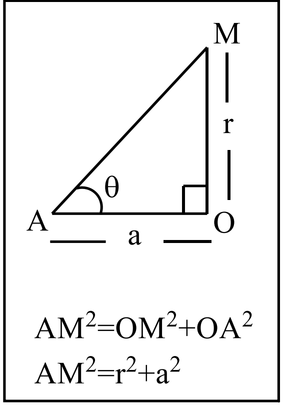
Consider a point M on the perpendicular bisector of the axis of dipole.

Let $\quad$ O = Midpoint of dipoles, +q and –q = two point charges

q = magnitude of charge, $\qquad$ 2a = dipole length

P = dipole moment acting from –q to +q,

$\hat{P}$ = unit vector



Magnitude of dipole moment is

$$P = q \times 2a \text{ ------ (1)}$$

From figure,

$AM^2 = r^2 + a^2$ and $BM^2 = r^2 + a^2$

$\therefore AM^2 = BM^2 = r^2 + a^2$

$$AM = BM = \left(r^2 + a^2\right)^{\frac{1}{2}}$$

Electric field at M due to +q charge is

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{AM^2}\, \hat{p}$$

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{\left(r^2 + a^2\right)}\hat{P} \quad \text{away from +q}$$

Electric field at M due to –q charge is

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{BM^2}\, \hat{p}$$

$$\vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{\left(r^2 + a^2\right)}\hat{P} \quad \text{towards –q}$$

Hence $\left|\vec{E}_{+q}\right| = \left|\vec{E}_{-q}\right|$ (magnitudes)

Resolving $\vec{E}_{+q}$ and $\vec{E}_{-q}$ in to two rectangular components each. Here, perpendicular components are equal and opposite. So they cancels each other. The components parallel to dipole axis are in same direction. So they add up

$\therefore$Total electric field at M

$$\vec{E} = -\left[\vec{E}_{+q}\cos\theta + \vec{E}_{-q}\cos\theta\right]$$

[–ve sign indicates that $\vec{E}$ is opposite to $\vec{P}$]

$$\vec{E} = -\left[\vec{E}_{+q} + \vec{E}_{-q}\right]\cos\theta$$

$$= -\left[\frac{1}{4\pi\varepsilon_0} \times \frac{q}{\left(r^2 + a^2\right)}\hat{P} + \frac{1}{4\pi\varepsilon_0} \times \frac{q}{\left(r^2 + a^2\right)}\hat{P}\right]\cos\theta$$

$$\vec{E} = -\left[\frac{2q}{4\pi\varepsilon_0\left(r^2 + a^2\right)}\cos\theta\right]\hat{P}$$

But from figure,

$$\cos\theta = \frac{AO}{AM} = \frac{a}{\left(r^2 + a^2\right)^{\frac{1}{2}}}$$

$$\therefore \vec{E} = -\left[\frac{2q}{4\pi\varepsilon_0\left(r^2 + a^2\right)} \times \frac{a}{\left(r^2 + a^2\right)^{\frac{1}{2}}}\right]\hat{P} \qquad \left(\because P = q \times 2a\right)$$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{P}{\left(r^2 + a^2\right)^{\frac{3}{2}}}\right]\hat{P}$$

If r>>a then $\left(r^2 + a^2\right)^{3/2} \approx \left(r^2\right)^{3/2} = r^3$

$$\therefore E = -\frac{1}{4\pi\varepsilon_0}\left[\frac{P}{r^3}\right]\hat{P}$$

This is the expression for electric field at a point on the equatorial plane.

Note (1) Electric field at a point on the dipole axis.

$$\vec{E}_{dipoleaxies} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2P}{r^3}\right)\hat{P}$$

It is acting along dipole moment

Note (2) Electric field at a point on the equatorial plane

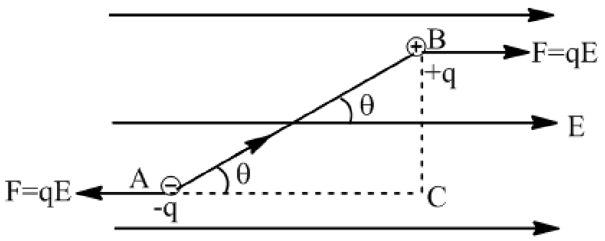
$$\vec{E}_{equatorial} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{P}{r^3}\right)\hat{P}$$

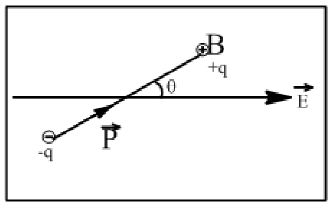
–ve sign indicates that net electric field is opposite to dipole moment

Note (3): From note (1) and (2). It is clear that, $E_{dipole\ axis} = 2\ E_{quatorial}$ (magnitude wise)

Note (4) : Force acting on positive charge is along the electric field direction. The force acting on negative charge is opposite to the field direction.

Obtain the expression for the torque acting on an electric dipole placed in a uniform electric field:





Consider an electric dipole placed in a uniform electric field 'E'.

Let +q and –q = two charges of dipole

AB = 2a = dipole length

θ = angle between $\vec{E}$ and dipole moment $\vec{P}$.

The fore acting on +q is

$F_{+q} = qE$ along $\vec{E}$

The force acting on –q is

$F_{-q} = qE$ opposite to $\vec{E}$

i.e. F_{+q} and F_{-q} are two equal and opposite force acting on the dipole. As a result, torque is acting on the dipole

but, torque = force × perpendicular distance between two forces.

$\tau = qE \times BC$ ----- (1)

From right angled triangle BCA

$$\sin\theta = \frac{BC}{AB} = \frac{BC}{2a}$$

$\therefore$ BC = 2a sinθ

$\therefore$ equation (1) becomes

$$\tau = q \times 2a \times E \times \sin\theta$$

$$\tau = P\ E \sin\theta \quad (\because q \times 2a = P)$$

This is the expression for torque acting on the dipole

Note-1: We have τ= P E sinθ

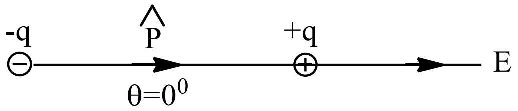
$$\vec{\tau} = \vec{P} \times \vec{E} \longrightarrow \text{Vector form}$$

i.e. Torque is the cross product of dipole moment and electric field, Torque is a vector.

S.I unit of torque is Nm (newtonmetre)

Note-2: If dipole is along the electric field

Then $\theta = 0^0$ Or $\theta = 180^0$



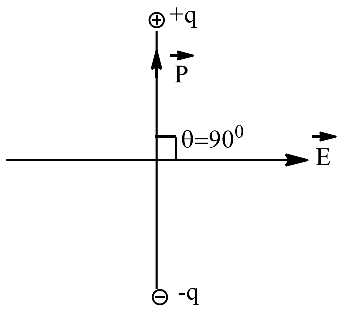
Then $\sin\theta = \sin 0^0 = \sin 180^0 = 0$

$\therefore \tau$ = PE × sinθ

τ= PE × 0

τ= 0 i.e no torque is acted on dipole (Torque is minimum)

Note-3:



If dipole is perpendicular to Electric field. Then, $\theta=90^0$.

Then $\sin\theta = \sin 90^0 = 1$

$\therefore \tau$ = P E sin θ

τ = PE

Torque is maximum when the dipole is placed perpendicular to the direction of field.

Linear charge density (λ):

It is defined as the charge per unit length.

i.e Linear charge density = $\frac{\text{charge}}{\text{length}}$

$$\lambda = \frac{\Delta q}{\Delta L}$$

S.I unit is coulomb/metre (c/m)

Surface charge density (σ):

It is defined as the charge per unit area

i.e. surface charge density = $\frac{\text{charge}}{\text{area}}$

$$\sigma = \frac{\Delta q}{\Delta S}$$

S.I unit is C/m^2

Volume charge density (ρ):

It is defined as the charge per unit volume

i.e. volume charge density = $\frac{\text{charge}}{\text{Volume}}$

$$\rho = \frac{\Delta q}{\Delta V}$$

S.I unit is C/m^3

Mention the expression for electric field due to continuous charge distribution

Electric field due to Δq charge present in small volume element ΔV is

$$\Delta E = \frac{1}{4\pi\varepsilon_0} \times \frac{\Delta q}{r^2} \longrightarrow (1)$$

But $\rho = \frac{\Delta q}{\Delta V}$ ρ = volume charge density

$$\therefore \Delta q = \rho \Delta V$$

- equation (1) become

$$\Delta E = \frac{1}{4\pi\varepsilon_0} \times \frac{\rho \Delta V}{r^2}$$

Electric field due to entire volume V is

$$E = \sum_{\text{all } \Delta V} \Delta E$$

$$E = \frac{1}{4\pi\varepsilon_0} \sum_{\text{all } \Delta V} \frac{\rho \Delta V}{r^2}$$

This is the expression for electric field due to continuous charge distribution.

State Gauss law:

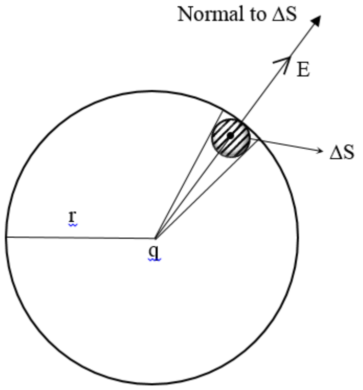
It states that "Total electric flux flow through the closed surface is equal to $\frac{q}{\varepsilon_0}$.

That is, Total electric flux $= \frac{q}{\varepsilon_0}$

Where, q=total charge closed by the surface
ε_0= permittivity of free space.

Deduce Gauss law:

Consider a sphere of radius ‘r’, divide the sphere into small area element as shown in figure.



Let, q = charge enclosed by a sphere,

r = radius of sphere.

Δs = small area element

Electric field at the surface is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Small electric flux flows through ΔS is

$$\Delta\phi = E\Delta S\cos\theta, \quad \text{but } \theta = 0°, \cos 0° = 1$$

$$\therefore \Delta\phi = E\Delta S$$

$$\Delta\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\Delta S$$

∴ Total electric flux through entire sphere.

$$\phi = \sum\Delta\phi$$

$$\phi = \sum\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}\Delta S$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}\sum\Delta S$$

But $\sum\Delta S = 4\pi r^2 =$ area of sphere

$$\therefore \quad \phi = \frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2}\times 4\pi r^2$$

$$\phi = \frac{q}{\varepsilon_0}$$

This is called Gauss law.

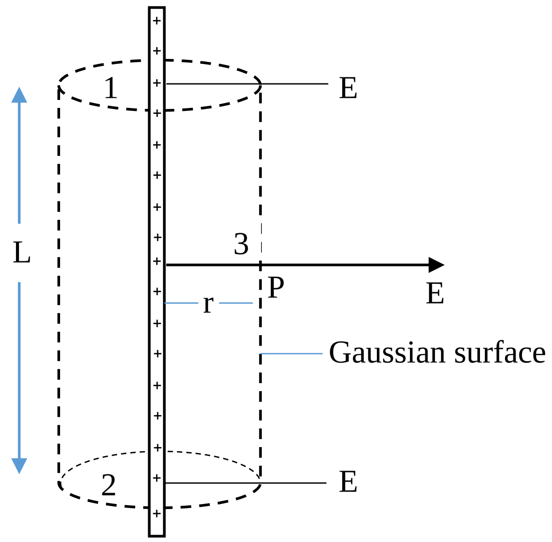
Significance of Gauss law:

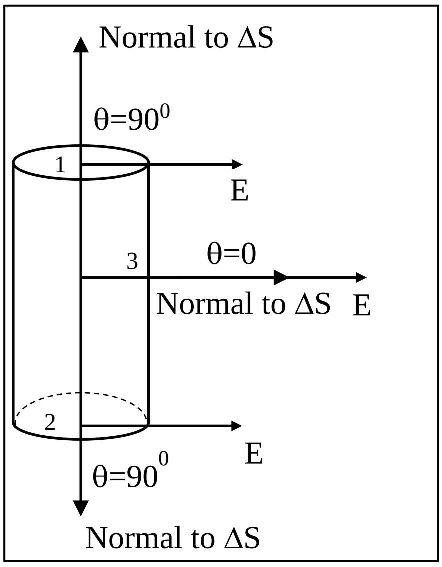
1. Gauss law is true for any closed surface, no matter what its shape or size.
2. According to Gauss law, $\phi = \frac{q}{\varepsilon_0}$ here q is the sum of all the charges enclosed by the surface.
3. The surface that we choose for the application of gauss law is called Gaussian surface.
4. Gauss law is based on inverse square dependence on distance contained in coulombs law.

Applications of Gauss law: Using Gauss law

1. Electric field due to infinite long straight uniformly charged wire can be found.
2. Electric field due to uniformly charged infinite sheet can be found.
3. Electric field due to a charged thin spherical shell can be found.

Derive an expression for electric field due to infinitely long straight uniformly charged wire using Gauss law:





Consider an infinitely long straight charged wire.

Let λ = Linear charge density of wire, E=Electric field at a point 'P'

r = Distance between wire and P.

Draw a cylindrical Gaussian surface of radius 'r' passing through point 'P' a shown in figure.

The electric field is radially outward. i.e. perpendicular to wire.

Electric flux through 1st circular end of a cylinder (Gaussian surface) is

$\Delta\phi_1 = E.\Delta S \cos\theta$

But $\theta = 90^0$ $\cos 90^0 = 0$

$\Delta\phi_1 = 0$

Similarly electric flux through 2nd circular end of cylinder (G.S.) is

$\Delta\phi_2 = 0$

Electric flux through curved surface of cylinder (Gaussian surface) is

$\Delta\phi_3 = E\Delta S \cos\theta$

$\theta = 0^0$, $\cos 0^0 = 1$

$\Delta\phi_3 = E\Delta S$

But, $\Delta S = 2\pi rL$ = Total surface area of cylinder, Where, L=length of cylinder.

$\Delta\phi_3 = E \times 2\pi rL$

Total electric flux is

$\phi = \Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3$

$\phi = 0 + 0 + E \times 2\pi rL$

$\phi = E \times 2\pi rL$ -------------- (1)

Let, q= charge enclosed by cylinder (Gaussian surface)

But $\lambda = \frac{\text{Charge}}{\text{Length}}$

$\lambda = \frac{q}{L}$

$q = \lambda L$

According to Gauss law

$\phi = \frac{q}{\varepsilon_0}$

$\phi = \frac{\lambda L}{\varepsilon_0}$ --------------- (2)

From equation (1) and (2)

$E \times 2\pi rL = \frac{\lambda L}{\varepsilon_0}$

$E = \frac{\lambda}{2\pi\varepsilon_0 r}$

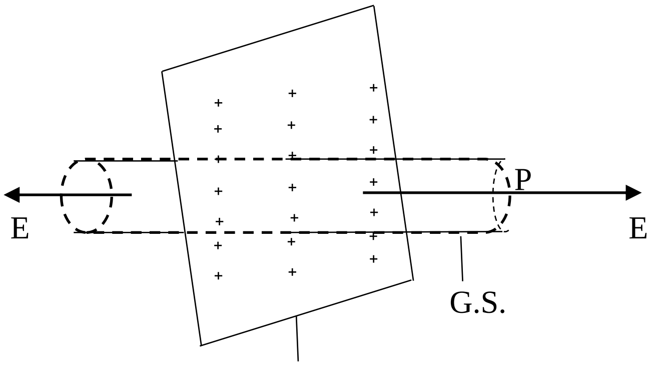
This is the expression for electric field due to a charged long wire.

Note: 1) In vector form, $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$,

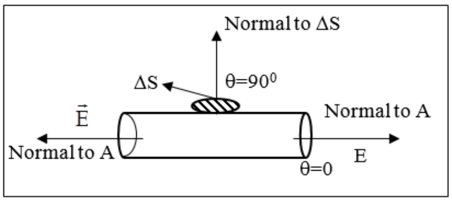
$\hat{r}$ = unit vector which gives directions of $\vec{E}$ and it is radially directed.

Note: 2) Electric field due to a wire is radially outward if charge is +ve (or λ is +ve). Electric field due to a wire is radially inward if charge is negative (or λ is negative):

Derive an expression for electric field due to a uniformly charged infinite plane sheet:



Plane sheet



Consider a uniformly charged infinite plane sheet.

Let, σ= surface charge density of plane sheet

E=Electric field at a point P due to plane sheet, it is directed outward.

Draw a cylindrical Gaussian surface, perpendicular to plane sheet passing through a point 'P' as shown in figure.

Let, A=cross sectional area of cylinder.

Electric flux through 1st circular end of cylinder (Gaussian surface) is

$\Delta\phi_1 = E\Delta S\cos\theta$

But, $\Delta S = A$, $\theta = 0^0$, $\cos 0^0 = 1$

$\therefore \quad \Delta\phi_1 = EA$

Similarly, electric flux through 2nd circular end of cylinder is

$\Delta\phi_2 = EA$

Electric flux through curved surface of cylinder is

$\Delta\phi_3 = E\Delta S\cos\theta$

$\theta = 90^0$, $\cos 90^0 = 0$

$\therefore \quad \Delta\phi_3 = E\Delta S \times 0$

$\Delta\phi_3 = 0$

$\therefore$ Total electric flux through cylinder (Gaussian surface) is

$\phi = \Delta\phi_1 + \Delta\phi_2 + \Delta\phi_3$

$\phi = EA + EA + 0$

$\phi = 2EA$ -------------- (1)

Charge enclosed by cylinder = q

But $\sigma = \dfrac{\text{Charge}}{\text{Area}}$

$\sigma = \dfrac{q}{A} \qquad \therefore \qquad q = \sigma A$

According to Gauss law

$$\phi = \frac{q}{\varepsilon_0}$$

$$\phi = \frac{\sigma A}{\varepsilon_0} \text{ ------------- (2)}$$

From equations (1) and (2)

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

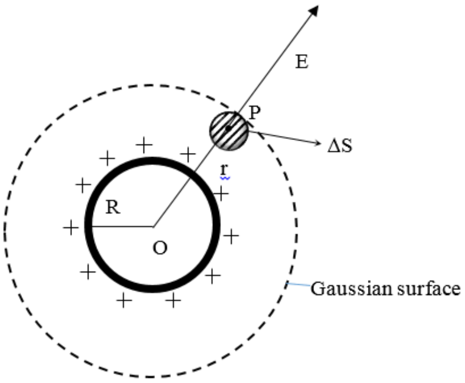
This is the expression for electric field due to plane sheet.

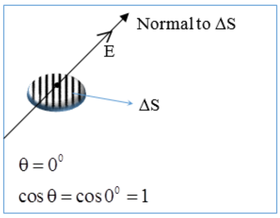
Note:

1. In vector form $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$. Where $\hat{n}$ =unit vector.
2. E is directed outward if σ is +ve (i.e. q is +ve) and E is directed inward if σ is negative (i.e. q is negative).
3. E is independent of distance from plane sheet.

Derive an expression for electric field due to a charged thin spherical shell using Gauss law:

Case 1: Electric field outside the shell:





Consider a charged thin spherical shell.

Let, R= radius of spherical shell, q=charge on spherical shell

E= electric field at a point 'P', σ=surface charge density

Area of spherical shell = $4\pi R^2$

$$\therefore \quad \sigma = \frac{\text{charge}}{\text{area}}$$

$$\sigma = \frac{q}{4\pi R^2}$$

$q=\sigma\times 4\pi R^2$----------- (1)

Consider a point P out side the shell. Imagine that a spherical Gaussian surface around the shell passing through point 'P' with the centre 'O' as shown in figure.

∴ r=radius of Gaussian surface

Let ΔS=small area element around point 'P'

∴ The electric flux through ΔS is

$\Delta\phi = E\Delta S \cos\theta$

But $\theta=0, \cos 0=1$

$\Delta\phi = E\Delta S$

Total electric flux through entire Gaussian surface is

$$\phi = \sum \Delta\phi$$

$$\phi = \sum E\Delta S$$

$$\phi = E\sum \Delta S$$ (Electric field E is same at all point on Gaussian surface)

But $\sum \Delta S = 4\pi r^2$ = area of Gaussian surface.

$$\phi = E \times 4\pi r^2 \text{ -------------- (2)}$$

According to Gauss law

$$\phi = \frac{q}{\varepsilon_0}$$ [from equation (1), $q=\sigma\times 4\pi R^2$]

$$\phi = \frac{\sigma 4\pi R^2}{\varepsilon_0} \text{ ----------- (3)}$$

From equations (1) and (2),

$$E \times 4\pi r^2 = \frac{\sigma 4\pi R^2}{\varepsilon_0}$$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

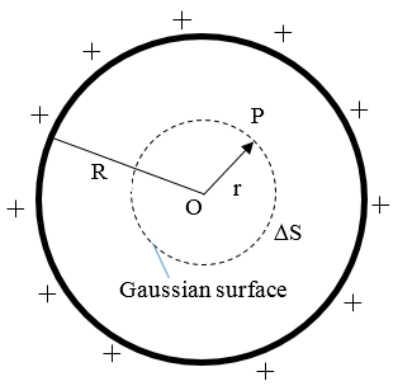
This is the expression for electric field at a point 'P'

Case 2:Electric field inside the shell:

Consider a point P inside the spherical shell. Consider a Gaussian surface passing through point 'P' as shown in figure.

Electric flux through the Gaussian surface is

$$\phi = E \times 4\pi r^2 \quad \text{(Refer equation 2)}$$



According to Gauss law, $\phi = \frac{q}{\varepsilon_0}$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Charge enclosed by Gaussian surface. i.e. q=0.

(No charge is present inside the shell)

$$\therefore E=0$$

i.e. Electric field due to a uniformly charged thin spherical shell is zero at all point inside the shell.

Note: $E = \frac{\sigma \times R^2}{\varepsilon_0 r^2}$

But $\sigma = \frac{q}{4\pi R^2}$

$$E = \frac{\frac{q}{4\pi R^2} \times R^2}{\varepsilon_0 r^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

This is the expression for electric field at a point out side the charged spherical shell.

One mark questions

1. What is an electric charge?
2. What are point charges?
3. Define polarity of charge.
4. What is gold leaf electro scope?
5. What is meant by earthing?
6. What is significance of earthing?
7. Define elementary charge.
8. What is meant by additivity of charge?
9. The net charge of a system of point charges -4, +3, -1 & +4 (S.I.units) =?
10. What is meant by conservation of charge?
11. What is quantisation of charge?
12. Mention the S.I. unit of charge. (March-2014)
13. State coulomb's law (March-2017)
14. Define one coulomb of charge. (March-2015)
15. State the principal of super position of electro statics.
16. Which principle is employed in finding the force between multiple charges?
17. Define electric field.
18. Is electric field a scalar/vector?
19. Mention the S.I. unit of electric field.
20. What is the direction of electric field due to a point positive charge
21. What is the direction of electric field due to a point negative charge?
22. What is a source charge?
23. What is a test charge?
24. How do you pictorially map the electric field around a configuration of charges?
25. What is an electric field line?
26. What is electric flux?
27. Mention the S.I.unit of electric flux.
28. What is an electric dipole?
29. What is the net charge of an electric dipole? Define dipole moment.
30. Is dipole moment a vector / scalar?

31. What is the direction of dipole moment?
32. What is the net force on an electric dipole placed in a uniform electric field?
33. When is the torque acting on an electric dipole placed in a uniform electric field maximum?
34. When is the torque acting on an electric dipole placed in a uniform electric field minimum?
35. State Gauss's law.
36. What is a Gaussian surface?
37. What happens to the force between two point charges if the distance between them is doubled?
38. If two charges kept in 'air' at a certain separation, are now kept at the same separation in 'water' of dielectric constant 80, then what happens to the force between them?
39. On a macroscopic scale is charge discrete or continuous?

Two mark questions

1. Explain construction of gold leaf electroscope.
2. Earthing is necessary for house, Why?
3. Write the expression for quantisation of charge and explain the terms in it.
4. State and explain Coulomb's law of electrostatics. (March-2014, July-2015, March-2017)
5. Write Coulomb's law in vector notation and explain the terms. (March-2015)
6. Write the pictorial representations of the force of repulsion and attraction, between two point charges.
7. Explain the principle of superposition to calculate the force between multiple charges.
8. Mention the expression for the electric field due to a point charge placed in vacuum.
9. Write the expression for the electric field due to a system of charges and explain it.
10. Draw electric field lines in case of a positive point charge.
11. Sketch electric field lines in case of a negative point charge.
12. Sketch the electric field lines in case of an electric dipole.
13. Sketch the electric field lines in case of two equal positive point charges.
14. Mention any two properties of electric field lines. (July-2015, March-2015, March-2017)
15. Write the expression for the torque acting on an electric dipole placed in a uniform electric field and explain the terms in it.
16. Define linear density of charge and mention its SI unit.
17. Define surface density of charge and mention its SI unit.
18. Define volume density of charge and mention its SI unit.
19. What is the effect of a non-uniform electric field on an electric dipole?

Three mark questions

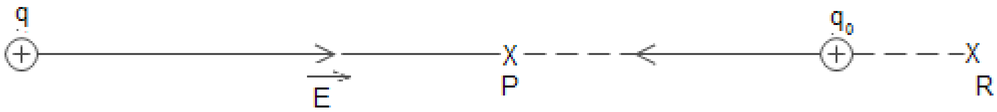
1. Mention three properties of electric charge. (July-2014)
2. Draw a diagram to show the resultant force on a charge in a system of three charges.
3. Why is the electric field inside a uniformly charged spherical shell, zero? Explain.
4. Obtain the expression for the torque acting on an electric dipole placed in a uniform electric field.

Five mark questions

1. Obtain an expression for the electric field at a point along the axis of an electric dipole.(Mar-16)
2. Obtain an expression for the electric field at a point on the equatorial plane of an electric dipole.(March-2015)
3. State Gauss's law. Obtain an expression for the electric field due to an infinitely long straight uniformly charged conductor. (July-2015)
4. State Gauss's law.Obtain an expression for the electric field due to a uniformly charged infinite plane sheet.
5. State Gauss's law.Obtain an expression for the electric field at an outside point due to a uniformly charged thin spherical shell. (March-2014, July-2014).

Chapter 2: ELECTROSTATIC POTENTIAL AND CAPACITANCE

Electrostatic potential energy:



Consider a source charge ‘q’,

Let E=electric field due to a charge q (q>0 i.e., q is the +ve charge)

q_0 =test charge ($q_0 > 0$ i.e., q_0 is +ve charge)

When we bring a test charge q_0 from a point R to P, then q charge produces repulsive force on ‘q_0’

∴ An external force is applied on charge ‘q_0’ to bring it from R to P.

i.e., external force = - electric force

i.e., $F_{ext} = -F_{ele}$

∴Net force on test charge q_0 =0, As a result the test charge moves with constant speed and with zero acceleration. The work done by the external force in moving a charge ‘q_0’ from R to P is

$$W_{RP} = \int_R^P \vec{F}_{ext}.\vec{dr}$$

Where $\vec{dr}$ =displacement

$$W_{RP} = -\int_R^P \vec{F}_{ele}.\vec{dr}$$

Let dr = small displacement of q_0 dw= small workdone ∴ $dw = F_{ext}.dr$, Total workdone= W= $\int dw = \int \vec{F}_{ext}.\vec{dr}$

-ve sign indicates that work done is against electrostatic repulsive force. This work done is stored as a potential energy.

At every point in the electrostatic field the charge ‘q’ possesses certain potential energy.

If R is at ∞, then $W_{\infty P} = -\int_\infty^P \vec{F}_{ele}.\vec{dr}$.

But, workdone= potential energy

$$W_{\infty P} = U_{\infty P}$$

$$U_{\infty P} = -\int_\infty^P \vec{F}_{ele}.\vec{dr}$$

∴ Potential energy of a charge (q_0) at a point is the work done by the external force to bring that charge from infinity to that point against the electric field.

Note:

1. Electrostatic potential energy is a scalar. S.I unit is joule.
2. Electrostatic potential energy is zero at infinity.

3. Coulomb force between two charges is a conservative force. Because the work done by an electrostatic force in moving a charge from one point to another depends only on the initial and final points and is independent of the path taken to go from one point to the other.

Electrostatic potential:

Electric potential at a point is the work done by the external force to bring a unit positive charge from infinity to that point against the electric field.

i.e., Electric potential $= \frac{\text{work done}}{\text{charge}}$

$$V = \frac{W}{q_0}$$

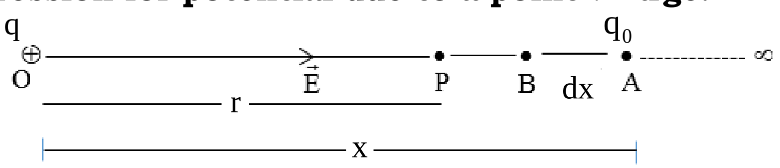
S.I unit is J/C or volt, potential is a scalar.

Note: 1Volt = 1 Joul / Coulomb

Potential difference: Potential difference between the two points is the work done by an external force to bring the unit positive charge from one point to another point against the electric field.
S.I. unit is J/C or volt.

Note: Potential goes on decreases along the direction of electric field.

Derive an expression for potential due to a point charge.



Let q = point charge at origin 'O'
V= Electric potential at point 'P'
A and B = any two points,
dx= distance between A and B
q_0 = unit positive charge (UPC) =+1C
E = electric field
r = distance between 'O' and 'P',
x=distance between O & A

According to coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{qq_0}{x^2}$$

$$\therefore F = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{x^2}$$ but q_0 =1 unit,

Small work done in bringing a unit positive charge 'q_0' from A to B is

$$dw = - F \times dx$$

-ve sign indicates that force is against field

$$\therefore dw = -\frac{1}{4\pi\varepsilon_0} \times \frac{q}{x^2} dx$$

To get total work done, integrating from x=∞ to x=r.

$$W = \int_{\infty}^{r} dw$$

$$= \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0} \times \frac{q}{x^2} dx$$

$$W = \frac{1}{4\pi\varepsilon_0} x \frac{q}{r}$$

But, $V = \frac{W}{q_0}$ but $q_0 = 1 = \text{upc}$

$$\therefore V = W$$

i.e., $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

$$W = \int_{\infty}^{r} dw = \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0} \times \frac{q}{x^2} dx$$

$$W = -\frac{q}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{1}{x^2} dx$$

$$= -\frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{x} \right]_{\infty}^{r}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

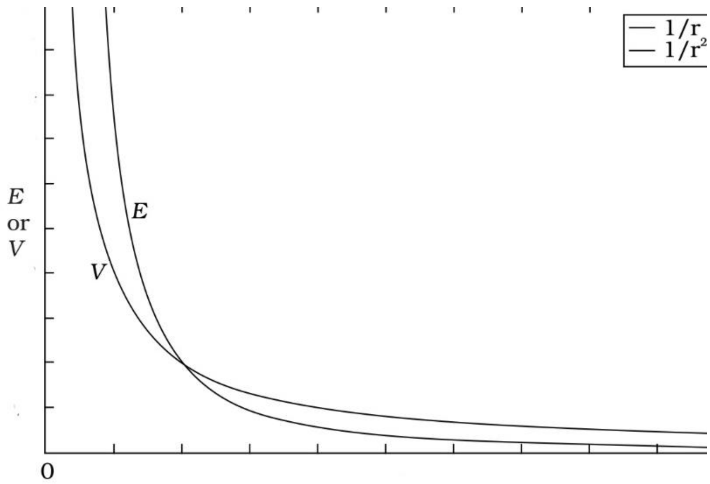
$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$= \frac{q}{4\pi\varepsilon_0} \times \left[\frac{1}{r} \right]$$

$$W = \frac{1}{4\pi\varepsilon_0} x \frac{q}{r}$$

This is the expression for electric potential.

Compare the variation of electric potential and electric field with distance:



The graph shows the variation of potential V with r and field E with r for a point charge q. They vary as $v\alpha \frac{1}{r}$ and $E\alpha \frac{1}{r^2}$.

Note 1:

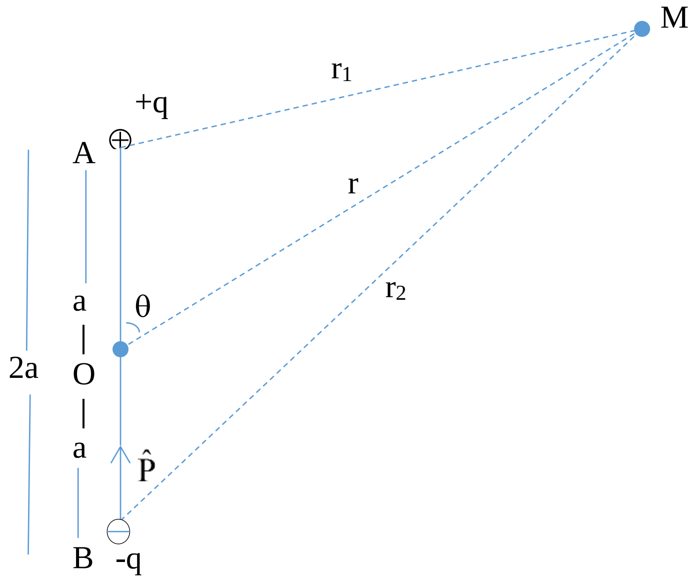
If $q>0$ (i e q is + ve) then $V>0$ (i e V is + ve)

if $q<0$ (i e q is -ve) then $V<0$ (i e V is -ve)

Note 2:

if $r=\infty$, then $V=0$ i. e., potential is zero at infinity.

Derive an expression for potential due to an electric dipole.



Consider a diploe of length 2a,

Let, O = centre of dipole,

M = any point at a distance r from O

r_1= distance between +q and M

r_2 = distance between –q and M

$\hat{p}$ = unit vector which gives the direction of dipole moment

θ=angle between $\hat{p}$ and OM .

potential due to +q at M is, $V_1 = +\frac{1}{4\pi\varepsilon_o} \times \frac{q}{r_1}$

potential due to -q at M is, $V_2 = -\frac{1}{4\pi\varepsilon_o} \times \frac{q}{r_2}$

Potential due to dipole at M is the sum of potentials due to charges +q and –q

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r_1} - \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r_2}$$

$$V = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] - - - - - - - (1)$$

From triangle AOM and BOM,

$r_1^2 = r^2 + a^2 - 2ar\cos\theta$ and $r_2^2 = r^2 + a^2 + 2ar\cos\theta$

$$= r^2\left[1+\frac{a^2}{r^2}-\frac{2a}{r}\cos\theta\right]$$

if $r >> a$, $\frac{a^2}{r^2}$ can be neglected

$$\therefore r_1^2 = r^2\left[1-\frac{2a\cos\theta}{r}\right]$$

$$\therefore r_1 = r\left[1-\frac{2a\cos\theta}{r}\right]^{\frac{1}{2}}$$

$$\frac{1}{r_1}=\frac{1}{r}\times\frac{1}{\left[1-\frac{2a\cos\theta}{r}\right]^{\frac{1}{2}}}$$

$$\frac{1}{r_1}=\frac{1}{r}\times\left[1-\frac{2a\cos\theta}{r}\right]^{-\frac{1}{2}}$$

$$=\frac{1}{r}\left[1+\frac{1}{2}\times\frac{2a\cos\theta}{r}\right] \quad (\because \text{using binomial theorem})$$

$$\frac{1}{r_1}=\frac{1}{r}+\frac{a\cos\theta}{r^2}----(2)$$

similarly $$\frac{1}{r_2}=\frac{1}{r}-\frac{a\cos\theta}{r^2}-----(3)$$

substituting (2) & (3) in eql (1)

$$V=\frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}+\frac{a\cos\theta}{r^2}-\left(\frac{1}{r}-\frac{a\cos\theta}{r^2}\right)\right]$$

$$=\frac{q}{4\pi\varepsilon_o}\left[\frac{1}{r}+\frac{a\cos\theta}{r^2}-\frac{1}{r}+\frac{a\cos\theta}{r^2}-\right]$$

$$V=\frac{q}{4\pi\varepsilon_o}\times\left(\frac{2a\cos\theta}{r^2}\right)$$

$$=\frac{1}{4\pi\varepsilon_o}\times\frac{q\times 2a\cos\theta}{r^2}$$

$$V=\frac{1}{4\pi\varepsilon_o}\times\frac{P\cos\theta}{r^2} \quad (\because p=q\times 2a)$$

This is the expression for potential due to dipole.

Note (1):

If $\theta = 0^0$or 180^0 Then $\cos 0^0 = +1$ or $\cos 180^0 = -1$

$$\therefore V=\pm\frac{1}{4\pi\varepsilon_o}\times\frac{P}{r^2}$$

This is the expression for potential at a point on the dipole axis.

Note (2):

If $\theta = 90^0$ then $\cos 90^0 = 0$

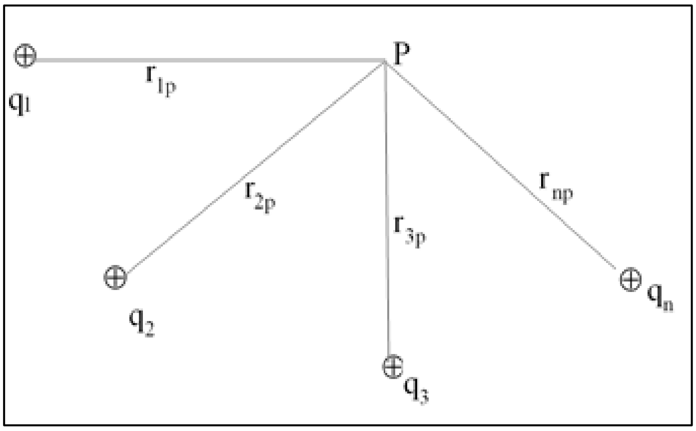
$\therefore$ V=0 i. e., potential at any point on equatorial line is zero.

Contrasting features of electrical potential of a dipole from that due to a single point charge:

1. The potential due to dipole depends on both distance and angle between position vector and direction of dipole moment but the potential due to single point charge depends only on distance.
2. $V \propto \frac{1}{r^2}$ for dipole, $V \propto \frac{1}{r}$ for single point charge.

To find potential due to a system of charges using super position principle:

Consider a system of charges q_1, q_2, q_3, ------------q_n with position vectors $\vec{r}_{1P}, \vec{r}_{2P}, \vec{r}_{3p}, -----\vec{r}_{np}$



The potential at 'p' due to q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1}{r_{1p}}$$

The potential at p due to q_2 is

$$V_2 = \frac{1}{4\pi\varepsilon_o} \times \frac{q_2}{r_{2p}}$$

Similarly $V_3 = \frac{1}{4\pi\varepsilon_o} \times \frac{q_3}{r_{3p}}$

And $V_n = \frac{1}{4\pi\varepsilon_o} \times \frac{q_n}{r_{np}}$

According to super position principle net potential at 'P' is

$$V = V_1 + V_2 + V_3 + ------- V_n$$

$$= \frac{1}{4\pi\varepsilon_o} \times \frac{q_1}{r_{1p}} + \frac{1}{4\pi\varepsilon_o} \times \frac{q_2}{r_{2p}} + \frac{1}{4\pi\varepsilon_o} \times \frac{q_3}{r_{3p}} + ------ \frac{1}{4\pi\varepsilon_o} \times \frac{q_n}{r_{np}}$$

$$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \frac{q_3}{r_{3p}} + ------ + \frac{q_n}{r_{np}}\right]$$

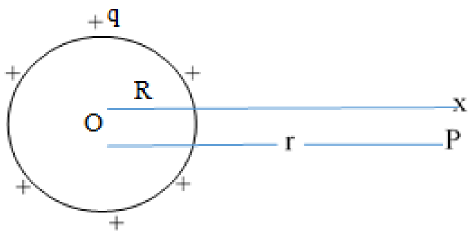
Note(1): Potential due to a charged spherical shell at a point outside the shell is

$$V = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{r} \qquad (r > R)$$

Note(2): Potential at the surface of spherical shell

i. e., r= R

$$V = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{R} \qquad r = \text{radius of shell}$$



Note(3) Potential inside the shells is equal to potential at the surface. Where as electric field inside the shell is zero.

Equipotential surface

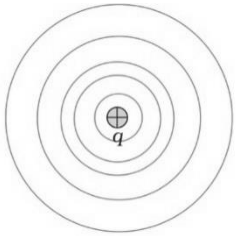
The surface with a constant value of potential at all points on the surface is called equipotential surface.

Properties of equipotential surface:

1. Potential difference between two points on the equipotential surface is zero. Therefore no work is required to move a test charge on the equipotential surface.
2. Electric field is normal to the equipotential surface.
3. Two equipotential surfaces never intersect each other.

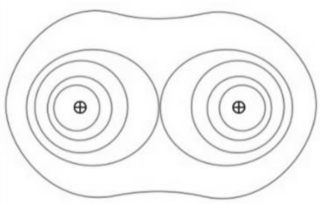
Shape of equipotential surfaces:

Ex1: equipotential surface for single charge is as shown.

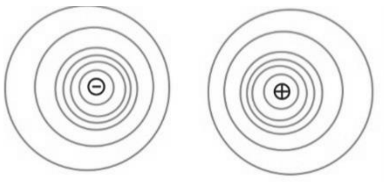


The equipotential surfaces are concentric spherical surfaces.

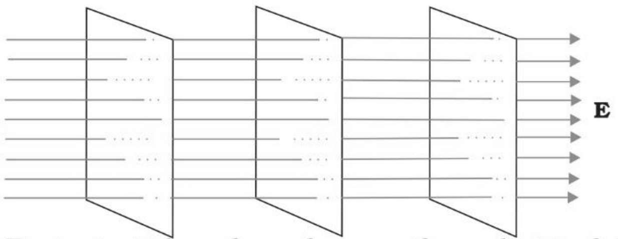
Ex2: for dipole



Ex3: for two identical +ve charges



Ex4: equipotential surface for a uniform electric field.



Equipotential surfaces for a uniform electric field

i.e., equipotential surfaces are planes.

Derive the relation between field and potential

Let: A and B = equipotential surfaces

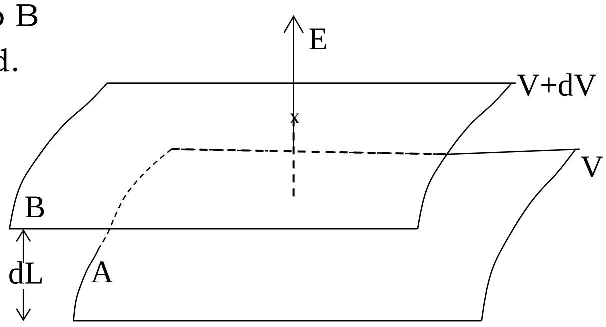
V_A=V= potential of A

V_B= V+dV = Potential of B

dL=perpendicular distance between A and B

$\vec{E}$ = electric field acting along A to B

E = | E | = magnitude of electric field.



Potential difference=V_A-V_B

= V- (V+dV)

Potential difference = –dV

Imagine unit positive charge (q_0=+1C) moves perpendicularly from B to A against field. direction

Then, workdone (W) = potential difference.

W= –dV

But $W = F \times dL$

W = E×dL

–dV= E×dL

$\therefore \quad E = -\frac{dV}{dL}$

$E = \frac{F}{q_o}$ but $q_o = 1 \quad \therefore E = F$

$W = F \times dL, \quad \therefore W = E \times dL$

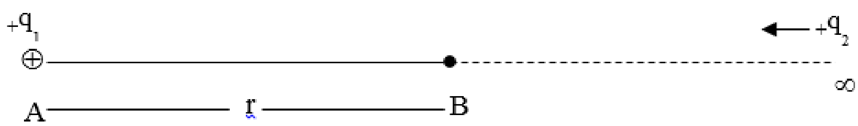
This is the relation between field and potential.

Note (1): According to the relation $E = -\frac{dV}{dL}$ the electric field is in the direction in which the potential decreases steepest.

Note (2): Magnitude of field is the change in magnitude of potential per unit displacement normal to the equipotential surface at the point

Note (3): $|E| = \frac{|dV|}{dL}$, Where, $|dV| = -dV$.

Potential energy of a system of two charges :



Let q_1= point charge at A

r= distance between A and B

Potential at B due to q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1}{r}$$

Bring a point charge q_2 from infinity to point B, then

$$\text{Potential} = \frac{\text{workdone}}{\text{charge}}$$

$$V_1 = \frac{W}{q_2}$$

$$\therefore W = V_1 \times q_2$$

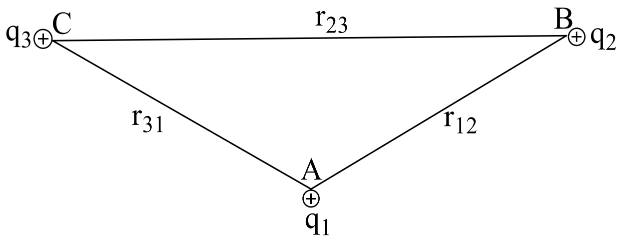
$$W = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1}{r} \times q_2$$

But workdone is stored as potential energy of system.
i.e., W=U

$$\therefore U = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1 q_2}{r}$$

This is the expression for potential energy of system of two charges.

Potential energy of system of three charges :



Consider three point charges q_1, q_2 and q_3 kept at a points A, B and C.

Let r_{12}, r_{23}, and r_{13} be the distance (Position vectors)

Workdone to bring q_2 from ∞ to B is

$$W_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_{12}}$$

Workdone to bring q_3 from ∞ to C against repulsive force of q_2 is

$$W_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q_3}{r_{23}}$$

Workdone to bring q_3 from ∞ to C against repulsive force q_1 is

$$W_3 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_3 q_1}{r_{31}}$$

Total workdone

$$W = W_1 + W_2 + W_3$$

$$W = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0} \times \frac{q_2 q_3}{r_{23}} + \frac{1}{4\pi\varepsilon_0} \times \frac{q_3 q_1}{r_{31}}$$

$$W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$$

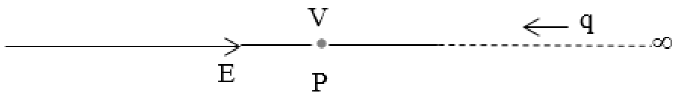
But, Workdone = Potential energy

W=U

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$$

This is the expression for potential energy of system of three charges.

Potential energy of a single charge in an external field



Let E=an external electric field at a point 'P'.

V=an external electric potential at a point 'P'.

q=any point charge.

The workdone in bringing a charge 'q' from infinity to point 'P' is

workdone = potential × charge

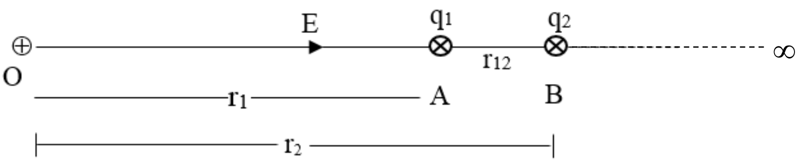
W=Vq

But workdone=potential energy

i.e., W=U

∴ $U=Vq$

Potential energy of a system of two charges in an external field



Let E= external electric field

V_1 = external potential at A

V_2 = external potential at B

Workdone to bring 'q_1' charge from ∞ to A is

$W_1 = V_1 \times q_1$

Workdone to bring 'q_2'charge from ∞ to B is

$W_2 = V_2 \times q_2$

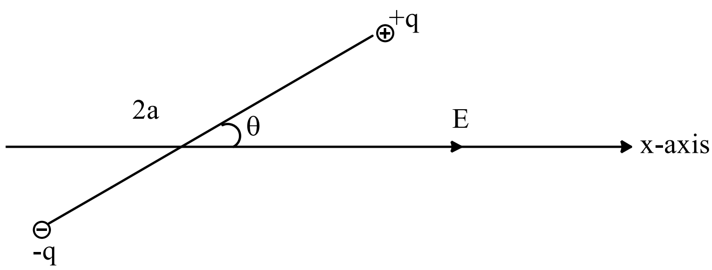
But to bring q_2, work is done not only against the external field E but also against the field due to q_1.

$$\therefore \quad W_3 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_{12}}$$

$\therefore$Potential energy of a system= W_1+W_2+W_3

$$U = \mathrm{v}_1 q_1 + \mathrm{v}_2 q_2 + \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r_{12}}$$

Potential energy of a dipole in an external field



Workdone to bring +q against external field E is

$W_1 = v_1 \times q$

Workdone to bring – q is

$W_2 = -v_1 \times q$

And, workdone to bring –q against field due to +q,

$$W_3 = \frac{-1}{4\pi\varepsilon_0} \times \frac{q^2}{2a}$$

$\therefore$Potential energy of a dipole = w_1+w_2+w_3

$$U = \mathrm{v}_1 q - \mathrm{v}_2 q - \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{2a}$$

$$U = q(\mathrm{v}_1 - \mathrm{v}_2) - \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{2a}$$

But v_1-v_2=-E x 2a cosθ

$$U = q(-E\,2a\cos\theta) - \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{2a} \qquad \text{(But } q\times 2a = p\text{)}$$

$$U = -PE\cos\theta - \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{2a}$$

Since second term in the above equation is insignificant constant for potential energy, we can drop that term,

$$\therefore \mathrm{U} = -\mathrm{PE}\cos\theta$$

This is the expression for potential energy of a dipole kept in an external field.

Note:

1. We have, $U = -PE\cos\theta$

   i.e. $U = -\vec{p}\cdot\vec{E}$

2. We have, $U = -PE\cos\theta$

   a) If, $\theta=0^0$, $\cos 0^0=1$,

   $\therefore$ U=–PE. Potential energy is minimum and dipole is in stable equilibrium.

   b) If, $\theta=180^0$, $\cos 180^0= -1$,

   $\therefore$ U=PE. Potential energy is maximum and dipole is in unstable equilibrium.

   c) If, $\theta=90^0$, $\cos 90^0= 0$,

   $\therefore$ U=0. Potential energy is minimum.

Electrostatics of conductors:

Conductors contain mobile charge carriers. In metallic conductors (ex: Cu, Al) free electrons are the mobile charge carriers (In metal, the valence electrons are the free electrons). They move randomly in different directions in the metal surface, but they cannot leave the metal. In the presence of an external electric field, the free electrons drift (move) against the direction of field.

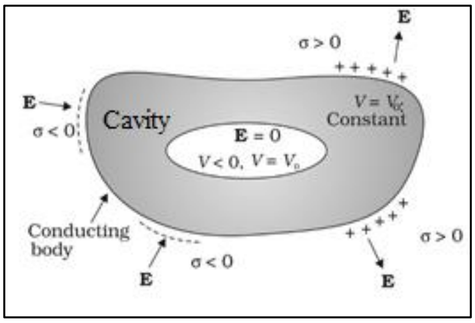
Note: In electrolytic conductors, (ex: NaCl) the charge carriers are both positive and negative ions. In this section, only metallic solid conductor is considered

Important results regarding electrostatic of conductor [metallic solid conductor]:

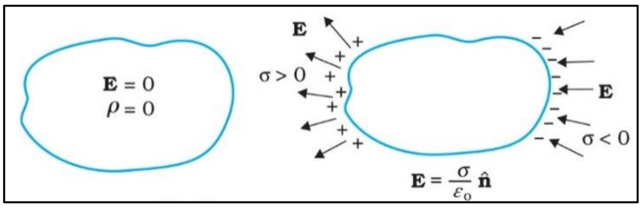
1) Electrostatic field is zero inside the conductor.
2) At the surface of charged conductor, electric field must be normal to the surface at every point.
3) In a charged conductor, excess charges reside only on the surface in static situation. i.e, no excess charge is present inside the conductor.
4) Electrostatic potential is constant throughout the volume of the conductor (i.e, inside the conductor) and is equal to the value at its surface.
5) Electric field at the surface of a charged conductor is, $E = \frac{\sigma}{\varepsilon_0}\hat{n}$

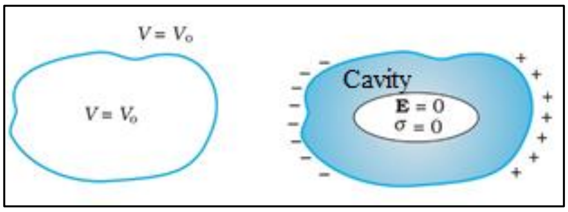
   Where σ = surface charge density, $\hat{n}$ = unit vector normal to the surface.
6) Any cavity in the conductor remains shielded from outside electric influence. This effect is known as electrostatic shielding.

Note-1: Advantage of electrostatic shielding:
This effect can be made use of in protecting sensitive instruments from outside electrical influence.



Note: 2) some important electrostatic properties of a conductor.





Non-polar molecules: The molecules in which centres of positive and negative charges coincide are called non-polar molecules. Ex: O_2, N_2, CO_2.

In non-polar molecule, the dipole moment is zero in the absence of external electric field.

Polar molecules: The molecules in which the centres of positive and negative charges are separated are called polar molecule: Ex: H_2O, HCl.

In polar molecule the permanent dipole moment is present even in the absence of external electric field.

Dielectrics:
Dielectrics are non-conducting substances and they donot have mobile charge carriers.

There are two types of dielectrics.

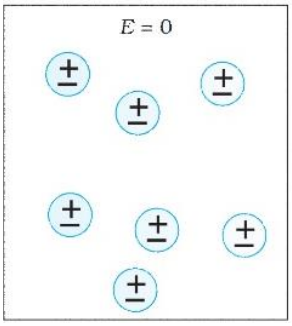
1) Non polar dielectrics and 2) polar dielectrics

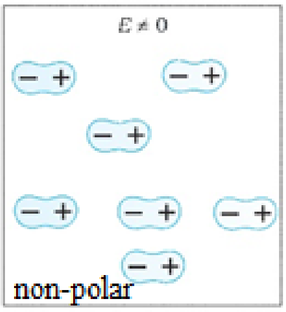
Non-polar dielectrics: The dielectric which contains non-polar molecules are called non-polar dielectrics.

Polar dielectrics: The dielectrics which contain polar molecules are called polar dielectrics.

Behaviour of non-polar dielectric in the absence of external electric field:

Non-polar dielectric contains a non-polar molecules the centres of positive and negative charges are coincided, therefore dipoles are not present. As a result, dipole moment is zero in the absence of external electric field.



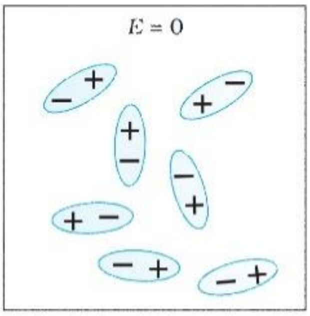


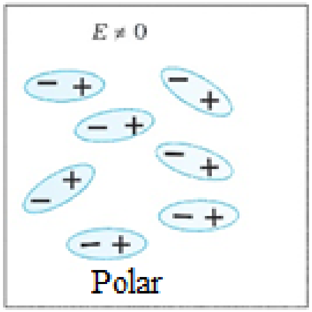
Behaviour of non-polar dielectric in the presence of external electric field:

When an external electric filed is applied to non-polar dielectric then dipoles are developed. As a result net dipole moment is produced in the presence of external electric field.

Behaviour of polar dielectrics in the absence of external electric field:

Polar dielectrics contain a polar molecule. The polar molecules have dipoles but the dipoles are arranged randomly in the absence of an electric field. As a result the net dipole moment is zero.





Behaviour of polar dielectric in the presence of external electric field:

When an external field is applied to polar dielectrics then dipoles are aligned in the direction of electric field. Therefore the net dipole moment is produced.

Note: Linear isotropic dielectrics: They are the substances for which the induced dipole moment is in the direction of the field and is proportional to the field strength.

Polarisation of Dielectric:

The dipole moment per unit volume is called polarization of dielectrics.

$$\text{Polarisation} = \frac{\text{Dipole moment}}{\text{Volume}}$$

S.I unit is coulomb/meter^2

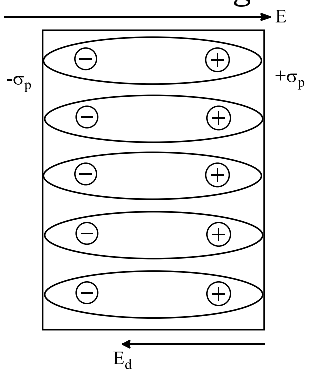
Note: For linear isotropic dielectrics, $P = \chi_e E$,

Where, P=Electric Polarisation,

'χ_e' = constant known as the electric susceptibility of the dielectric medium.

Note:

Consider a rectangular dielectric slab as shown



$-\sigma_p$ and $+\sigma_p$ Are surface charge density

E = external field

E_d = induced field due to induced dipoles.

$\therefore$ Net field = E - E_d

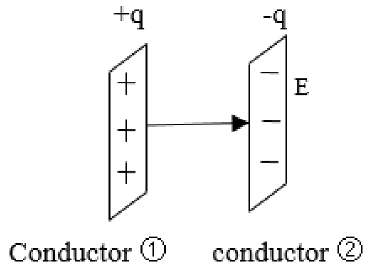
Dielectric strength: The dielectric can withstand the electric field up to some maximum value. If this value exceeds, then insulating property will break down and dielectric conducts the charge.

The Maximum electric field that a dielectric medium can withstand without breakdown of insulating property is called dielectric strength.

S.I Unit of dielectric strength is V/m

Note: For air, dielectric Strength = 3×10^6 V/m

Capacitor: Capacitor is a system of two conductors separated by an insulator to store electric charge and electric energy.



Let + q = charge on conductor ①, – q = charge on conductor ②

V_1 = potential of conductor ①, V_2= potential of conductor ②

$V=V_1 - V_2$ = potential difference E = Electric field

In general, q = charge of the capacitor,

V = potential difference between two conductors.

Capacitance of a capacitor:

It is found that, potential difference is directly proportional to charge

i.e $V \alpha q$ or, $q \alpha V$

$q = CV$

Where C = constant called capacitance of the capacitor.

$$\therefore \boxed{C = \frac{q}{V}}$$

Capacitance of a capacitor is defined as the ratio of charge on a capacitor to the potential difference between two conductors of it.

S.I unit is coulomb/volt (C/V) or farad (F)

Define 1 farad:

W.K.T., $C = \frac{q}{V}$

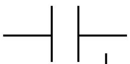
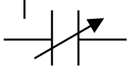
If, q=1C, V= 1 volt, then C= 1 farad

Then, 1 farad $= \frac{1 \text{ coulomb}}{1 \text{Volt}}$

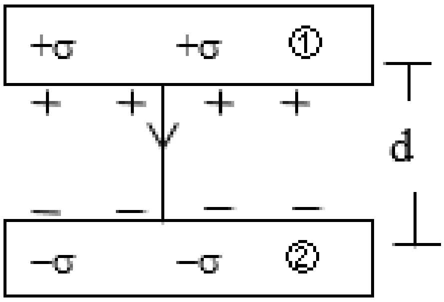
$1F = 1\ CV^{-1}$

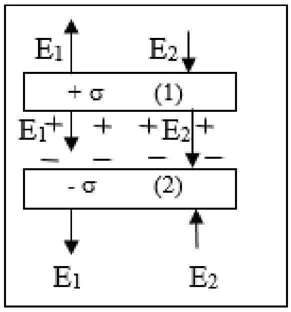
The capacitance is said to be one farad if one coulomb of charge is stored when the potential difference is one volt.

Note:

1. capacitance depends on geometrical configuration (i.e. Shape, size, separation) and nature of insulator (dielectric)
2. A capacitor with fixed capacitance is represented as 
3. A capacitor with variable capacitance is represented as 

Derive an expression for capacitance of Parallel Plate capacitor (If the space between the two plates is vacuum):





A Parallel plate capacitor consists of two parallel metal plates separated by a small distance. The space between two plates to be vacuum

Let, A=Area of each plate

d= distance between two plates

+ q and –q = charges on two plates

+ σ and –σ = surface charge densities

But $\sigma = \frac{q}{A}$

Electric Filed due to plate ① is $E_1 = \frac{\sigma}{2\varepsilon_0}$

Electric field due to plate ② is $E_2 = \frac{\sigma}{2\varepsilon_0}$

In the region above the plate ① E_1 and E_2 are in opposite direction.

∴ net field above the plate ① is $E = E_1 - E_2$

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

$$E = 0$$

Similarly, E_1 and $E_2$are in opposite direction in the region below the plate ②.

∴ Net field E =0

But in the inner region between plates ① and ②, E_1 and E_2 are in same direction.

∴Net field, $E = E_1 + E_2$

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$= \frac{2\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$ But $$\sigma = \frac{q}{A}$$

$\therefore \quad E = \dfrac{q}{\varepsilon_0 A} \rightarrow (1)$

We have $E = \dfrac{V}{d}$ - - - -> (2)

$$\left[\because E = -\frac{dV}{dL}\right]$$

$$\left[|E| = \frac{dV}{dL} = \frac{V}{d}\right]$$

From equation (1) & (2)

$$\frac{q}{\varepsilon_0 A} = \frac{V}{d}$$

Or $\dfrac{q}{V} = \dfrac{\varepsilon_0 A}{d}$ But $\dfrac{q}{V}$ =C=Capacitance of capacitor

$$\therefore \quad C = \frac{\varepsilon_0 A}{d}$$

This is the expression for capacitance of a parallel plate capacitor

Note: In case of a parallel plate capacitor, near the outer boundaries of the plates, the field lines bend outward at the edges, this effect is called Fringing of the field.

Derive the expression for Capacitance of Parallel capacitor when dielectric medium is present:

Case 1: Consider a Parallel plate Capacitor. The space between the plate is Vacuum.

Let q = Charge on capacitor

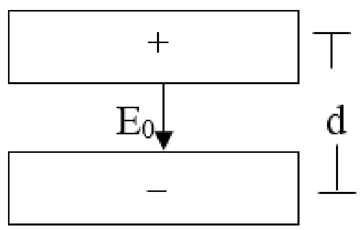
V_0 = Potential Difference

E_0 = Electric Field between plates

σ = Surface charge density

A = Area of each plate

d = distance between two plates

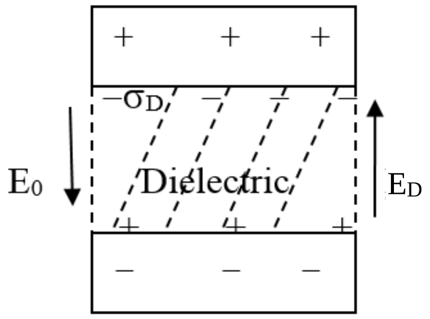


$$\therefore \quad E_0 = \frac{\sigma}{\varepsilon_0}$$

Case 2: When a dielectric medium is filled between two plates of a capacitor, then dipoles are induced.

Let, σ_D = Surface charge density due to polarization of dielectric

E_D = electric field due to polarisation



But, $E_D = \dfrac{\sigma_D}{\varepsilon_0}$

$\therefore$ Net electric field

$$E = E_0 - E_D$$

$$= \frac{\sigma}{\varepsilon_0} - \frac{\sigma_D}{\varepsilon_0}$$

$$E = \frac{\sigma - \sigma_D}{\varepsilon_0}$$

It is found that $\sigma - \sigma_D \ \alpha \ \sigma$

$$\therefore \sigma - \sigma_D = \frac{\sigma}{K}$$

Where, K = dielectric constant (K>1)

$$\therefore \quad E = \frac{\sigma}{\varepsilon_0 K} \text{ ----------- } ①$$

We know that, $E = \dfrac{V}{d}$ ----------- ②

$$\frac{\sigma}{\varepsilon_0 K} = \frac{V}{d}$$

$$\sigma = \frac{V\varepsilon_0 K}{d} \qquad \text{But} \quad \sigma = \frac{q}{A}$$

$$\therefore \quad \frac{q}{A} = \frac{V\varepsilon_0 K}{d}$$

$$\frac{q}{V} = \frac{\varepsilon_0 KA}{d}$$

$$C = \frac{\varepsilon_0 KA}{d} \qquad \because \left(\frac{q}{V} = C\right)$$

This is the expression for capacitance of a parallel plate capacitor when dielectric is present.

Note 1: we have $C = \dfrac{\varepsilon_0 KA}{d}$.

Where $\varepsilon_0 K = \varepsilon$ = permittivity of the dielectric medium.

$$\therefore K = \frac{\varepsilon}{\varepsilon_0}$$

Where K= dielectric constant, 'K' has no unit.

Note 2 : For vacuum, K = 1, K>1 for any dielectric medium.

Note 3 :We have $C_0 = \frac{\varepsilon_0 A}{d}$ -----① for vacuum

And $C = \frac{\varepsilon_0 KA}{d}$ For dielectric medium

Or $C = \frac{\varepsilon_0 A}{d} \times K$

$C = C_0 K$ from eqn ①

Or $K = \frac{C}{C_0}$

Define dielectric constant: Dielectric constant is the ratio of capacitance of capacitor when space between the plates is filled with dielectric to the capacitance of same capacitor when space between the plates is vacuum.

Equivalent capacitor and equivalent capacitance (Effective capacitance):

A single capacitor which produces same effect as that of set of capacitor is called equivalent capacitor and its capacitance is called equivalent capacitance.

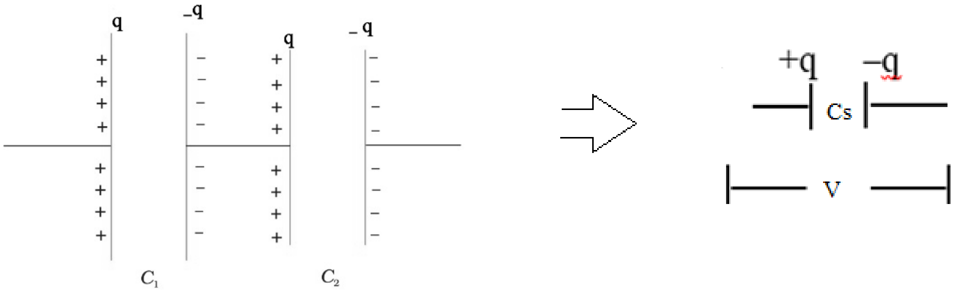
Combination of capacitor:

There are two combination.

i) Series combination, ii) Parallel combination

Series combination: A set of capacitors is said to be in series if they are connected end to end such that charge on each capacitor remain same.

Derive an expression for effective capacitance of a series combination:



Consider two capacitor of capacitance C_1 and C_2 connected in series as shown in diagram. Let +q and –q be the charge on two plates of a capacitors.

In series combination, charge on each capacitor is same (say 'q').

Let V_1 = potential difference across C_1

V_2 = potential difference across C_2

V = potential difference across combination

C_s= effective capacitance of series

In series, V = V_1 + V_2 ------- ①

q=CV

$\therefore V = \frac{q}{C}$

But $V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$ and $V = \frac{q}{C_s}$

∴ equ ① becomes

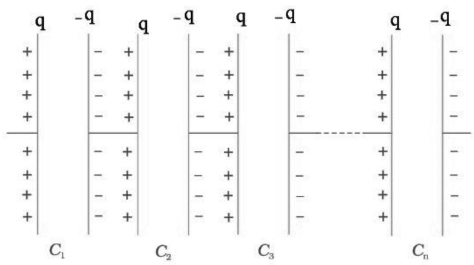
$$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\frac{q}{C_s} = q\left[\frac{1}{C_1} + \frac{1}{C_2}\right]$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

This is the Expression for effective capacitance of two Capacitors in series.

For n –Capacitors in Series



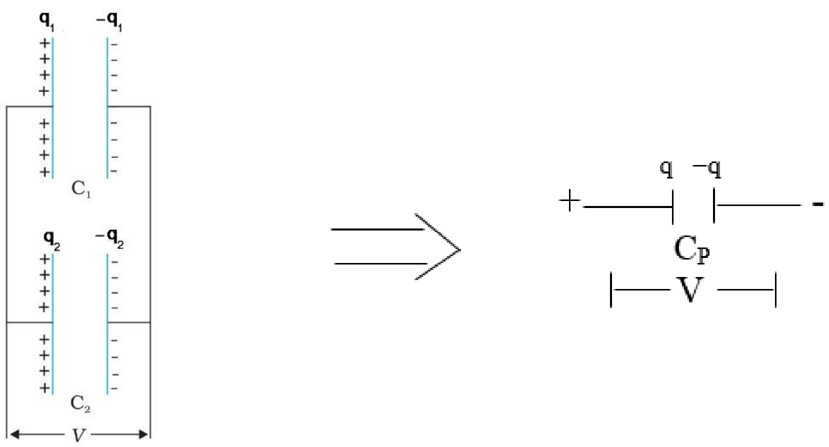
V = V_1 +V_2 + V_3 + --------- + Vn

$$= \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} + ------- + \frac{q}{C_n}$$

$$\therefore \quad \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C3} + ------- + \frac{1}{C_n}$$

This is the Expression for effective capacitance of 'n' capacitors in series

Derive an expression for effective capacitance of a parallel combination



Consider two capacitor of capacitance C_1 and C_2 connected in parallel as shown in diagram. Potential difference across each capacitor is same in parallel.

Let q_1 = charge on capacitor C_1

q_2 = charge on capacitor C_2

q = total charge of combination

V = potential difference

C_p = effective capacitance of parallel

In parallel, $q = q_1 + q_2$ -------①

But $q_1 = C_1V$, $q_2 = C_2V$ and $q = C_pV$

∴ eqn ① becomes

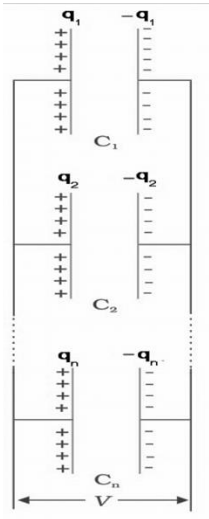
∴ q = CV
V is same

$C_pV = C_1V + C_2V$

$C_pV = V(C_1 + C_2)$

∴ $C_p = C_1 + C_2$

This is the expression for effective capacitance of two capacitors in parallel.



For 'n' capacitors

$q = q_1 + q_2 + q_3 + \text{-----------} + q_n$

$C_pV = C_1V + C_2V + C_3V + \text{-------} + C_nV$

$C_p = C_1 + C_2 + C_3 + \text{-------} + C_n$

This is the expression for effective capacitance of n capacitors in parallel.

Note:

1. In series combination, charge on each capacitor remains constant.
2. In parallel combination, potential difference across each capacitor remains constant.
3. If C_1 and C_2 connected in series.

   then, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1C_2}$, $\quad \therefore C_s = \frac{C_1C_2}{C_1 + C_2}$.

   For three capacitors

   $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2C_3 + C_3C_1 + C_1C_2}{C_1C_2C_3}$, $\quad \therefore C_s = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$
4. For 'n' capacitors in series

   $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C3} + \text{-------} + \frac{1}{C_n}$

   For 'n' identical capacitors, $C_1=C_2=C_3=\ldots\ldots=C_n=C$,

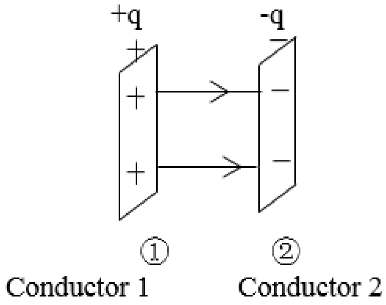
   $\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \text{-------} + \frac{1}{C}$, $\quad \frac{1}{C_s} = n \times \frac{1}{C}$, $\quad C_S = \frac{C}{n}$
5. If 'n' identical capacitor are connected in parallel

   $C_P = C_1 + C_2 + C_3 + \ldots\ldots + C_n$

   For 'n' identical capacitor, $C_P=C+C+C+\ldots\ldots+C$

   $C_P=nC$

Derive an expression for energy stored in a capacitor



Consider a capacitor of capacitance C. The capacitor consists of two conductors, initially these are uncharged. When a positive charge is transferred from conductor ② to ① bit by bit, finally conductor ① gets charge '+q' and conductor ② has –q. In transferring positive charge from conductor ② to ①, work will be done externally.

At intermediate situation,

Let, +Q and -Q' be the charge on conductor ① and ②

V'=Potential difference,

$\therefore$ Potential difference, $V' = \frac{Q}{C}$

At this stage,

dQ = a small charge is transferred from conductor ② to ①.

dW=Small work done to transfer dQ

but, dW = V' dQ

$$\therefore \quad dW = \frac{Q}{C} dQ$$

$$V' = \frac{dW}{dQ} \qquad \therefore dW = V'dQ$$

$\therefore$ total work done to transfer a charge from zero to q is

$$W = \int_0^q dW$$

$$W = \frac{q^2}{2C}$$

$$W = \frac{1}{C}\int_0^q Q\,dQ$$

$$= \frac{1}{C}\left[\frac{Q^{1+1}}{1+1}\right]_0^q$$

$$= \frac{1}{C}\left[\frac{Q^2}{2}\right]_0^q$$

$$= \frac{1}{2C}\left[q^2 - 0\right]$$

$$W = \frac{q^2}{2C}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Work done is stored as a potential energy

i.e. work done = potential energy

W = U

$$\therefore \quad U = \frac{q^2}{2C}$$

This is the expression for energy stored.

Note:

1. W.K.T. $U = \frac{q^2}{2C}$ --------------- (1)

   but, q=CV

$$U = \frac{(CV)^2}{2C}$$

$$U = \frac{CV^2}{2}$$

$$U = \frac{1}{2}CV^2 \quad \text{---------------} \ (2)$$

   but, $C = \frac{q}{V}$

$$U = \frac{1}{2} \times \frac{q}{V} \times V^2$$

$$U = \frac{1}{2}qV \quad \text{---------------} \ (3)$$

$$\therefore \quad U = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

Energy density: The energy stored per unit volume is called Energy density.

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

S.I. Unit is Joule/meter^3.

Derive an expression for energy density:

We have, $U = \frac{q^2}{2C}$ ....(1)

But, $\sigma = \frac{q}{A}$,

$\therefore$ q=σA ----------------(2)

But, $E = \frac{\sigma}{\varepsilon_0}$

$$\sigma = \varepsilon_0 E$$

Equation (2) becomes

$$q = \varepsilon_0 EA \quad \text{--------------} (3)$$

Substituting Equation (3) in (1)

$$U = \frac{\varepsilon_0^2 E^2 A^2}{2 \times \frac{\varepsilon_0 A}{d}}$$

$$U = \frac{\varepsilon_0 E^2 Ad}{2}$$

$$\frac{U}{Ad} = \frac{\varepsilon_0 E^2}{2}$$

Ad = Area X distance

Ad = volume of region between plates

$$\frac{U}{Ad} = energy/volume$$

$$\frac{U}{Ad} = \text{energy density}$$

$$U = \frac{(\varepsilon_0 EA)^2}{2C}$$

$$U = \frac{\varepsilon_0^2 E^2 A^2}{2C} \text{ --------------(4)}$$

But, $C = \frac{\varepsilon_0 A}{d}$

Above equation (4) becomes

$\therefore$ Energy density = $\frac{\varepsilon_0 E^2}{2}$

This is the expression for energy density.

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} + \frac{q}{r}\right]$$

****

One mark questions:

1. What do you mean by the conservative nature of electric field?
2. Define Electrostatic Potential.
3. What is the SI unit of Electric Potential?
4. What are the equipotential surfaces of a point charge?
5. Draw the Equipotential surfaces for a point charge.
6. Give the condition for equipotential surface in terms of the direction of the electric field.
7. Define Electrostatic Potential energy of a system of charges.
8. Write the expression for potential energy of two point charges in the absence of external electric field.
9. Write the expression for potential energy of two point charges in the presence of external electric field.
10. Write the expression for Electric field near the surface of a charge conductor.
11. What happens when Dielectrics are placed in an electric field?
12. What is electric polarisation?
13. What is Parallel plate capacitor?
14. Mention the expression for capacitance of a Parallel plate capacitor without any dielectric medium between the plates.
15. Mention the expression for capacitance of a Parallel plate capacitor with a dielectric medium between the plates.
16. Define Dielectric constant of a substance.
17. What is an electric dipole (March-2016)?
18. What is the electric field strength inside a charged spherical conductor?
19. What is a capacitor? (July-2014)

Two mark questions:

20. How does the electric field and electric potential vary with distance from a point charge?

21. How does the electric potential at a point due to an electric dipole vary with distance measured from its centre? Compare the same for a point charge.
22. Using superposition principle, write the expression for electric potential at a point due to a system of charges.
23. What is an equipotential surface? Give an example.
24. Explain why the equipotential surface is normal to the direction of the electric field at that point.
25. Explain why Electric field inside a conductor is always zero.
26. Explain why Electrostatic field is always normal to the surface of charged conductor.
27. Explain why Electric charges always reside on the surface of a charge conductor.
28. Explain why Electrostatic potential is constant throughout the volume.
29. What is Electrostatic shielding? Mention one use of it.
30. What are non-polar Dielectrics? Give examples.
31. What are polar Dielectrics? Give examples.
32. Define capacitance of a capacitor. What is SI unit?
33. On what factors does the capacitance of a parallel plate capacitors depends? (March-2017)

Three marks questions:

34. Write the expression for electric potential at a point due to an electric dipole and hence obtain the expression for the same at any point on its axis and any point on its equatorial plane.
35. Obtain the relation between the electric field and potential. (July-2015, July-2014)

    OR

    Show that electric field is in the direction in which the potential decreases steepest.
36. Derive the expression for potential energy of two point charges in the absence of external electric field. (March-2016)
37. Mention the expression for potential energy of an electric dipole placed in an uniform electric field. Discuss its maximum and minimum values.
38. Derive the expression for effective capacitance of two capacitors connected in series.
39. Derive the expression for effective capacitance of two capacitors connected in parallel.
40. Derive the expression for energy stored in a capacitor. (March-2016, March-2017)

Five marks questions:

41. Derive the expression for electric potential at a point due to a point charge.
42. What are Dielectrics? Mention the types of Dielectrics.
43. Derive the expression for capacitance of a Parallel plate capacitor without any dielectric medium between the plates. *(or Parallel plate air capacitor).*
44. What is Van De Graff generator? Write its labelled diagram. What is the principle of its working? Mention its use.

Chapter: 3 CURRENT ELECTRICITY

Current electricity: It is a branch of physics which deals with the study of electric charge in motion.

Electric Current: The net electric charge passing per unit time is called electric current

i.e electric Current = $\frac{\text{net charge}}{\text{time taken}}$

$$I = \frac{q}{t}$$

S.I unit is coulomb/sec or ampere (A)

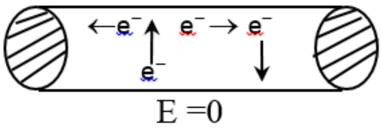
Note:

1. Electric current is a scalar quantity
2. We have I = $\frac{q}{t}$, but q =ne ∴ I = $\frac{ne}{t}$, where n = number of free electrons
3. I = $\frac{q}{t}$, this Formula is for steady current (i.e for constant current).
4. If current is variable (not steady), then, we take instantaneous current.
   Let Δq = small charge flows
   Δt = small time interval
   And $\Delta t \to 0$

Then instantaneous current is defined as

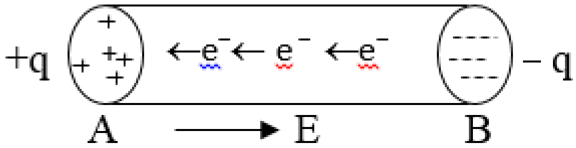
$$I = \lim_{\Delta t \to 0}\left(\frac{\Delta q}{\Delta t}\right)$$

Electric Current in Conductor:



e^- = free electron

The solid conductors have large number of free electrons. In the absence of applied electric field, The free electron move in a random direction due to thermal energy. Therefore, number of free electrons travelling in any direction will be equal to the number of free electrons travelling in the opposite direction. That is net flow of free electron in a specific direction is zero. Therefore, in the absence of electric field net current flows through the conductor is zero.



Take positive and negatively charged circular dielectric plates. Now, attach them on the two ends A and B of a conductor. then end A is positively charged (+q) and end B is negatively charged (– q). Therefore end A is at higher potential (+ve) and end B is at lower potential (–ve). As a result electric field is acted from A to B. Therefore, force is acted on free electron against the electric field. So all the free electrons move in a direction opposite to electric field (ie from lower potential to higher potential). As a result net current flows through the conductor. During the motion, the free electrons collide with fixed ions and atoms due to thermal excitation, and they lost kinetic energy in the form of heat.

Drift Velocity (v_d)**:** The average velocity with which the free electrons move in a conductor under the influence of electric field is called drift velocity.

The SI Unit of drift velocity is ms^{-1}.

Mobility (μ): The ratio of magnitude of drift velocity to the applied electric field is called mobility.

$$\mu = \frac{v_d}{E}$$

where μ = Mobility, v_d = drift velocity, E = applied electric field.

S.I Unit is m^2/Vs

Note: Mobility is positive.

Relaxation time (τ): The time interval between the two successive collisions of free electrons in a conductor is called relaxation time.

State and explain ohm's law:

It states that "The electric current flowing through a conductor is directly proportional to potential difference across its ends if temperature and dimensions of conductor kept constant."

i.e. current ∝ potential difference

or, potential difference ∝ current.

$$V \propto I$$

$$\boxed{V = RI}$$

Where R = resistance of the conductor.

Note: Ohm's law was discovered by G S Ohm in 1828.

Electrical resistance:

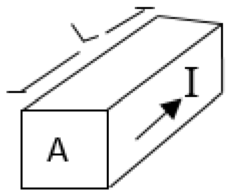
According to ohm's law, V = RI $\therefore$ $R = \frac{V}{I}$

Resistance of a conductor is the ratio of potential difference across its ends to the current flows through it.

S.I unit is Volt/ampere (or) ohm (Ω)

Note: one ohm=1volt/ampere.

Explain how resistance depends on length and cross-sectional area of conductor (dimensions of conductor):



Consider a conductor of length L and cross-sectional area 'A'.

Let I = current, V = potential difference. R = resistance of a conductor

According to ohm's law, R = $\frac{V}{I}$.

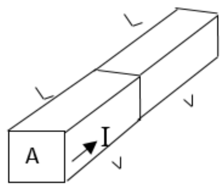
Case ①:

Consider two identical conductors in contact side by side as shown below

Now, same current I flows through each conductor and potential difference across each conductor is V.

$\therefore$ Total potential difference across the combination is $V^{|}$=V+V=2V

Current through the combination is, $I^{|} = I$



$\therefore$ Resistance of the combination is

$$R^{|} = \frac{V^{|}}{I^{|}}$$

$$= \frac{2V}{I}$$

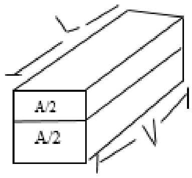
$$R^{|} = 2R \qquad \left(\therefore \frac{V}{I} = R\right)$$

i.e. $R \propto L$ --------→ ①

Resistance is proportional to length of conductor.

Case ②:

Now divide the conductor into two parts by cutting lengthwise as shown
Length of each slab =L, cross sectional area of each slab = A/2
Current through each slab = $I^{|} = I/2$, Potential Difference = $V^{|} = V$.
Resistance of each slab = R^1



$$\therefore \quad R^{|} = V^{|} / I^{|}$$

$$= \frac{V}{I/2}$$

$$= \frac{2V}{I}$$

$$R^{|} = 2R \quad (\because V/I = R)$$

$$\therefore R \propto \frac{1}{A} \quad \text{--------}\rightarrow ②$$

from eq ① and ② $R \propto \frac{L}{A}$

$R = \rho\frac{L}{A}$, Where, ρ = resistivity of a conductor

Note-1 : a) Resistance of a conductor is directly proportional to Length of a conductor ($R \propto L$)
b) Resistance of a conductor is inversely proportional to Cross - sectional area of a conductor $\left(R \propto \frac{1}{A}\right)$.
c) Nature of material of a conductor.
Note-2: Resistance of a conductor is directly proportional to temperature of a conductor. ($R \propto T$)

Resistivity of a conductor

We have $R = \frac{\rho L}{A}$

If L =1m, and A = $1m^2$, then R = ρ or ρ = R

Resistivity of a conductor is the resistance of conductor of length 1m and cross-sectional area $1m^2$

S.I Unit of resistivity is Ωm (ohm-metre).

Note-1: Resistivity does not depend on dimension (length and area) of a conductor but it depends on nature of material of conductor.
Note-2: Resistivity of a conductor is directly proportional to temperature. ($\rho \propto T$)

Conductivity: The reciprocal of resistivity is called conductivity.

i.e Conductivity = $\frac{1}{resistivity}$

$$\sigma = \frac{1}{\rho}$$

S.I Unit is ohm^{-1} m^{-1} (Ω^{-1} m^{-1}) or siemen m^{-1}

Current density (J): The current per unit area taken normal to the current is called current density.
Current density = $\frac{current}{Area}$

$$J = \frac{I}{A}$$

S.I Unit is ampere/metre^2 (A/m^2).
Current density is a vector.

Ohm's law in terms of electric field and current density.

we have V=RI but $R = \frac{\rho L}{A}$

$$\therefore V = \frac{\rho L}{A} \times I = \frac{\rho L \times I}{A}, \quad but \frac{I}{A} = J$$

$$V = \rho LJ \quad \text{but } \therefore V = EL \qquad (\because E = \frac{V}{L})$$

$$\therefore EL = \rho LJ$$

$$\therefore E = \rho J \dashrightarrow (1)$$

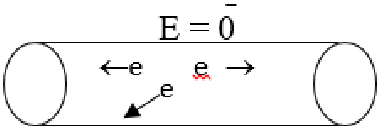
$$\text{or} \quad J = \frac{1}{\rho}E \qquad \text{but } \frac{1}{\rho} = \sigma = Conductivity$$

$$J = \sigma E \dashrightarrow (2)$$

eq: (1) and (2) represent Ohm's law

Note: in vector form, $\vec{E} = \rho\vec{J}$ and $\vec{J} = \sigma\vec{E}$.

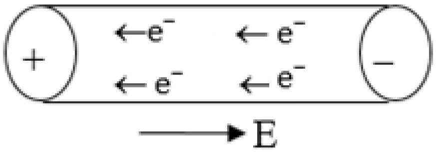
Drift of Electron and the origin of resistivity of conductor (Deduce ohm's law):
(Obtain the expression for conductivity):



e^- = free electron

Consider a conductor of cross sectional area `A'. In the absence of electric Field, The free electrons move in random direction. They will suffer collisions with fixed ions. After collision, their average velocity will be zero, since their directions are random.

i.e. $\frac{1}{N}\sum_{i=1}^{n} v_i = 0$, Where, N=number of electrons, v_i=velocity of i^{th} electron



When an electric field is applied, all the free electrons are drifted towards positive potential,

∴ force on electron is, $F = -eE$ -------> ①

Where, –e = charge on electron, E=electric field.

but $F = ma$ ----------- ②

Where, a =acceleration, m = mass of electron,

form eq: ① & ②, $ma = -eE$

$\therefore a = \frac{-eE}{m}$ ------- ③

$E = \frac{F}{q}$, $F = qE$,
but $q = -e$ for electron

W.k.t, $v = v_0 + at$ ----------- ④

Where,

v_0 = velocity of i^{th} electron immediately after last collision at time `t'

v = final velocity

Collisions of electrons do not occur at regular intervals, therefore the time t_i is not constant. So consider average time and average velocity.

i.e $(t)_{average} = \tau$ = relaxation time

∴ $(v)_{average} = v_d$ = drift velocity and $(v_0)_{average} = 0$

∴ eq. (4) becomes

$v_d = 0 + a\tau$

$v_d = a\tau$ but $a = \frac{-eE}{m}$ (from eqn ③)

$\therefore \quad v_d = \frac{-eE}{m} \times \tau$ --------------- ⑤

$$|v_d| = \frac{eE\tau}{m}$$

This is the expression for drift velocity.

Let, Δx = small length of conductor, Δt = small time interval.

Then, $v_d = \frac{\Delta x}{\Delta t}$

$\because Velocity = \frac{distance}{time}$

$V = \frac{d}{t}$

$\therefore \Delta x = v_d \Delta t$

Volume of conductor = $\Delta x \times A$

(Volume=length×Area)

Volume of conductor = $v_d \Delta t A$

Let, n = number of free electrons per unit volume.

Total free electrons transported = n × volume

Total free electrons transported = $n \times v_d \Delta t A$

Total Charge transported = number of electrons × Charge on electron

$\therefore$ total charge transported = $n \times v_d \Delta t A \times (-e)$

$\Delta q = -neAv_d\Delta t$ (This is the charge transported opposite to $\vec{E}$)

$\therefore$ The total charge transported along $\vec{E}$ is

$$\Delta q = -(-neAv_d\Delta t)$$

$$\Delta q = neAv_d\Delta t$$

But current, $I = \frac{\Delta q}{\Delta t}$

$$I = \frac{neAv_d \Delta t}{\Delta t}$$

$I = neAv_d$ --------→ ⑥

This is the expression for drift current.

But $|v_d| = v_d = \frac{eE}{m}\tau$

$\therefore$ eq: ⑥ becomes

$$I = neA\left(\frac{eE}{m}\tau\right)$$

But $\frac{I}{A} = J = Current\ Density$

$$I = \frac{ne^2A\tau}{m} \times E$$

$$\frac{I}{A} = \frac{ne^2\tau}{m} \times E$$

$$J = \frac{ne^2\tau}{m} \times E \quad \text{-------}\blacktriangleright \quad ⑦$$

According ohm's law; $J = \sigma E$ ----------- ⑧

Where, σ = Conductivity

from eq ⑦ and ⑧, It shows that eq: ⑦ represent the ohm's law

$$\therefore \quad \sigma = \frac{ne^2\tau}{m}.$$

This is the expression for conductivity.

Note: Different forms of ohm's law: a) $V = RI$, b) $\vec{J} = \sigma\vec{E}$, c) $\vec{E} = \rho\vec{J}$.

Expression for resistivity:

We have, $\sigma = \frac{ne^2\tau}{m}$,

but $\rho = \frac{1}{\sigma}$,

$\therefore \quad \rho = \frac{m}{ne^2\tau}$

Where ρ = resistivity (specific resistance)

m = mass of electron

e = charge of electron

n = number of free electron per unit volume

τ = relaxation time

m and e are constant. `n' and τ varies

Factors on which resistivity depends

W.K.T., $\rho = \frac{m}{ne^2\tau}$

Where, m and e are constant.

1) Resistivity inversely proportional to number of free electrons per unit volume ($\rho \propto \frac{1}{n}$)

2) Resistivity inversely proportional to relaxation time ($\rho \propto \frac{1}{\tau}$).

3) Resistivity proportional to absolute temperature ($\rho \propto T$).

Note:

1. Relaxation time $\alpha \frac{1}{Temperature}$

   $\tau \;\alpha\; \frac{1}{T}$

   but $\rho \;\alpha\; \frac{1}{\tau} \;\alpha\; T$

   i.e, As temperature of conductor increases resistivity also increases.
2. The conductivity (electric current) arises from mobile charge carriers,
   a) In metals, mobile charge carriers are free electrons.
   b) In an ionized gas, mobile charge carriers are electrons and positively charged ions.
   c) In an electrolytes and ionic crystals, mobile charge carriers are positively and negatively charged ions.

Expression for mobility of electron

We know that

$$\text{Mobility} = \frac{driftvelocity}{electric\ field}$$

$$\mu = \frac{v_d}{E}$$

But $v_d = \frac{eE\tau}{m}$

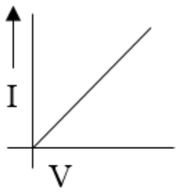
$$V_d = -\frac{eE\tau}{m}$$

$$|v_d| = \frac{eE\tau}{m}$$

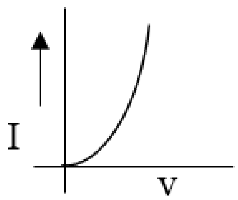
$\therefore \mu = \frac{eE\tau/m}{E}$ $\mu = \frac{e\tau}{m}$ This is the expression for mobility.

Note:

1. The devices which obey ohm's law are called ohmic devices.
   Ex: Conductors, ammeter voltmeter etc.
   for Ohmic devices, V α I,
   A graph of V versus I is linear as shown.

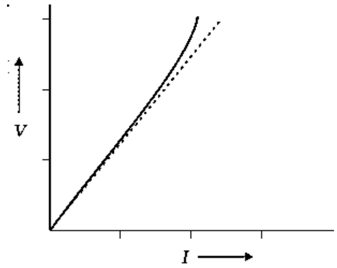


2. The devices which donot obey ohm's law are called non ohmic devices
   Ex: Super Conductors, semiconductor devices (diode)
   For non-ohmic devices, V is not proportional to I. The graph is not linear.

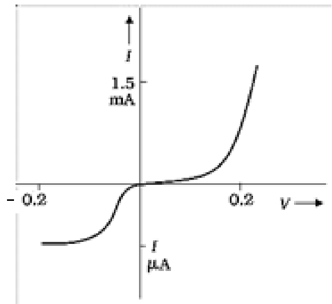


Limitations of ohm's law

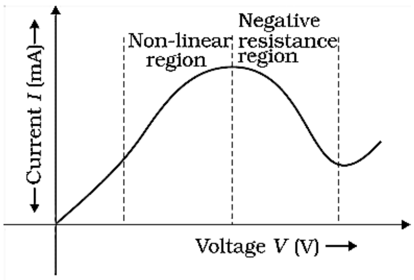
1. In some materials and devices V ceases to be proportional to I (i.e proportionality of V and I stops)
   (Solid line is for good conductors)



2. In some devices like diode, the relation between V and I depends on the sign of V.
   It is found that magnitude of I is not same for same magnitude of positive and negative V



3. In some material like GaAs, the relation between V and I is not unique, i.e there is more than one value of V for the same I as shown



Note: Ohm's law fails if

1) V depends on I non linearly.
2) The relation between V and I depends on the sign of V for the same absolute value of V.
3) The relation between V and I is non unique.

Resistivity of various materials: The materials are classified as conductors, semiconductors and Insulators, depending on their resistivities.

The material having low resistivity in the range of $10^{-8}\Omega m$ to $10^{-6}\Omega m$ are called conductors (Metals).

The Materials having resistivity 10^{18} times greater than metals are called insulators.

The materials having the resistivity in between metals and insulators are called semiconductors.

in general, resistivity of conductors < resistivity of semi conductors < Resistivity of Insulators.

Note :

1. Resistivity of metals increases with increasing temperature.
2. Resistivity of semiconductors increases with decreasing temperature.
3. Resistivity of some materials.

Conductor		Semiconductor		Insulator	
Material	Resistivity (Ω m)	Material	Resistivity (Ω m)	Material	Resistivity (Ω m)
Silver	$1{\cdot}6\times10^{-8}$	Carbon (Graphite)	3.5×10^{-5}	Pure Water	2.5×10^{5}
Copper	1.7×10^{-8}	Germanium	0.46	Glass	$10^{10}–10^{14}$
Alluminium	2.7×10^{-8}	Silicon	2300	Hard Rubber	$10^{13}–10^{16}$
Tungsten	5.6×10^{-8}			NaCl	10^{14}
Iron	10×10^{-8}			Fused quartz	10^{16}
Nichrome (alloy)	100×10^{-8}				
Manganin (alloy)	48×10^{-8}				

Resistor: The resistor is a device used to oppose the flow of current in the circuit.

There are two types 1) wire bound resistor and 2) Carbon resistor

Wire bound resistor: It is made by winding the wires of an alloy like manganin, nichrome or constantan.

These materials are usually preferred because their resistivity is insensitive to temperature.

The range of resistances of wire bound resistors is small (up to few hundred ohms).

Carbon resistor: It is a compact and inexpensive. Thus they are widely used in electronic circuits. Carbon resistors are available in the higher range resistances (kilo ohm, mega ohm)

Carbon resistors are small in size and hence their values are given using a colour code

Colour code of resistor: It is a technique used to find the value of resistance of given resistor.

It consists of four colour bands on the resistor as shown.

$R = xy \times 10^Z \pm$ tolerance

Where R = resistance

x and y = I and II colour bands and they are two significant figures

z = III colour band

Tolerance = IV colour band

colour	First and second Band value (x and y)	Thrid band Value (10^Z)	Z	Tolerence
Black	0	10^0	0	
Brown	1	10^1	1	
Red	2	10^2	2	
Orange	3	10^3	3	
yellow	4	10^4	4	
Green	5	10^5	5	
Blue	6	10^6	6	
Violet	7	10^7	7	
Grey	8	10^8	8	
White	9	10^9	9	
Gold	-	10^{-1}	-1	5%
Silver	-	10^{-2}	-2	10%
No colour	-	-	-	20%

Note: B B R O Y Goes to Bombay Via Gate Way with Gold and Silver
0 1 2 3 4 5 6 7 8 9 -1 -2

Ex ① : If four colours are orange, blue, yellow, and gold, then find resistance.

Solution : orange-blue-yellow-gold
x y z tolerance.

$R = xy \times 10^z \pm$ tolerance

$R = 36 \times 10^4 \pm 5\%$

Ex ② : colour code is Brown – Black – Red – Silver, find R

Solution : $R = xy \times 10^z \pm$ tolerance

$R = 10 \times 10^2 \pm 10\%$

Ex ③: colour code is Green-Blue – Black – no colour. find R?

Solution: $R = xy \times 10^z \pm$ tolerance

$R = 56 \times 10^0 \pm 20\%$

Write the colour code for the following resistance value :-

1. $65 \times 10^6 \pm 5\%\ \Omega$
2. $80K \pm 10\%\ \Omega$
3. $150\ \Omega$

Note :

We have $\rho = \frac{m}{ne^2\tau}$ ----→ ②

For a given metal, m, n, and e are constants

$\therefore \quad \rho = \frac{1}{\tau}$ Where τ = relaxation time

As temperature of metal increases, number of collision increases, therefore relaxation time decreases.

i.e $\tau \propto \frac{1}{T}$ or $T \alpha \frac{1}{\tau}$

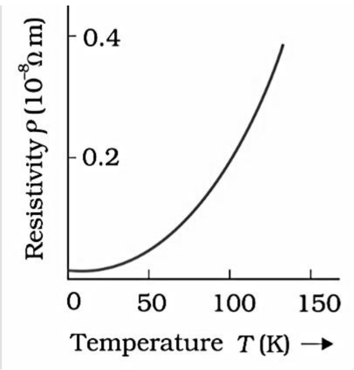
$\therefore\ \rho \propto T$, where T = Temperature

Note: In metals number of free electrons per unit volume does not depend on temperature but in semiconductor and insulator number of free electrons per unit volume depend on temperature.

Temperature dependence of resistivity of metals:

Resistivity of metals is directly proportional absolute temperature ($\therefore\ \rho \propto T$)

The variation of ρ with T is as shown



Resistivity of a conductors increases with increase in temperature. The resistivity of metal is given by

$$\rho_T = \rho_0\left[1+\alpha\left(T-T_0\right)\right] \quad \text{----→ ①}$$

Where, ρ_T = resistivity at temperature T

ρ_0 = resistivity at temperature T_0

α = temperature co-efficient of resistivity

Temperature co-efficient of resistivity of conductor (α of a conductor)

It is defined as the fractional increase in resistivity per unit increase in temperature.

$$\alpha = \frac{\Delta\rho / \rho_0}{\Delta T}$$

$$\alpha = \frac{\Delta\rho}{\rho_0 \Delta T}$$

S.I unit of α is K^{-1},

Practical unit of α is $°C^{-1}$

α is positive for metals

Note:

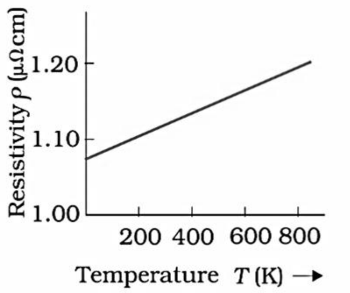
1. we have, $R_T = R_0\left[1+\alpha\left(T-T_0\right)\right]$

   R_T = resistance of conductor at temperature T

   R_0 = resistance of conductor at temperature T_0

   α = temperature co-efficient

2. For some alloy like Nichrome (alloy of Nickel, Chromium and Iron), Manganin and constantan exhibit a weak dependence of resistivity with temperature. That is resistance and resistivity would change very little with temperature. And temperature co-efficient value is very small. Thus, these materials are widely used in wire bound resistors. Since their resistance values would change very little with temperature. The variation of resistivity of alloy with temperature is as shown.

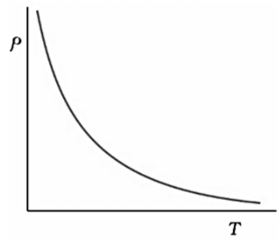


3. In insulator, ρ decreases with T increases.

Temperature dependence of resistivity of Semiconductors:

For semiconductors, resistivity decreases with increase in temperature as shown

i.e. '∝' is negative for semiconductor



In semiconductor as temperature increases, the number of free electrons and holes per unit volume (n) increases.

Therefore 'ρ' decreases $(\because \rho \; \alpha \; \frac{1}{n})$

Note: α values for different materials

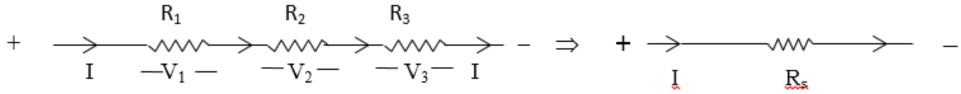
Material	Temperature coefficient of resistivity (α) (per ^0C)
Conductors	
Silver	0.0041
Copper	0.0068
Aluminium	0.0043
Tungsten	0.0045
Iron	0.0065
Platinum	0.0039
Mercury	0.0009
Nichrome (alloy of Ni, Fe, Cr)	0.0004
Manganin (alloy)	0.002×10^{-3}
Semiconductors	
Carbon (graphite)	– 0.0005
Germanium	– 0.05
Silicon	– 0.07

Series combination of resistors: The two or more resistors are said to be in series if only one of their end points is joined such that current through each resistor remains same.

Parallel combination of resistors: The two or more resistors are said to be in parallel if one end of all the resistors is joined together, and similarly the other ends joined together such that potential difference across each resistor is same.

Equivalent resistance: A single resistor which produces same effect as that of set of resistors is called equivalent resistor and its resistance is called equivalent resistance.

Derive an expression for equivalent resistance of three resistors connected in serried



Consider three resistors of resistances R_1, R_2 and R_3 connected in series as shown in diagram. In series combination, current remains same.

Let I = Current flows through R_1, R_2 and R_3,

V_1= Potential difference across R_1, V_2= Potential difference across R_2,

V_3= Potential difference across R_3, V = Potential difference across Combination.

R_s = Equivalent resistance of combination (R_{eq})

∴ In series $V = V_1 + V_2 + V_3$ ------ ①

According to ohm's law V = IR

For 1st resistor, $V_1 = IR_1$,

For 2nd resistor, $V_2 = IR_2$

For 3rd resistor, $V_3 = IR_3$

For combination $V = IR_s$ ∴ eq: ① becomes

$IR_s = IR_1 + IR_2 + IR_3$

$IR_s = I(R_1 + R_2 + R_3)$

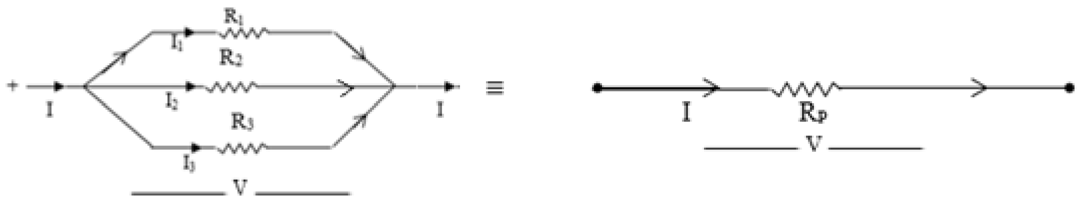
$R_s = R_1 + R_2 + R_3$

This is the expression for equivalent resistance of set of three resistors in series.

For 'n' resistors in series

$R_s = R_1 + R_2 + R_3 + \text{----------} + R_n$

Derive an expression for equivalent resistance of three resistors connected in parallel



Consider three resistors of resistances R_1, R_2, and R_3 connected in parallel as shown in diagram, In parallel, Potential difference across each resistor is same.

Let I_1= Current flows through R_1, I_2= Current flows through R_2

I_3= Current flows through R_3 I = total Current in the combination.

V = Potential difference,

R_p = equivalent resistance in parallel (Req)

In Parallel combination, $I = I_1 + I_2 + I_3$ ---------(1)

According to ohm's law, $I = \frac{V}{R}$

For 1st resistor, $I_1 = \frac{V}{R_1}$,

For 2nd resistor, $I_2 = \frac{V}{R_2}$

For 3rd resistor, $I_3 = \frac{V}{R_3}$

For combination $I = \frac{V}{R_p}$

$\therefore$ equation (1) becomes

$$\frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{V}{R_p} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\therefore \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

This is the expression for equivalent resistance of three resisters connected in parallel

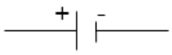
For 'n' number of Resistors in parallel.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{---------} + \frac{1}{R_n}$$

Electric cell:

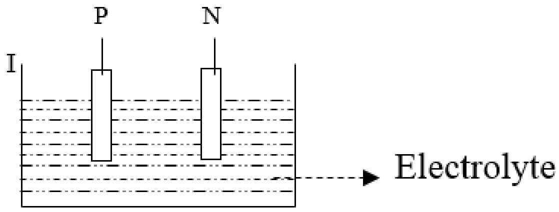
The device used to maintain a steady current in the circuit is called electric cell.

Ex: Electrolytic cell, Daniel cell, Leclanche cell, dry cell.

Electrical symbol of cell is 

EMF of a cell:

Consider an electrolytic cell. It has two electrodes called positive (P) and negative (N) electrodes as shown.



Two electrodes are immersed in electrolytic solution. Now electrodes exchange the charges with the electrolyte.

Let V_+ = potential difference between ‘P’ and electrolyte.

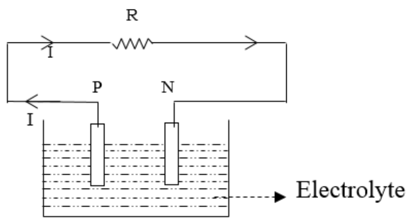
$-V_-$ = Potential difference between N and electrolyte ($\therefore$negative electrode produce negative potential)

$\therefore$ Potential difference between P and N is called emf

i.e. $\text{emf} = V_+ - (-V_-)$

$E = V_+ + V_-$ (For open circuit)

Note-1:



Let, R = external resistance in the circuit,

I=current flows through the circuit,

V=potential difference across R.

but $I = \frac{V}{R}$

If $R = \infty$ then $I = \frac{V}{\infty} = 0$

Circuit is said to be open circuit if no current flows through the cell.

Note-2: circuit is said to be closed circuit if current flows through the cell.

Define emf (E) of a cell : EMF is the potential difference between the positive and negative electrodes of a cell in an open circuit.

The S.I Unit is volt

Note1: Emf is measured during open circuit. i.e. when no current flows through the cell.

Note2: EMF = Electro motive force

Terminal potential difference(TPD): TPD is the potential difference between the positive and negative electrodes of a cell in a closed circuit.

In general TPD is the potential difference across external resistance.

SI unit is volt.

Note: TPD is measured during closed circuit. i.e. When current flows through

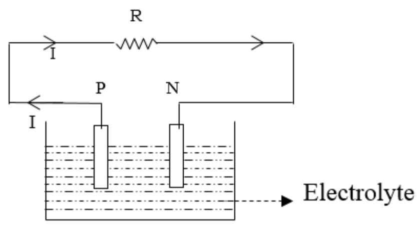
For closed circuit $R \neq \infty$ $\therefore$ $I \neq 0$

Internal resistance of a cell (r) : The resistance of the cell which opposes the flow of current through it is called internal resistance of a cell.

For an ideal cell, r=0.

Note: Internal resistance of dry cell > Internal resistance of electrolytic cell.

Relation between emf and TPD:



Let, R = external resistance in the circuit,

E= emf of a cell

I=current flows through the circuit,

r=internal resistance of a cell.

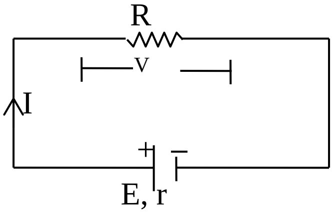
Potential difference across R is, V= IR = Terminal potential difference

Potential difference in cell = Ir

$\therefore$ E=V+Ir

Note: If r=0, then E=V,

Derive an expression for current through simple circuit



Consider a simple circuit

Let, R = external resistance E = emf of a cell

r = internal resistance of a cell V = potential difference across R

I = current flows through R.

For a cell, E=V+Ir --------------- ①

According to ohm's law, V = IR ----------- ②

From eq: ① and ②

E=IR+Ir

E=I(R+r)

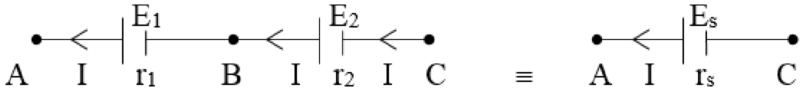
$\therefore \quad I = \frac{E}{R + r}$

This is the expression for current flows through the circuit by the cell

Note:

1. If R = 0, then cell draws maximum current. i.e $I_{max} = \frac{E}{r}$.
2. For closed circuit, V = E – Ir.
   Now, V is called terminal potential difference (terminal p. d)
   Terminal potential difference = potential difference across R.
   Where, R=external resistance.

Obtain the expression for equivalent emf and internal resistance of two cells connected in series.



Consider two cells in series

Let, E_1 and E_2 = emf s of two cells,

r_1 and r_2 = Internal resistances of two cells

E_s = Equivalent emf

r_s = equivalent internal resistance

V_A = potential at a point A

V_B = potential at a point B,

V_c=potential at a point C

Potential difference between A and B is,

$$V_{AB} = V_A - V_B = E_1 - Ir_1 \qquad (\because \; V = E - Ir)$$

Potential difference between B and C is,

$$V_{BC} = V_B - V_C = E_2 - Ir_2$$

The potential difference between A and 'C' of the combination is

$$V_{Ac} = V_A - V_c$$
$$= V_A - V_B + V_B - V_c$$
$$= E_1 - Ir_1 + E_2 - Ir_2$$
$$= E_1 + E_2 - Ir_1 - Ir_2$$
$$V_{AC} = E_1 + E_2 - I(r_1 + r_2) \quad \dashrightarrow \quad (1)$$

but V_{AC} = emf of equivalent cell

$$\therefore V_{AC} = E_S - Ir_s \quad \dashrightarrow \quad (2)$$

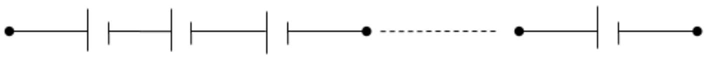
$\therefore$comparing eq (1) and (2)

$E_s = E_1 + E_2$ This is the expression for equivalent emf.

and $r_s = r_1 + r_2$, This is the expression for equivalent internal resistance

Note:

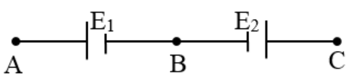
1. For n cells in series



The $E_s = E_1 + E_2 + E_3 + \text{------} E_n$

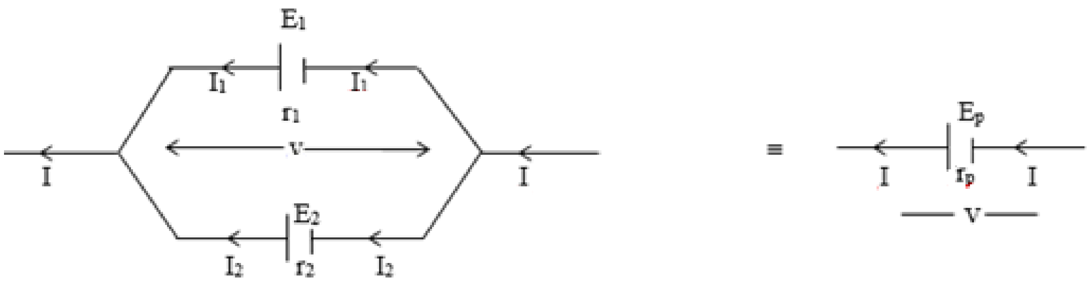
and $r_s = r_1 + r_2 + r_3 + \text{-------} r_n$

2. If negative terminal of 1st cell is connected to –ve of 2nd cell as shown.



then $E_s = E_1 - E_2$ and $r_s = r_1 + r_2$

Obtain the expression for equivalent emf and equivalent internal resistance of two cells connected in parallel



Consider two cells in parallel

Let E_1 and E_2 = emfs of two cells

r_1 and r_2 = internal resistances of two cells

E_p = equivalent emf r_p= equivalent resistance

v = Potential difference across each cell,

I_1 = current flows from 1st cell I_2 = current flows from 2nd cell

I = total current in the combination

In parallel, $I = I_1 + I_2$ ---------- ① W.K.T. $V = E - Ir$

$\therefore \quad Ir = E - V$ or $I = \frac{E - V}{r}$

for 1st cell, $I_1 = \frac{E_1 - V}{r_1}$

V is the same in parallel

for 2nd cell $I_2 = \frac{E_2 - V}{r_2}$ $\therefore$ eq: ① becomes

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$I = \frac{E_1}{r_1} - \frac{V}{r_1} + \frac{E_2}{r_2} - \frac{V}{r_2}$$

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \text{ ---------- ②}$$

but for equivalent cell

$$I = \frac{E_P - V}{R_P} \qquad I = \frac{E_P}{R_P} - V\left(\frac{1}{r_P}\right) \text{ ---------- ③}$$

Comparing equ ③ and ④

$$\frac{E_p}{r_p} = \frac{E_1}{r_1} + \frac{E_2}{r_2} \quad \text{and} \quad \frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2}$$

This is the expression for equivalent emf and internal resistance in parallel.

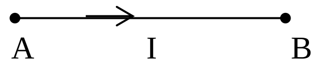
Note 1) For 'n' cells in parallel

$$\frac{E_p}{r_p} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + - - - - - - - + \frac{E_n}{r_n} \quad \textit{and} \quad \frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + - - - - - - + \frac{1}{r_n}$$

Note 2): If negative terminal of second cell is connected to +ve terminal of 1st cell. Then $E_2 \rightarrow -E_2$

i.e. $\frac{E_p}{r_p} = \frac{E_1}{r_1} - \frac{E_2}{r_2}$ and $\frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2}$

Electrical energy:



Consider a conductor AB in which current I flows from A to B.

V_A=Potential at A, V_B=Potential at B. And $V_A > V_B$ ($\because$ I flows from A to B)

Potential difference = $V_A - V_B$

Let Δq = amount of charge flows in Δt time

Then $I = \frac{\Delta q}{\Delta t}$ $\therefore \Delta q = I \Delta t$

Potential energy of a charge at A is $U_A = \Delta q \times V_A$

Potential energy of charge at B is $U_B = \Delta q \times V_B$

$\therefore$ Change in potential energy = Final potential energy – Initial Potential energy

Potential Energy = charge × Potential

$$\Delta U = U_B - U_A$$
$$= \Delta q V_B - \Delta q V_A$$
$$= \Delta q (V_B - V_A)$$
$$= \Delta q(-V) \quad (V_B - V_A = -V \quad \because V_A > V_B)$$
$$= -\Delta q V$$
$$\Delta U = -I \Delta t V$$

But, change in kinetic energy = – change in potential energy

$$\Delta K = -\Delta U$$
$$\Delta K = -(-I\Delta t V)$$
$$\Delta K = I\Delta t V$$

i.e The charges gain kinetic energy when they move through the conductor. But during motion, they collide with fixed ions and atoms of the conductor. There fore they lose kinetic energy. This energy is dissipated as heat in the conductor

i.e Energy dissipated = Loss of kinetic energy.

$$\Delta w = IV\Delta t$$

Electrical power: The energy dissipated per unit time is called electrical power

i.e. electrical power = $\frac{\text{energy dissipated}}{\text{time taken}}$

$$P = \frac{\Delta w}{\Delta t}$$

$$= \frac{IV\Delta t}{\Delta t}$$

$$P = IV$$

S.I unit is power is watt or J/s

Note:

1. We have $P = IV$

   But $V = IR$ ∴ $P = I^2 R$

   and $I = \frac{V}{R}$ ∴ $P = \frac{V^2}{R^2} \times R$

   $$P = \frac{V^2}{R}$$

   i.e $P = IV = I^2 R = \frac{V^2}{R}$
2. Energy dissipated in the external resistance is given by chemical energy of the electrolytic cell or dry cell. i.e. cell supplies the power.
3. **Transmission of electrical power through transmission line**

   Let P= power to be transmitted

   P_c = power loss in cable

   I = current flows through the cable

   then R_c = resistance of the cable

   V = voltage across cable, but $P=IV$, $I = \frac{p}{V}$

The power dissipated in the cable wire (Transmission line is)

$$P_C = I^2 R_C \quad \text{or} \quad P_C = \frac{P^2 R_c}{V^2} \quad \text{i.e } P_C \propto \frac{1}{V^2}$$

i.e Power loss in transmitted cable is inversely proportional to square of voltage.

Hence, to minimise the power loss, high voltage is provided in the transmission lines.

Electrical network: The different combinations of resistors and cells is called electrical network or circuit.

Node: The point in an electrical network at which the current finds more than two branches is called node or junction.

Mesh: The closed path for the flow of current in the circuit is called mesh or loop.

Kirchhoff's rules:

Junction rule (Kirchoff's current law)

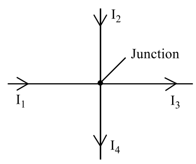
It states that "at any junction of circuit, the sum of the currents entering the junction is equal to the sum of currents leaving the junction.

i.e, at the junction, total current entering = total current leaving.

Let, I_1 and I_2 = Entering currents

I_3 and I_4= Leaving currents

$\therefore \quad I_1+I_2=I_3+I_4$

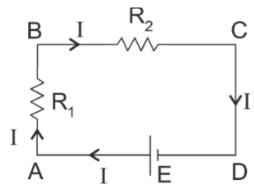


Note:

1. According to Junction rule, there is no accumulation of charges at the junction.
2. Junction rule is consequence of conservation of charge.

Loop rule (Kirchhoff's voltage law):

It states that "the algebraic sum of changes in the potential around any closed loop involving resistors and cells is zero"



Applying Kirchhoff's loop rule to the loop ABCDA,

$$-IR_1 - IR_2 + E = 0$$

$$\sum IR = 0$$

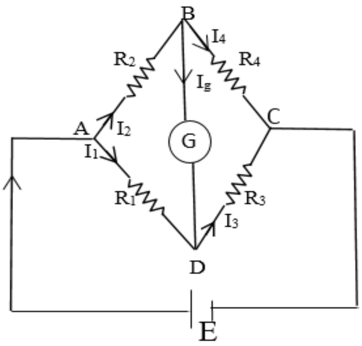
Note:

1. In a closed loop, we come back to the initial point. There fore total change in the potential must be zero.
2. Loop rule is consequence of conservation of energy.

Note: Sign Convention for applying loop rule

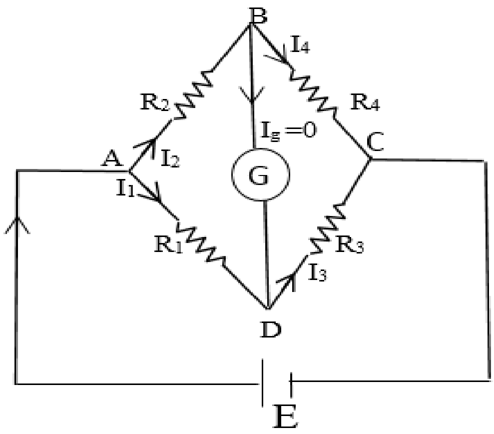
1. IR product is negative if we move along the direction of current and positive if we move opposite to the direction of current.
2. Emf of a cell is positive if potential raises (move from N to P) and emf is negative if potential drops (P to N) (N=negative terminal, P = positive terminal)

Wheatstone bridge: It is an electrical network in which four resistors are connected in the form of quadrilateral. A cell is connected between the diagonally opposite points. A galvanometer is connected between other two opposite points as shown.



AC line = Source arm (∵cell is connected).
BD line = galvanometer arm
I_g = current through galvanometer,
for balanced bridge, $I_g = 0$
then galvanometer shows null deflection.

Obtain the balance condition of bridge for null deflection of galvanometer:



Consider a Wheatstone bridge
Let, R_1, R_2 R_3 and R_4 are four resistances,
I_1, I_2, I_3 and I_4 are the branch currents,
I_g=Current through galvanometer,
G = resistance of galvanometer,
for null deflection, $I_g = 0$
applying Kirchhoff' junction rule at B and D
then, $I_2 = I_4$ and $I_1 = I_3$
Applying Kirchhoff's loop rule to the closed loop A D B A

$- I_1 R_1 + 0 \times G + I_2 R_2 = 0$ ($I_g = 0$)

$- I_1 R_1 + I_2 R_2 = 0$

$I_1 R_1 = I_2 R_2$ $\frac{I_1}{I_2} = \frac{R_2}{R_1}$ ----→ ①

For closed loop C B D C

$I_4 R_4 - 0 \times G - I_3 R_3 = 0$ ($I_g = 0$)

$I_3 R_3 = I_4 R_4$ $\therefore \frac{I_3}{I_4} = \frac{R_4}{R_3}$ But $I_3 = I_1$ and $I_4 = I_2$ $\therefore \frac{I_1}{I_2} = \frac{R_4}{R_3}$ ----→ ②

from eq: ① and ②

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

This is the balance condition of Wheatstone bridge.

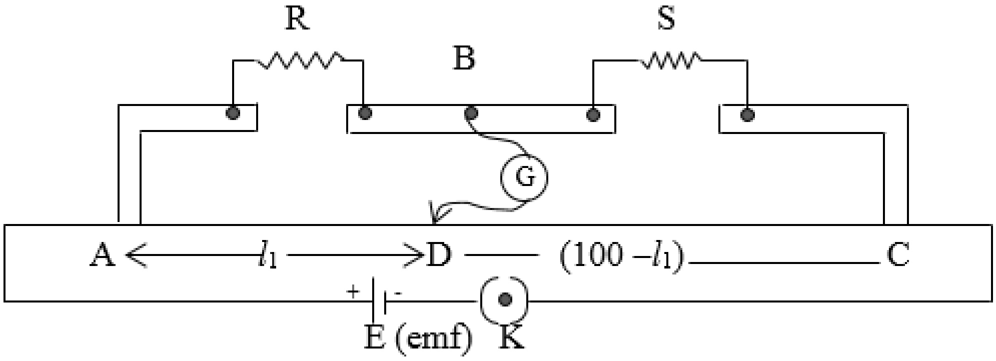
Note:

We have $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ If R_4 is unknown then, $R_4 = \frac{R_2}{R_1} \times R_3$

$\therefore$ R_4 can be found.

$\therefore$ Wheatstone bridge is used to find the unknown resistance.

Metre bridge: It is a device works on the principle of balanced Wheatstone bridge.



The metre bridge is as shown in figure. It consists of 1m wire clamped between A and C. The metallic strip has two gaps. Two resistances R and S are connected across two gaps. One end of galvanometer is connected to the point B, and other end is connected to a jockey. The jockey can slide over the wire to make electrical connection.

R is an unknown resistance and 'S' is known resistance. Slide the jockey from A till the galvanometer shows zero deflection. Say at a point 'D' the galvanometer shows zero deflection. The portion AD of wire has a resistance Kl and portion DC of wire has a resistance $K(100-l)$ where, k= resistance of wire per unit cm. l=Balancing length.

The four arms AB, BC, DA and CD form a Wheatstone bridge

$\therefore$AB = R, BC =S, DA =Kl and CD = $K(100-l)$,

at balance condition, $$\frac{R}{S} = \frac{Kl}{K(100-l)}$$

$$\frac{R}{S} = \frac{l}{(100-l)}$$ where l is in cm.

Note-1: Meter bridge is used to find value of unknown resistance.

Note-2: The length of meter bridge wire at which galvanometer shows zero deflection is called balancing length.

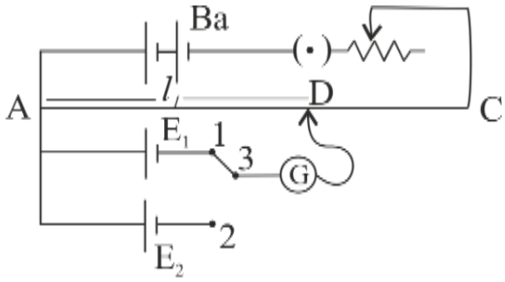
Potentiometer: It is a device used to compare the emfs of two cells.

The potentiometer has the advantage that it draws no current from the cell whose emf is being measured.

Applications of potentiometer:

1) It is used to find the unknown emf of the cell (To compare ems of two cells)
2) It is used to find internal resistance of a cell

To find the unknown emf of the cell (To compare emf of two cells)



Potentiometer consists of long uniform wire 'AC'. The two cells of emfs E_1 and E_2 are connected as shown .

The points 1 and 3 are connected and now the jockey is moved along the wire till the galvanometer shows zero deflection (say at 'D')

Now apply Kirchhoff's loop rule to the closed loop ADG31A,

$-\phi l_1 + 0 + E_1 = 0$

$\therefore\ E_1 = \phi\, l_1$

Similarly, when the points 2 & 3 are connected,

Then, $E_2 = \phi\, l_2$ Where, l_1 and l_2 are balancing lengths.

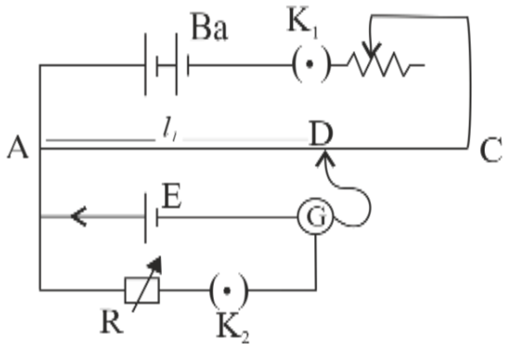
Eq: ① / ②

$$\frac{E_1}{E_2} = \frac{\phi l_1}{\phi l_2}$$

$$\therefore \frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If E_2 is known, then unknown E_1 can be calculated

To find internal resistance of a cell



The circuit connections are made as shown

Let, E = emf of a cell

r = internal resistance of cell is to be determined, K_1 and K_2 = plug keys,

R = External resistance

When K_2 is opened,

Then $E = \phi\, l_1$ -------- ①

Where, ϕ =Potential drop per unit length

l_1 = AD = balancing length at which galvanometer shows zero deflection.

When, K_2 is closed, then cell sends current through resistance 'R' and balancing length is l_2

Then, $V = \phi\, l_2$ -------► ② Where, V = terminal potential difference

$\therefore$ eq: ①/ ②, $\frac{E}{V} = \frac{\phi l_1}{\phi l_2}$

$$\frac{E}{V} = \frac{l_1}{l_2} \text{ -------► ③}$$

But, V = IR and E=V+Ir =IR+Ir = I(R+r)

$\therefore$ eq: ③ becomes

$$\frac{r}{R} = \frac{l_1}{l_2} - 1$$

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

This is the expression for internal resistance.

$$\frac{I(R+r)}{IR} = \frac{l_1}{l_2}$$

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$1 + \frac{r}{R} = \frac{l_1}{l_2}$$

$$\frac{r}{R} = \frac{l_1}{l_2} - 1$$

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

One Marks Questions:

1. Define mobility of electron. (March-2014, March-2017)
2. Define drift velocity of electrons (July-2014)
3. A resistor is marked with colours red, red, orange and gold. Write the value of its resistance.(March-2015)
4. What is the condition for the balanced state of wheat stone's network? (July-2015)
5. What is current Electricity?
6. What is electric current? or What is strength of current?
7. What is instantaneous current?
8. What is current density.
9. Define resistance of a conductor.
10. Define 'ohm'

11. Mention the factors on which the resistance of a conductor depends.
12. How the resistance of a conductor does very with its length and area of cross section?
13. Define resistivity of a conductor?
14. How does the resistivity of a conductor vary with temperature?
15. How does the resistivity of a semiconductor vary with temperature?
16. Define conductance of a conductor.
17. Define conductivity of a conductor.
18. Mention the relation between resistance and conductance.
19. Define drift velocity.
20. Define relaxation time.
21. Define mobility of free electron.
22. What is potential difference in a circuit?
23. What is emf of a cell?
24. What is internal resistance of cell?
25. What is terminal potential difference?
26. When does the terminal potential difference become equal to e.m.f of cell.
27. What is equivalent resistor?
28. What is equivalent resistance?
29. When is the resistors are said to be connected in series?
30. When is the resistors are said to be connected in parallel?
31. When is the cells are said to be connected in series?
32. When is the cells are said to be connected in parallel?
33. What is electrical energy?
34. Mention formula for electrical energy dissipated in a resistor.
35. What is electrical power?
36. Mention the expression for a power dissipated in a resistor.
37. Mention the expression for a power dissipated in a circuit.
38. What is power loss?
39. What is junction in an electrical mesh.
40. What is the condition for balance of Wheatstone bridge?
41. What is metre bridge?
42. What is the principle of metre bridge?
43. What is potentiometer?
44. What is the principle of potentiometer.

Two Marks Questions:

1. State and explain Ohm's law. (March-2017)
2. Name the factors on which the resistance of the conductor depends upon.
3. On what factors does resistivity of a conductor depends?
4. Write the limitations of Ohm's. (July-2014) (March-2015) (July-2015)
5. What are ohmic and non-ohmic device? Give an example for each.
6. Represent graphically.
   i) The variation of resistivity of a metallic conductor with temperature.
   ii) The variation of resistivity of a semiconductors with temperature.
7. On what factors does the internal resistance depends?

8. State and explain Kirchhoff's current law.
9. State Kirchhoff's voltage law.
10. Draw Wheatstone's bridge circuit and write the condition for the balance. (March-2014)
11. Mention the applications of potentiometer.

Three Marks Questions:

1. Establish the relation between drift velocity of electrons and electric field. (March-2016)(July-2016)
2. Distinguish between resistance and resistivity.
3. Distinguish between emf and p.d.
4. Derive an expression for current through a simple circuit.
5. Distinguish between series and parallel combinations of resistors.
6. Explain construction and working of metre bridge with a neat diagram.
7. With a circuit diagram briefly explain how to compare emf of two cells.
8. With a circuit diagram explain how internal resistance can be determined.
9. Derive the relation between current density and conductivity. (or) Derive $\vec{J} = \sigma\vec{E}$

Five Marks Questions:

1. Derive an expression for conductivity of a conductor in terms of relaxation time. (July-2015) (Or) Derive $\sigma = \frac{ne^2\tau}{m}$
2. Derive an expression for current drawn by an external resistance and terminal potential difference.
3. Derive an expression for equivalent/ effective resistance of two resistors are connected in series.
4. Derive an expression for equivalent/ effective resistance of two resistors are connected in parallel. (March-2014, March-2015)
5. Distinguish between series and parallel combination of resistors.
6. Two cells of different emf's and different internal resistance are connected in series. Find the expressions for the equivalent emf and equivalent internal resistance of the combination.
7. Two cells of different emf's and different internal resistances are connected in parallel. Find the expression for the equivalent emf and equivalent internal resistance of the combination.
8. State and explain Kirchhoff's rule of network.
9. Deduce the condition for balance of a wheat stone bridge using Kirchhoff's laws. (March-2010, July-2016, March-2017).

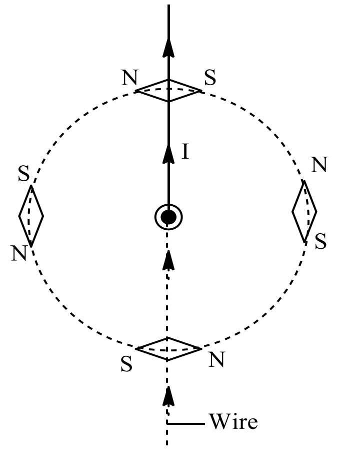
Chapter-4: MOVING CHARGES AND MAGNETISM

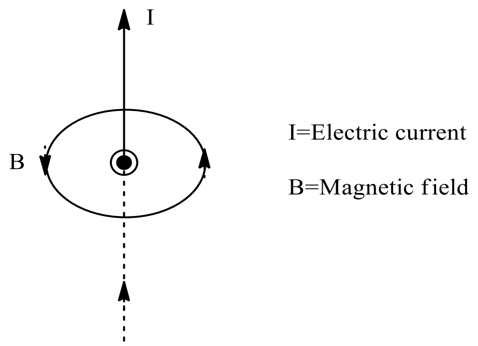
Introduction:

A moving charge or current produces a magnetic field in the surrounding space. It was discovered by H.C. Oersted.

Oersted observed that a magnetic compass needle is deflected when it is kept near a wire carrying an electric current.

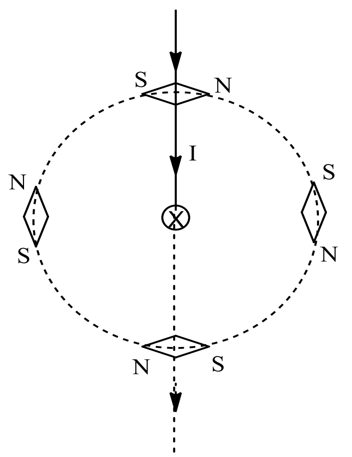
Consider a wire (conductor) which is perpendicular to plane of paper. When a current emerges out of the plane of paper, then orientation of magnetic needle is as shown.

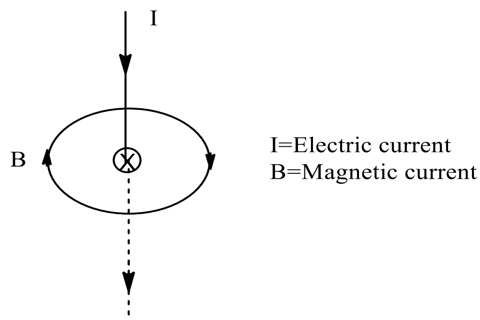




(The needle is aligned tangential to the imaginary circle as shown)

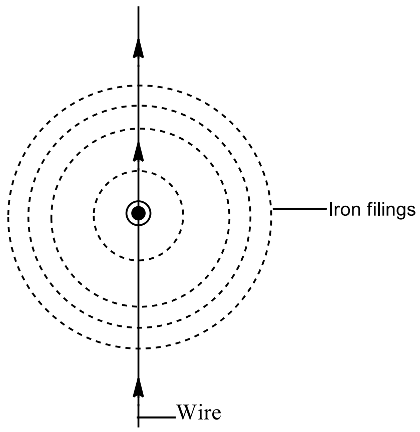
When a current moves into the plane of paper, (i.e. current direction reverses) then orientation of needle also reverses as shown.





Note:

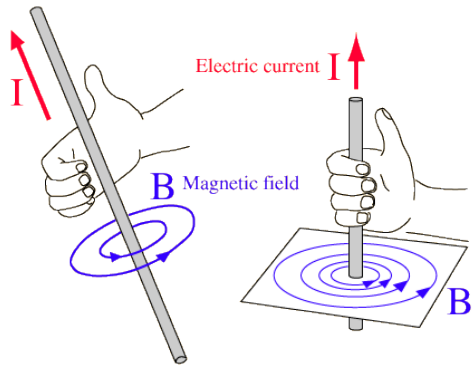
1. When iron filings are sprinkled around the current carrying wire, they arrange themselves in concentric circles with the wire as the axis.
2. The deflection of needle increases on increasing the current or bringing the needle close to the wire.
3. The laws obeyed by electricity ad magnetism were unified by James Maxwell and they together called Electro-magnetism. (In 1864)
4. In this Chapter, we adopt the following convention (i) A current or field (electric or magnetic) emerging out of the plane of paper is depicted by a dot [⊙]. (ii) A current or field going into the plane of paper is depicted by a cross [⊗].



Rules to determine the direction of magnetic field:

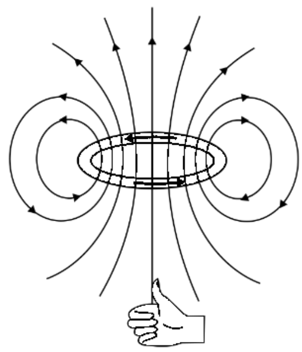
1. Right hand thumb rule to determine the direction of field due to long wire:

It states that "grasp the wire in right hand such that thumb in the direction of the current, then the fingers curl around the wire gives the direction of magnetic field.



2. Right hand thumb rule for circular wire:

It states that "curl the palm of right hand around the circular wire with the fingers pointing in the direction of the current. Then thumb gives the direction of magnetic field.



3. Maxwell's right handed screw rule:

It states that "if right handed screw is rotated such that the tip advances, the direction of the tip gives the direction of electric current, and the direction in which the head of the screw is rotated gives the direction of magnetic field."

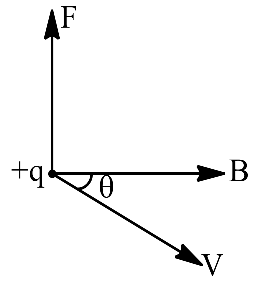
Note-1: Electric field $(\vec{E})$ can convey energy and momentum.

$\vec{E} = \frac{\vec{F}}{q}$ (or) $\vec{F} = q\vec{E}$ Where, q=charge, $\vec{F} =$ force.

Note-2: A static charge produces only the electric field $\vec{E}$. But a moving charge or current produces both electric field $(\vec{E})$ and magnetic field $(\vec{B})$. Magnetic field is a vector. It varies with distance (r). $\therefore$ It is denoted by $\vec{B}(r)$.

Magnetic force on moving charge:

Consider a charge 'q' moving in an external magnetic field $(\vec{B})$



Magnetic force on charge is given by

$$\vec{F} = q\left[\vec{v} \times \vec{B}\right]$$

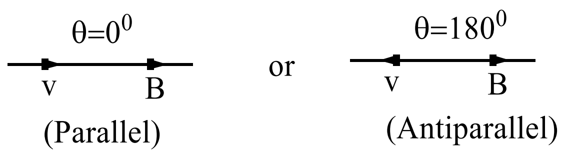
$$\vec{F} = q\left[vB \sin\theta \, \hat{n}\right]$$

$$\therefore \quad \boxed{\vec{F} = qvB \sin\theta \, \hat{n}} \qquad \left[\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}\right]$$

Where, q=charge on a particle, v=velocity of charged particle
B=magnetic field, θ=angle between v and B
$\hat{n}$ = unit vector which gives the direction of force.

Features of magnetic force:

1. We have F=qvB sinθ (considering magnitude only), Magnetic force depends on q, v, B and θ.

2. 

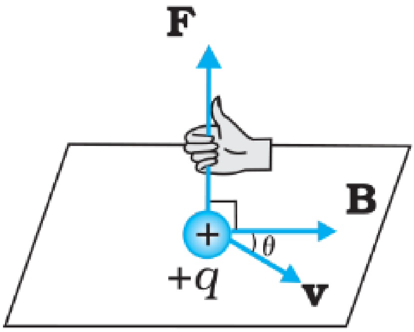
If $\theta=0^0$ or $\theta=180^0$ Then $\sin\theta=\sin 0^0=\sin 180^0=0$

$\therefore$ F=0 i.e. Magnetic force is zero

If the charge moves parallel or antiparallel to magnetic field, then force is zero.

3. The magnetic force is zero if charge is at rest. (F=0, If v=0)

4. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$ and its direction is given by Screw rule or right hand rule for vector product.

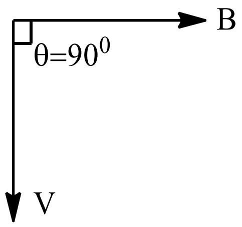


Note: Force on negative charge is opposite to that on a positive charge.

Magnetic field [B]:

We have F=qvBsinθ [considering magnitude only]

$$\therefore \quad B = \frac{F}{qv\sin\theta}$$



If $\theta=90^0$, $\sin 90^0=1$

Then, $B = \frac{F}{qv}$

Magnetic field is defined as the force acting on unit charge moving perpendicular to field with unit velocity.

S.I. unit of magnetic field is tesla (T).

Define one tesla:

W.K.T. $B = \frac{F}{qv\sin\theta}$

If, F=1N, q=1C, $v=1ms^{-1}$ and $\theta=90^0$

Then, $B = \frac{1N}{1C \times 1ms^{-1}}$

B=1 tesla

The magnetic field is said to be one tesla if one newton force acting on one coulomb charge moving perpendicular to field with a velocity $1ms^{-1}$.

Note-1: A smaller unit of magnetic field is gauss (G), $1G=10^{-4}T$.

Note-2:

Physical situation	Magnitude of B (in tesla)
Surface of a neutron star	10^{8}
Typical large field in a laboratory	1
Near a small bar magnet	10^{-2}
On the earth's surface	10^{-5}
Human nerve fibre	10^{-10}
Interstellar space	10^{-12}

Lorentz force:

Consider a point charge 'q' moving with velocity 'v' in the presence of external electric field $\vec{E}$ and magnetic field $\vec{B}$.

The force acting on 'q' due to $\vec{E}$ is

$$\vec{F}_{electric} = q\vec{E}$$

The force acting on 'q' due to $\vec{B}$ is,

$$\vec{F}_{magnetic} = q\left[\vec{v}\times\vec{B}\right]$$

$\therefore$ Total force acting on 'q' is,

$$\vec{F} = \vec{F}_{electric} + \vec{F}_{magnetic}$$
$$\vec{F} = q\vec{E} + q\left[\vec{v}\times\vec{B}\right]$$
$$\vec{F} = q\left[\vec{E} + \left(\vec{v}\times\vec{B}\right)\right]$$

This force is given by H.A. Lorentz so it is called Lorentz force.

Note: $\vec{F} = q\left[\vec{E}(r) + \left(\vec{v}\times\vec{B}(r)\right)\right]$

$\vec{E}(r)$ and $\vec{B}(r)$ means electric field (E) and magnetic field (B) depends on distance (r)

Magnetic force on a current carrying conductor:

Consider a straight rod carrying current kept in an external magnetic field 'B'.

Let, L = Length of rod, I = Current flows

A= Cross sectional area of rod, volume = AL

V_d=drift velocity

n = Number of free electrons per unit volume.

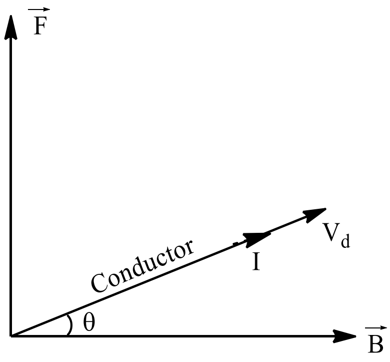
∴ Total free electrons = n × Volume

Total free electrons = n × AL

Total charge = n AL×e

q=neAL,

e=charge on electron



∴Magnetic force on a rod is

$$\vec{F} = q\left(\vec{v}_d \times \vec{B}\right) \qquad F = q\left(\vec{v} \times \vec{B}\right)$$

$$\vec{F} = neAL\left(\vec{v}_d \times \vec{B}\right)$$

$$\vec{F} = neAv_d\left(\vec{L} \times \vec{B}\right)$$

$$\vec{F} = I\left(\vec{L} \times \vec{B}\right) \qquad \left(\because I = neAv_d\right)$$

Note:

1. Force acting on current carrying conductor in vector form

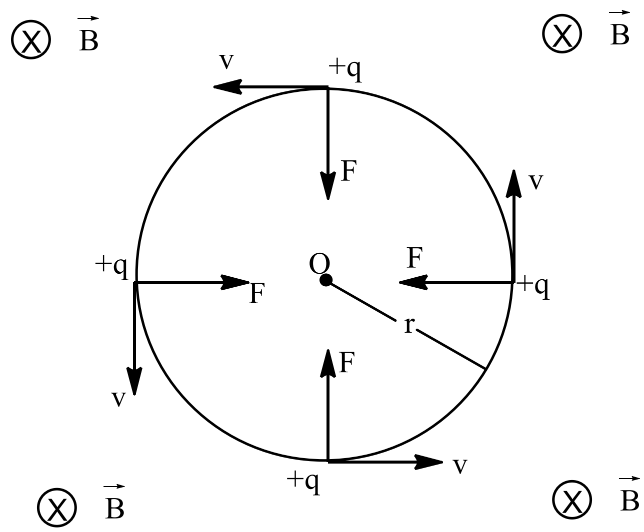
$$\vec{F} = I\left(\vec{L} \times \vec{B}\right)$$

$$\vec{F} = ILB\ \sin\theta\ \hat{n}$$

Magnitude of force is F=ILB sinθ where, $I=neAv_d$

2. If the conductor has number of strips of length 'dL'. Then $F = \sum I\left(\overrightarrow{dL} \times \vec{B}\right)$

Motion of a charged particle in an external magnetic field when velocity of charged particle is perpendicular to magnetic field: (Derive an expression for radius of a circle described by the charged particle):



When a charged particle enters perpendicular to magnetic field. It describe a circular path. Because a magnetic force is acted on a charge like a centripetal force.

The magnetic force is

$F = qvB \sin\theta$ but $\theta = 90^0$, $\sin 90^0 = 1$

$\therefore$ $F = qvB$

For circular motion,

Magnetic force =Centripetal force

$$qvB = \frac{mv^2}{r} \qquad \left(\text{But, Centripetal force} = \frac{mv^2}{r}\right)$$

$$qB = \frac{mv}{r}$$

$$r = \frac{mv}{qB}$$

Where, r = radius of the circular path, m = mass of a charged particle
q= charge on a particle, v= velocity of a charged particle,
B= magnetic field

Note:

1. We have, $r = \frac{mv}{qB}$

   $\therefore \quad v = \frac{qBr}{m}$, Where, v=velocity of a charged particle.

2. We have v= r ω, where, ω=angular frequency

$$\therefore \quad \omega = \frac{v}{r} = \frac{qBr/m}{r}$$

$$\omega = \frac{qB}{m}$$. This is the expression for angular frequency.

Angular frequency (ω) is independent of velocity and energy of a charged particle.

3. $T = \frac{2\pi}{\omega}$ $\left(\because \omega = \frac{2\pi}{T}\right)$

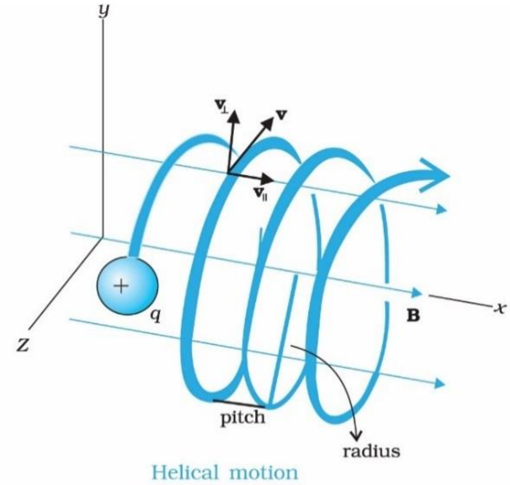
$\therefore T = \frac{2\pi m}{qB}$. This is the expression for time period of revolution.

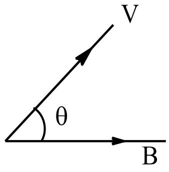
4. We know that, $\nu = \frac{1}{T}$, where $\nu = \text{frequency}$.

$$\therefore \quad \nu = \frac{qB}{2\pi m}$$ This is the expression for frequency of revolution.

5. w.k.t. **ω=2πγ,** Where, γ=frequency, ω=angular frequency

6. If the charged particle enters the field with certain angle [$\theta \neq 0^0$, $\theta \neq 180^0$, $\theta \neq 90^0$], then the charged particle describes helical motion.





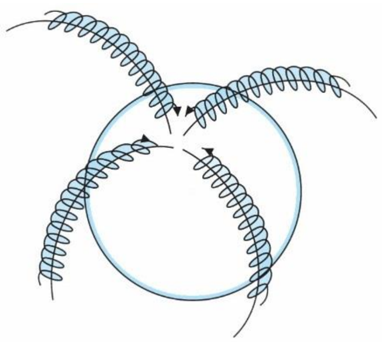
Let v_H =Component of velocity parallel to B,

v_T =Component of velocity perpendicular to B

The distance moved along the magnetic field in one rotation is called **pitch** (P), but $v_H = \frac{P}{T}$ T=time period

$\therefore$ $P = V_H T$, but $T = \frac{2\pi m}{qB}$, $\therefore$ $P = \frac{2\pi m v_H}{qB}$

7. The radius of circular component of helical motion is called the radius of helix.
8. In polar regions like Alaska and Northern Canada, a splendid display of colours is seen in the sky. This is called Aurora Boriolis. This can be explained on the basis of helical motion of charged particles like electrons and protons near the polar region.

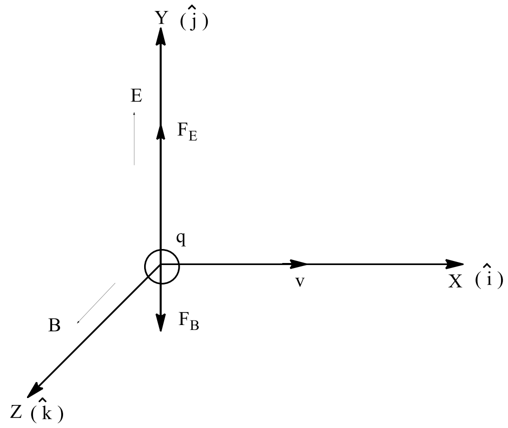
   During a solar flare, a large number of electrons and protons are ejected from the sun. Some of them get trapped in the earth's magnetic field and move in helical paths along the field lines. The field lines come closer to each other near the magnetic poles; see figure. Hence the density of charges increases near the poles. These particles collide with atoms and molecules of the atmosphere. Excited oxygen atoms emit green light and excited nitrogen atoms emits pink light. This phenomenon is called *Aurora Boriolis* in physics.

Motion of charge in combined electric and magnetic fields (Crossed fields) [Velocity selector]:

Let $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields acting perpendicular to each other.

Let, v=velocity of charge 'q' and it is perpendicular to both E and B.

i.e. $\vec{v} = v\hat{i},\ \vec{E} = E\hat{j},\ \vec{B} = B\hat{k}$



$\therefore$ Electrical force on charge is

$\vec{F}_E = qE\hat{j}$ along +ve y-axis

Magnetic force on charge is

$\vec{F}_B = q\left[v\hat{i} \times B\hat{k}\right]$ $\left(\because \vec{F}_B = q\left(\vec{v} \times \vec{B}\right)\right)$

$\vec{F}_B = qvB\left[\hat{i} \times \hat{k}\right]$ $\left(\hat{i} \times \hat{k} = -\hat{j}\right)$

$\vec{F}_B = qvB\left[-j\right]$

$\vec{F}_B = -qvB\hat{j}$ along –ve y-direction

$\therefore$ F_E and F_B are acting on a charge 'q' in opposite direction.

If $F_B = F_E$

i.e. $qvB = qE$

$\therefore$ $v = \frac{E}{B}$, This is the velocity selector.

At this condition net force on charge is zero. Therefore the charge will move in the crossed fields undeflected (without deflection).

Note:

1. If E and B are perpendicular to each other then they are called crossed fields.
2. The condition, $v = \frac{E}{B}$ can be used to select charged particles of a particular velocity out of a beam containing charges moving with different velocities.
3. In crossed fields by adjusting the fields E and B, we can get $F_B = F_E$.
   $\therefore$ E and B can be used as a velocity selector.
4. Only the charged particles with speed, $v = \frac{E}{B}$, pass undeflected in crossed fields. This method was employed by J.J. Thomson to measure the "Charge to mass ratio" of electron.
5. This principle also employed in mass spectrometer.

6. Mass-spectrometer is a device that separates charged particle according to their "charge to mass ratio."

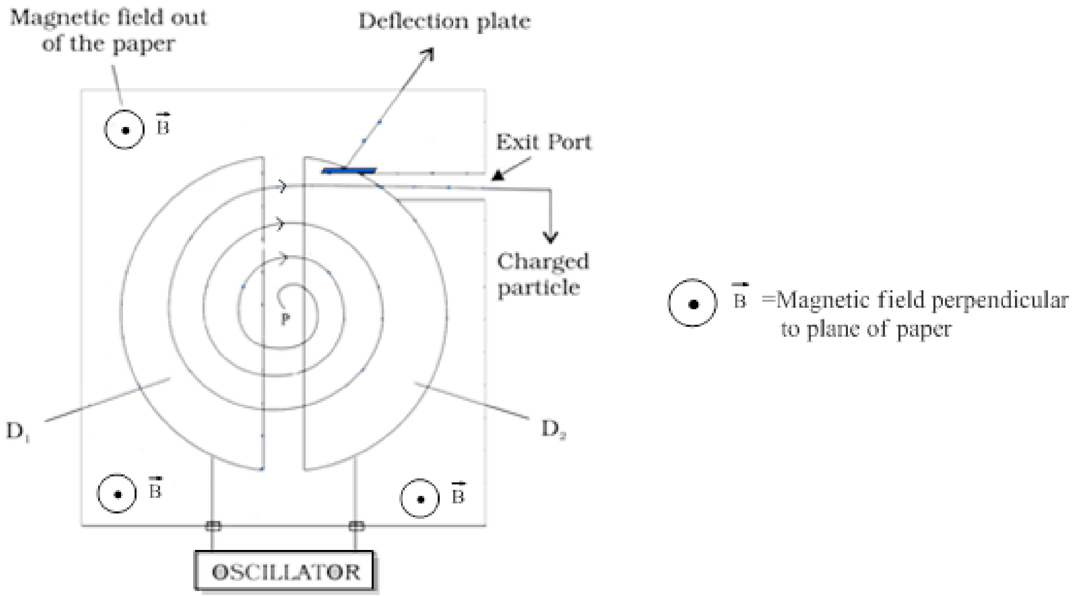
Cyclotron: Cyclotron is a machine used to accelerate positively charged particles or ions to get high energies.

It was invented by E.O. Lawerence and M.S. Livingston.

Principle of cyclotron: The frequency of revolution of the charged particle in a magnetic field is independent of its energy.

Construction and working of cyclotron:

The schematic diagram of cyclotron is as shown. It consists of two semicircular disc like metal containers 'D_1' and 'D_2'. They are called dees, because they look like the letter 'D'. The whole assembly is evacuated to minimise collisions between the charged particles and air molecules. A high frequency alternating voltage (oscillator) is applied to the dees D_1 and D_2. The positive ions or protons are released at the centre 'P'. The electric field 'E' and magnetic field B are applied, perpendicular to each other, so they are called crossed fields. Inside the dees the electric field is absent but the magnetic field is present. The electric field is present between the gap of D_1 and D_2.



Working (Thoery): Positive ion or proton is produced at 'P', if at that time, D_1 is at higher potential (positive potential) and D_2 is at lower potential (negative potential), then ion will be accelerated towards D_2. The ion inside the D_2 moves with constant speed $[\because E = 0]$. But due to perpendicular magnetic field, the ion will describe semicircular path in D_2 in a time interval $\frac{T}{2}$.

but, $T = \frac{2\pi m}{qB}$

Where, T=Period of revolution of ion, m= mass of ion, q= charge of ion

and, $\gamma_c = \frac{1}{T}$ Where, γ_c=cyclotron frequency (frequency of ion)

$\therefore \quad \gamma_c = \frac{qB}{2\pi m}$

The frequency of applied voltage (γ_a) is adjusted so that when the ions arrive at the edge of D_2, the D_1 is at lower potential. It is possible if,

$\gamma_a = \gamma_c$.

This is called resonance condition.

Now the ions are accelerated across the gap of D_2 and D_1 and ions enter into D_1 with greater speed. As a result, energy of ions increases. Therefore radius of their path increases.

The maximum velocity of ions is given by

$v = \frac{qBR}{m}$ Where, R=radius of dees=radius of last circular orbit.

$\therefore$ Kinetic energy of the ion is given by

$$E_K = \frac{1}{2}mv^2$$

$$E_k = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2$$

$$E_k = \frac{q^2B^2R^2}{2m}$$

W.K.T.

$r = \frac{mv}{qB}$

$v = \frac{qBr}{m}$

$v = \frac{qBR}{m} \quad (\because r = R)$

Note:

1. The operation of cyclotron is based on the fact that the time for one revolution of ion is independent of its speed and radius of orbit.
2. In cyclotron, the positive ions or positively charged particles like proton, α-particles are accelerated.

Uses of cyclotron: It is used

1. It is used to implant ions into solid to modify its properties and synthesise new materials.
2. To investigate nuclear structure.
3. To produce radio active substances which are used in diagnosis and treatment in medical field.

Note:

1. Electric permittivity (ε) is a physical quantity that describes how an electric field affects the dielectric medium. It is determined by the ability of a material to polarise in the presence of electric field.

   $$\varepsilon=\varepsilon_0 K,$$

   Where, ε_0=permittivity of free space,
   K=dielectric constant (relative permittivity of medium)
2. Magnetic permeability (μ) is the ability of a substance to acquire magnetism in the presence of applied magnetic field. It is a measure of the extent to which the magnetic field can penetrate the substance.
3. An infinitesimal (very small) element of conductor carrying a current is called current element.

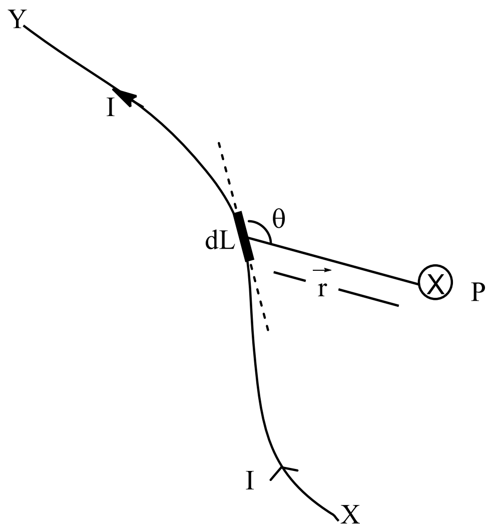
   Current element = IdL.
   I=current, dL=length of element
   Current element is a vector. S.I. unit is ampere. metre (Am)

Biot-Savart law (Expression for magnetic field due to a current element):
Consider a finite conductor 'XY' carrying a current I. consider a current element of length dL.

Let, P=the point at a distance 'r'.
$\vec{r}$ = Displacement vector $\left(\vec{r} = r\,\hat{r}\right)$
θ=angle between $\overrightarrow{dL}$ and $\vec{r}$.
dB=magnetic field at a point 'P'

According to Biot-Savart law, the magnitude of magnetic field at a point produced by a current element is

1. directly proportional to current (i.e. $dB \propto I$).
2. directly proportional to length of element ($dB \propto dL$)
3. directly proportional to $\sin\theta$ ($dB \propto \sin\theta$).
4. inversely proportional to square of the distance $\left(dB \propto \frac{1}{r^2}\right)$.



$$\therefore \quad dB \propto \frac{IdL\sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \times \frac{IdL\sin\theta}{r^2}$$

Where, μ_0= permeability of free space (vacuum)

Note:

1. Biot-Savart law in vector form, is

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{IdL\sin\theta}{r^2} \times \hat{r}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{IdL\sin\theta}{r^2} \times \frac{r\,\hat{r}}{r}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{I\left(dLr\sin\theta\right)\vec{r}}{r^3}$$

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \times \frac{I\left(\overrightarrow{dL} \times \vec{r}\right)}{r^3}$$

Where, $\overrightarrow{dL} \times \vec{r} = dLr\sin\theta\hat{r}$, $\hat{r} =$ unit vector.

2. Direction of magnetic field is perpendicular to plane containing $\overrightarrow{dL}$ and $\vec{r}$. It is given by right hand screw rule.

3. $\frac{\mu_0}{4\pi} = 10^{-7}\ Tm/A$, T=tesla, m=metre, A=Ampere.

$\mu_0 = 4\pi \times 10^{-7}\ Tm/A$

Similarities and differences between Biot-Savart law (in magnetic field) and Coulomb's law (in electric field):

1. Both are long range, since both depend inversely on the square of distance from the source to the point.
2. Principle of superposition applies for both laws.
3. The electric field is produced by a scalar source, namely the electric charge. The magnetic field is produced by a vector source. $I\overrightarrow{dL}$ (current element).
4. The electric field is along the displacement vector joining the source and the point. The magnetic field is perpendicular to the plane containing displacement vector $(\vec{r})$ and current element (IdL).
5. There is an angle dependence in Biot-Savart law which is not present in Coulomb's law.

   If $\theta=0^0$, $\sin\theta=0$, $\therefore dB=0$.

   i.e. magnetic field at a point in the direction of $\overrightarrow{dL}$ is zero (along the $\overrightarrow{dL}$ is zero).

Note: Relation between ε_0, μ_0 and C **(Speed of light):**

Consider, $\varepsilon_0\mu_0 = \varepsilon_0 \times 4\pi\times10^{-7} = 4\pi\varepsilon_0 \times 10^{-7} \quad (\because \mu_0 = 4\pi\times10^{-7})$

$$\varepsilon_0\mu_0 = \frac{1}{9\times10^9}\times10^{-7}$$

$$\varepsilon_0\mu_0 = \frac{1}{9\times10^9\times10^7}$$

$$\varepsilon_0\mu_0 = \frac{1}{9\times10^{16}}$$

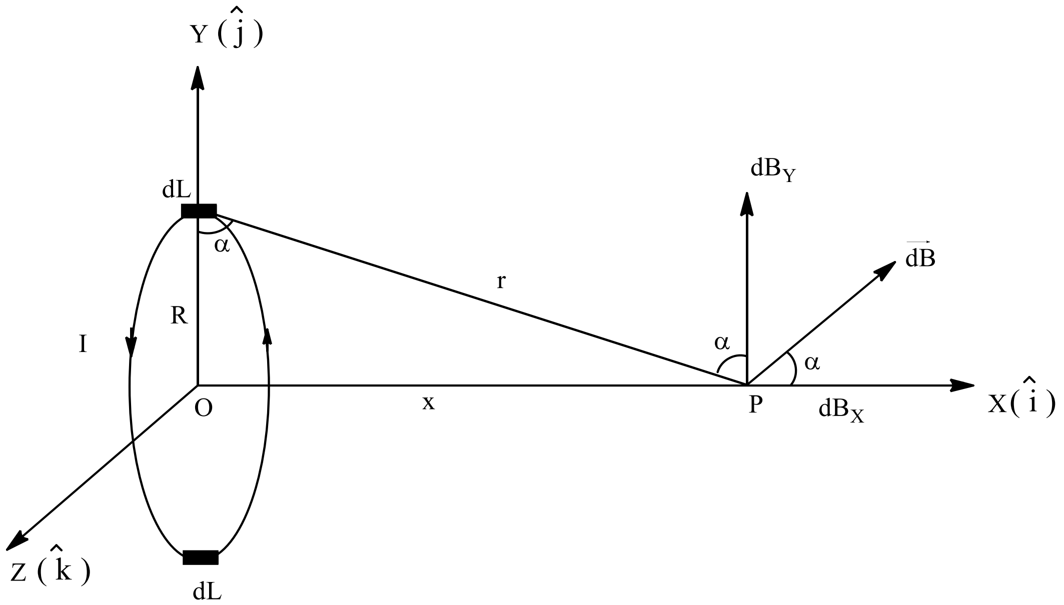
$$\varepsilon_0\mu_0 = \frac{1}{\left(3\times10^8\right)^2}$$

$$\varepsilon_0\mu_0 = \frac{1}{C^2}$$ Where, $C=3\times10^8$=speed of light

Or $$C^2 = \frac{1}{\varepsilon_0\mu_0}$$

$$C = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

Derive an expression for the magnetic field due to a circular current loop on its axis:



Consider a circular loop carrying a steady current 'I'. The loop is in free space and lie in y-z plane.

Let, R = radius of loop, O= centre of loop
x=distance between 'O' and point 'P', dL= length of current element of loop
r = distance between 'dL' and 'P',

dB=magnetic field at 'P' due to current element.

According to Biot-Savart law, the magnitude of magnetic field

$$dB = \frac{\mu_0}{4\pi} \times \frac{IdL\sin\theta}{r^2}$$

but, $\theta=90^0$, $\therefore$ $\sin\theta=\sin 90^0=1$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} \times \frac{IdL}{r^2} \text{ ----------------- (1)}$$

The $\overrightarrow{dL}$ (Current element) is in Y – Z plane and $\vec{r}$ (displacement vector) is in X – Y plane. There fore angle between $\overrightarrow{dL}$ and $\vec{r}$ is 90^0

From figure, $r^2=x^2+R^2$ $\left[\therefore r = \left(x^2 + R^2\right)^{\frac{1}{2}}\right]$

$\therefore$ equation (1) becomes

$$dB = \frac{\mu_0}{4\pi} \times \frac{IdL}{\left(x^2 \times R^2\right)} \text{ --------------- (2)}$$

The direction of dB is perpendicular to the plane containing $\overrightarrow{dL}$ and $\vec{r}$. Resolving the dB into two components.

$dB_x = dB\cos\alpha \rightarrow$ along x-axis (x-component)

$dB_y = dB\sin\alpha \rightarrow$ along y-axis (y-component)

dB_y component due to current element dL is cancelled by the another dB_y component due to diametrically opposite current element dL.

Thus only the dB_x components survive.

i.e. $dB_X = dB\cos\alpha$

$$dB_X = \frac{\mu_0}{4\pi} \times \frac{IdL}{\left(x^2 + R^2\right)} \times \cos\alpha$$

From figure, $\cos\alpha = \frac{R}{r} = \frac{R}{\left(x^2 + R^2\right)^{\frac{1}{2}}}$

$$\therefore \quad dB_x = \frac{\mu_0}{4\pi} \times \frac{IdL}{\left(x^2 + R^2\right)} \times \frac{R}{\left(x^2 + R^2\right)^{\frac{1}{2}}}$$

$$dB_x = \frac{\mu_0}{4\pi} \times \frac{IdLR}{\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

The total magnetic field at 'P' due to entire loop is given by

$$\int dB_x = \int \frac{\mu_0}{4\pi} \times \frac{IdLR}{\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 IR}{4\pi\left(x^2 + R^2\right)^{\frac{3}{2}}} \int dL$$

but, $\int dL = 2\pi R$ = circumference of loop

$$\therefore \quad B = \frac{\mu_0 IR}{4\pi\left(x^2 + R^2\right)^{\frac{3}{2}}} \times 2\pi R$$

$$B = \frac{\mu_0 IR^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$ acting along the axis of a coil.

Note-1: in vector form, $\vec{B} = \frac{\mu_0 IR^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}\hat{i}$

Where, $\hat{i}$ = unit vector acting along x-axis

Note-2: Magnetic field at the centre of loop

Here, x=0 $\vec{B}=\frac{\mu_0 IR^2}{2\left(0+R^2\right)^{\frac{3}{2}}}\hat{i}$, $\vec{B}=\frac{\mu_0 IR^2}{2R^3}\hat{i}$

$$\vec{B}=\frac{\mu_0 I}{2R}\hat{i}$$

Note-3: Magnetic field due to current loop is always acting along its axis.

Note-4: If the coil has N number of turns, then $B=\frac{\mu_0 NIR^2}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$

At the centre of coil, $B=\frac{\mu_0 NI}{2R}$.

Amperes's circuital law:

Ampere's circuital law states that "the line integral of magnetic field around the closed path is equal to 'μ_0' times the total current passing through the surface".

i.e. $\oint \vec{B}\cdot\vec{dL}=\mu_0 I$

where, μ_0=permeability of free space, I=total current through the surface,

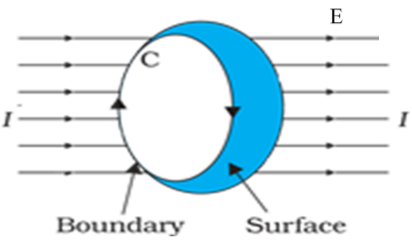
dL=length of line element of boundary,

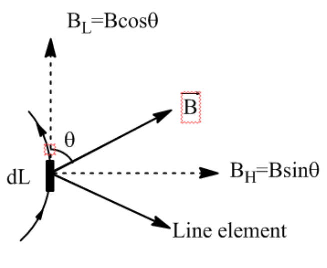
B=magnetic field due to line element.

Note:

Aperes's circuital law:

Consider an open surface with boundary as shown.





Dividing the boundary in to number of small line elements of length dL.

Let, B=magnetic field at line element. B_L=tangential component of B

B_L=Bcosθ

Consider a product, $B_L\,dL = B\cos\theta\, dL = B\, dL\cos\theta$

$$B_L dL = \vec{B}\cdot\overrightarrow{dL} \qquad \left[\vec{B}\cdot\overrightarrow{dL} = \text{dot product of } \vec{B} \text{ and } \overrightarrow{dL}\right]$$

Sum of all the $\vec{B}\cdot\overrightarrow{dL}$ products due to entire curve (boundary) is called line integral.

It is denoted by $\oint \vec{B}\cdot\overrightarrow{dL}$

i.e., $\oint \vec{B}\cdot\overrightarrow{dL} = \text{line integral of } \vec{B} \text{ around the closed path}$

Note:

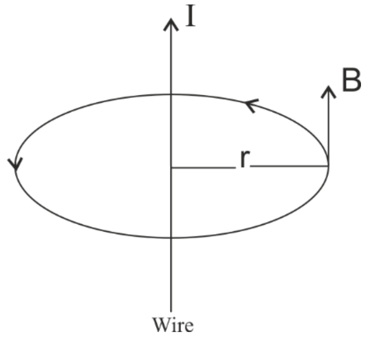
1. The closed curve or loop for which ampere circuital law is applied is called amperian loop.
2. Ampere circuital law is alternate method for Biot-Savart law.
3. Simplified form of Ampere circuital law is

$$BL = \mu_0 I_e$$

Where, L=Length of the loop, I_e=enclosed current.

Derive an expression magnetic field due to straight infinite current carrying wire using Ampere's circuital law:

Let, I=current flows through straight wire,
B= magnetic field at a distance 'r'



The boundary of loop chosen is a circle,
$\therefore$ r=radius of circle
B is tangential to circumference of circle.

According to Ampere circuital law,

$$BL = \mu_0 I_e \text{ ------ (1)}$$

But, L=2πr=circumference of circle, I_e=I

$\therefore$ equation (1) becomes

$$B\times 2\pi r = \mu_0 I$$

$$\therefore \quad B = \frac{\mu_0 I}{2\pi r}$$

This is the expression for magnetic field due to long straight conductor (wire).

Some important points regarding magnetic field due to long straight conductor:

1. We have, $B = \frac{\mu_0 I}{2\pi r}$

   i.e. $B \propto I$ and $B \propto \frac{1}{r}$

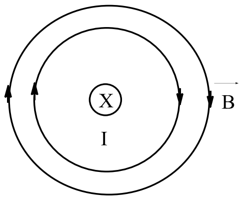
   The field is directly proportional to the current and inversely proportional to the distance from the wire.
2. The field direction at any point on the circle is tangential.
3. The magnitude of field at every point on the circle is same in another words the magnetic field possess cylindrical symmetry.

Note-1:

⊗→ Cross mark shows that the current going into the plane of the paper away from the observer.

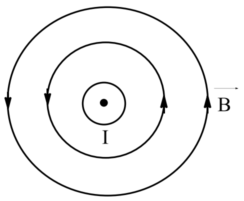
⊙→ Dot mark shows that the current coming out of the plane of the paper towards the observer.

Note-2:



Field direction is clockwise. If the current is directed away from the observer.

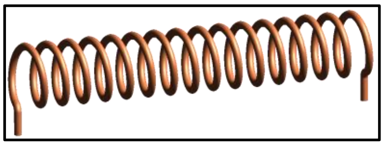
Note-3:



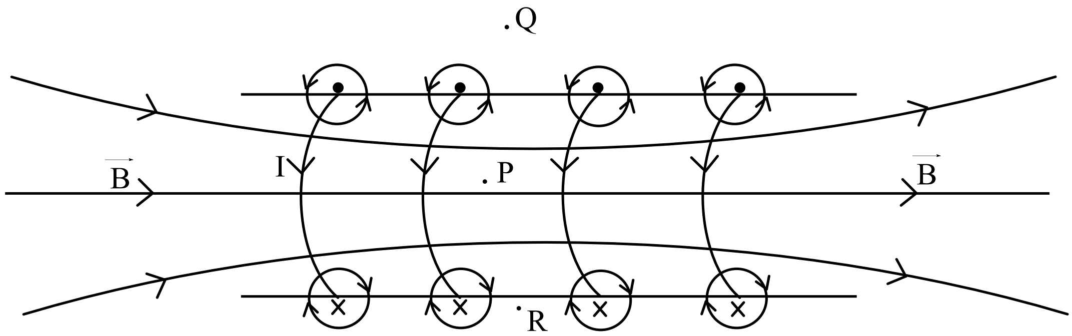
Field direction is anticolockwise if the current is directed towards the observer.

Solenoid:

The solenoid consists of a long wire wound in the form of a helix. In the solenoid, each turn can be regarded as a circular loop. In very long solenoid, the length is very long compared to its radius.



When current is passed through the solenoid then magnetic field is generated. The net magnetic field is the vector sum of the fields due to all the turns.



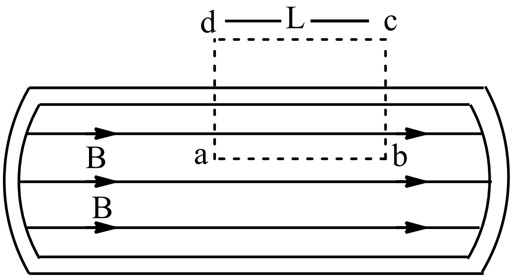
⊗=cross mark, ⊙=dot mark, I=current, B=Magnetic field

The magnetic field lines for a finite solenoid is as shown. It is clear from the circular loops that, the field between two neighbouring turns vanishes. i.e. at R, magnetic field B=0.

Field at interior midpoint 'P' is uniform, strong and along the axis of a solenoid. Field at exterior point 'Q' is weak.

Note: If the solenoid is made very longer then field outside the solenoid is zero and the field inside is uniform and parallel to axis.

Derive an expression for Magnetic field due to a very long solenoid:



Consider a very long solenoid. The field inside the solenoid is parallel to axis. Consider a rectangular amperian loop 'abcd'. Along cd, the magnetic field is zero. Along bc and ad, the field component is zero. The field along 'ab' is 'B'.

Let, n=number of turns per unit length of loop, L=length of loop,

Rotal turns=nL

I=current flows in each turn

∴ Total current enclosed is, I_e=InL.

∴ According to ampere circuital law

$$BL = \mu_0 I_e \text{ ------ (1)}$$

∴ $B \times L = \mu_0 nLI$ (But, I=InL)

$B = \mu_0 nI$

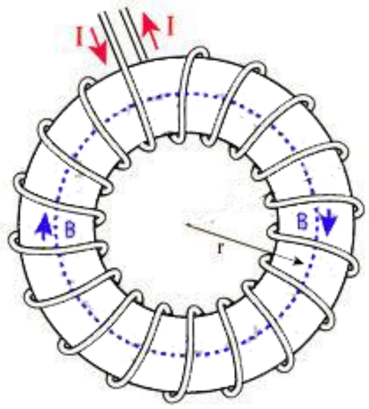
This is the expression for magnetic field produced by a solenoid.

Note: If N=Total number turns, L=length of solenoid, then number of turns per unit length is, $n = \frac{N}{L}$.

Uses of solenoid:

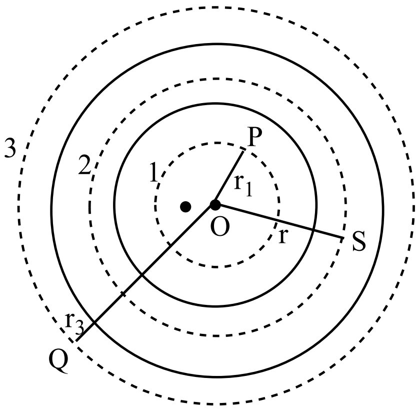
It is used to obtain a uniform magnetic field. It is used in television and synchrotron.

Note: A large field is obtained by inserting a soft iron core inside the solenoid.



Toroid: The toroid is a hallow circular ring on which a large number of turns of wire are closely wound.

Derive an expression for Magnetic field due to toroid:



Consider a toroid, imagine three amperian loops 1, 2 and 3 as shown. r_1, r and r_3 are the radii.

$\therefore$ Circumference of loop 1 is $L_1=2\pi r_1$

Similarly $L_2=2\pi r$, and $L_3=2\pi r_3$.

According to Ampere's circuital law

$$BL=\mu_0 I_e$$

Where, I_e = current enclosed in the loop

For 1st loop , $B_1L_1=\mu_0 I_e$

But $I_e=0$

$\therefore$ $B_1\times 2\pi r_1=\mu_0\times 0$

$B_1=0$

Similarly for 3rd loop, $B_3=0$

For 2nd loop, $B_2L_2==\mu_0 I_e$

but $I_e=NI$

where, N=number of turns

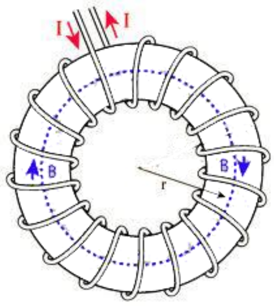
$B\times 2\pi r=\mu_0 NI$

$$\therefore \quad B=\frac{\mu_0 NI}{2\pi r}$$

This is the expression for magnetic field due to toroid.

Note:

1. At the points P and Q the magnetic field due to toroid is zero. But magnetic field is present inside the toroid.
2.



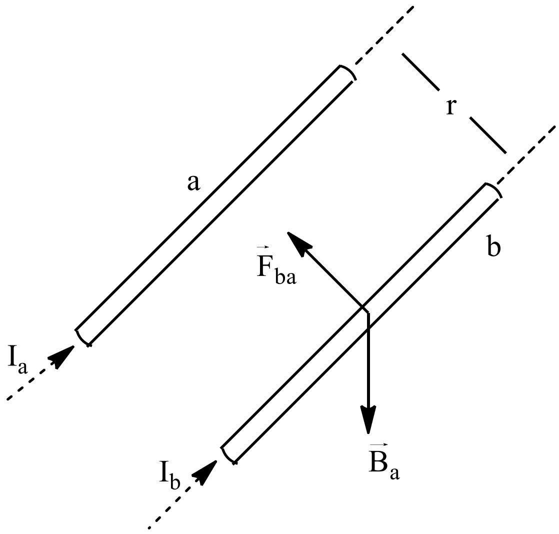
$$B=\mu_0 I\times\frac{N}{2\pi r}$$

But, $n=\frac{N}{2\pi r}$ = number of turns per unit length.

$\therefore$ $B=\mu_0 nI$

Obtain the expression for force between two parallel current carrying conductors:

Consider two long parallel conductors ‘a’ and ‘b’ separated by a distance ‘r’ and carrying currents ‘I_a’ and ‘I_b’. the conductor ‘a’ produces the magnetic field B_a at all points along conductor ‘b’. The right hand rule tells that the direction of $\vec{B}_a$ is down ward [when the conductors are placed horizontally]. Magnitude of B_a is given from Ampere’s circuital law is



$$B_a = \frac{\mu_0 I_a}{2\pi r}$$

The force acting on a conductor ‘b’ due to an external magnetic field is given by

$F_{ba}=B_aI_bL$ ($\because$ F=BILsinθ, θ=90^0, sin90^0=1, F=BIL)

Where, L=length of small segment of conductor ‘b’

$$\therefore \quad F_{ba} = \frac{\mu_0 I_a I_b L}{2\pi r}$$

Similarly force on ‘a’ conductor due to ‘b’ is

$$F_{ab} = \frac{\mu_0 I_a I_b L}{2\pi r}$$

But F_{ba} and F_{ab} equal and opposite

$$\therefore \quad \vec{F}_{ba} = -\vec{F}_{ab}$$

In general $|\vec{F}_{ba}| = |\vec{F}_{ab}| = F_{ba}$ $\quad \therefore \quad F_{ba} = \frac{\mu_0 I_a I_b L}{2\pi r}$

$$\frac{F_{ba}}{L} = \frac{\mu_0 I_a I_b}{2\pi r} \qquad f_{ba} = \frac{\mu_0 I_a I_b}{2\pi r}$$

Where, $f_{ba} = \frac{F_{ba}}{L}$ = force per unit length.

This is the expression for force per unit length between two parallel current carrying conductors.

Note:

1. If the currents in the two parallel conductors are in same direction, then two conductors attract each other.
2. If the currents in the two parallel conductors are in opposite direction, then two conductors repel each other.

   i.e. Parallel currents attract and antiparallel currents repel.
3. Ampere is the S.I. unit of electric current.

Define ampere:

We have $f_{ba} = \frac{\mu_0 I_a I_b}{2\pi r}$

If $I_a = I_b = 1A$, $r = 1m$

then, $f_{ba} = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi}$

$f_{ba} = 2 \times 10^{-7}$ N/m

One ampere is that current which when flowing in each of the two very long straight parallel conductors and kept one metre apart in vacuum, would produce on each conductor a force equal to 2×10^{-7} Newton per metre of length.

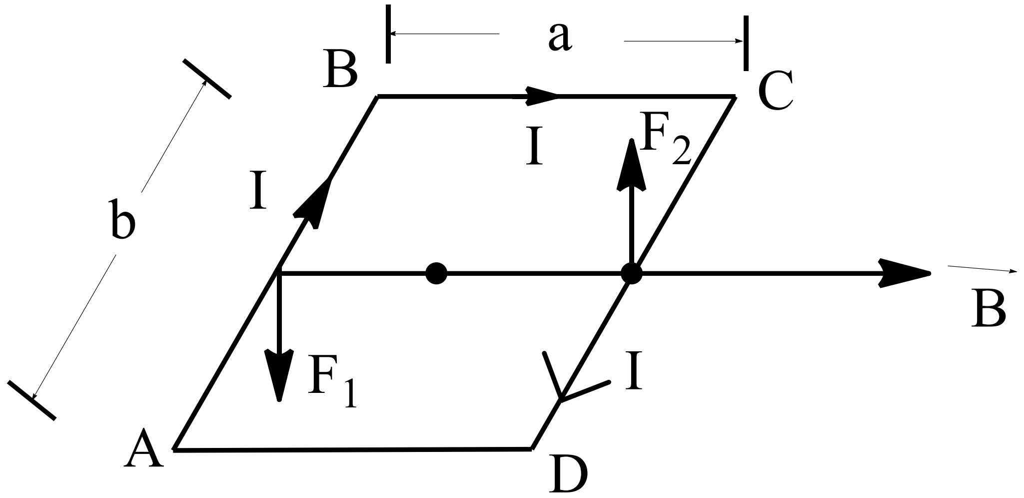
Note: Define one coulomb interms of ampere:

When a steady current of 1A is set up in a conductor, then quantity of charge flows through it in one second is one coulomb.

Derive an expression for torque on current loop in a uniform magnetic field:

Case (i): When the plane of the loop is parallel to magnetic field.

Consider a rectangular loop ABCD in a uniform magnetic field $\vec{B}$. Let I be the current flows through the coil.



Force acting on a straight conductor is F=BIL sinθ ----------- (1)

The two arms BC and DA are parallel to field

i.e. $\theta=0^0$, $\therefore F=0$

No force acts on the AD and BC.

The two arms AB and CD are perpendicular to the field. i.e. $\theta=90^0$.

Force on AB arm is

$F_1=BIb$ $(F=BIL\sin\theta,\ L=b,\ \theta=90^0\ \therefore F_1=BIb)$

Force on CD arm is

$F_2=BIb$

F_1 and F_2 are equal and opposite

Thus net force on loop is zero. But there is a torque on the loop due to pair of forces F_1 and F_2. Therefore torque rotates the loop.

We have,

$\tau=F_1\times a$ (Torque=either of force × Perpendicular distance between them)

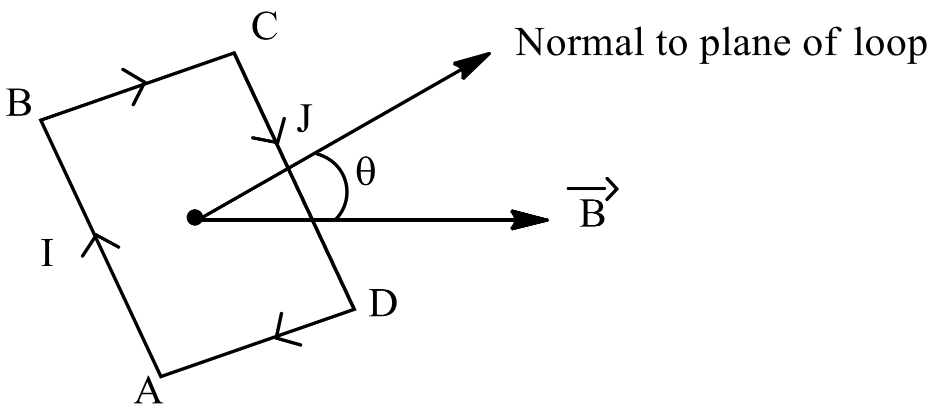
$\tau = F_1 \times a$

$\tau=BIb\times a$

$\boxed{\tau=BIA}$ Where, A=ab = Area of loop.

This is the expression for torque on a rectangular loop.

Case 2 : When the plane of the loop is not along the field:



In this case net force on BC and DA is zero. The force on the arm AB is

$F_1 = BIb \sin\theta$

The force on the arm CD is

$F_2 = BIb \sin\theta$

F_1 and F_2 are equal and opposite but they are not collinear. Therefore, torque is produced.

∴ Torque is given by

$\tau = F_1 \times a$

$\tau = BIb \sin\theta \times a = BIba\sin\theta$

$\tau = BIA \sin\theta$ ∵ ba=A=Area of loop.

If the loop has 'N' number of turns,

Then, $\boxed{\tau = NBIA \sin\theta}$

Note: $\theta = 90^0$ ∴ $\sin\theta = \sin 90 = 1$

∴ $\tau = NBIA$

For one turn, N=1, then $\tau = BIA$

Magnetic moment of current loop:

It is defined as the product of current in the loop and area of loop.

i.e. magnetic moment = current × area

m = IA

Magnetic moment is a vector quantity

S.I. unit is Am^2 (ampere $metre^2$)

If the loop has 'N' number of turns, then

m = NIA

Derive an expression for torque in terms of $\vec{m}$ and $\vec{B}$:

We have, $\tau = IAB\sin\theta$ If N=1 (for one turn)

but $IA = m$

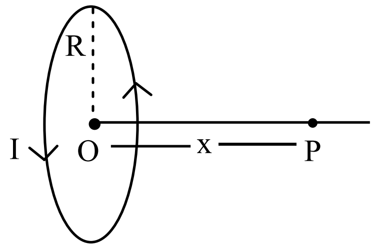
$\therefore$ $\tau = mB\sin\theta$

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Show that circular current loop is equivalent to a magnetic dipole (short bar magnet):

Magnetic field due to magnetic dipole (short bar magnet) at a point is

$$B = \frac{\mu_0}{4\pi} \times \frac{m}{x^3} \text{ ----------- (1)}$$



Where, m=dipole moment, x=large distance,

Consider a current carrying circular loop.

I=current, R=radius of loop, x=distance from 'O' to 'P'

Then, magnetic field at 'P' is

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

If, X>>R, then $\left(x^2 + R^2\right)^{\frac{3}{2}} = \left(x^2\right)^{\frac{3}{2}} = x^3$

$$\therefore \quad B = \frac{\mu_0 I R^2}{2x^3}$$

$$B = \frac{\mu I \pi R^2}{\pi \times 2x^3}$$

But, $\pi R^2 = A$ = Area of loop

$$B = \frac{\mu_0 IA}{2\pi x^3}$$

But, IA=m=magnetic moment

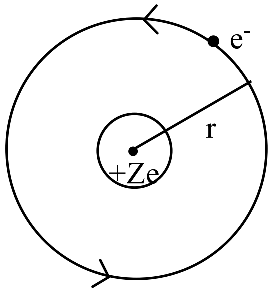
$$\therefore \quad B = \frac{\mu_0 m}{2\pi x^3}$$

$$B = \frac{\mu_0}{4\pi} \times \frac{2m}{x^3} \text{ ------------ (2)}$$

From above equations (1) and (2), it can be shown that a planar current loop is equivalent to a magnetic dipole.

Expression for magnetic dipole moment of a revolving electron:

Consider, an electron revolving around the nucleus of an atom.



Let, Charge on electron= e,
Time period of revolution = T
Orbital velocity = v, Current = I,
Radius of circular path = r

But, $I = \frac{e}{T}$ ----------- (1) $\left(\because I = \frac{q}{t},\ q = e,\ t = T\right)$

and $v = \frac{2\pi r}{T}$ $\left(\text{Velocity} = \frac{\text{Distance}}{\text{Time taken}} \quad v = \frac{2\pi r}{T}\right)$

$\therefore$ $T = \frac{2\pi r}{v}$ ----------- (2)

Substituting equation (2) in (1)

$I = \frac{e}{\frac{2\pi r}{v}}$ $\therefore I = \frac{ev}{2\pi r}$ ----------- (3)

We know that, magnetic moment due to electron is

$\mu_L = IA$ (We have m=IA, here m=μ_L)

But, A=πr^2 = Area of circle

$\therefore$ $\mu_L = \frac{ev}{2\pi r} \times \pi r^2$ $\left(\because I = \frac{ev}{2\pi r}\right)$

$\mu_L = \frac{evr}{2}$

$\mu_L = \frac{e}{2m_e}(m_e vr)$

Where, m_e= mass of the electron

But, $m_e vr$= L = Orbital angular momentum.

$\therefore$ above equation becomes

$\mu_L = \frac{e}{2m_e} \times L$

$$\boxed{\mu_L = \frac{eL}{2m_e}}$$

This is the expression for magnetic moment of revolving electron.

Note:

1) Magnetic moment in vector form

$$\vec{\mu_L} = -\frac{e}{2m_e}\vec{L}$$

Negative sign indicates that the angular momentum of electron is opposite in direction to the magnetic moment.

2) For positive charge (+e), $\vec{\mu_L}$ and $\vec{L}$ are in same direction.

Gyromagnetic ratio: It is the ratio of magnetic moment to the orbital angular momentum.

Gyromagnetic ratio = $\frac{\mu_L}{L}$.

S.I. unit is Coulomb/kg.

Expression of Gyromagnetic ratio:

We have, $\mu_L = \frac{eL}{2m_e}$ (considering magnitude only)

$$\frac{\mu_L}{L} = \frac{e}{2m_e}$$

The ratio $\frac{\mu_L}{L}$ is called gyromagnetic ratio.

∴ Gyromagnetic ratio $= \frac{e}{2m_e}$

Note:

1) For an electron, gyromagnetic ratio = $\frac{e}{2m_e}$

i.e. $$\frac{\mu_L}{L} = \frac{1.6\times10^{-19}}{2\times9.11\times10^{-31}}$$

$$\frac{\mu_L}{L} = 8.8\times10^{10}\ c/kg$$

Bohr magneton (μ_B):

The smallest value of magnetic moment of revolving electron is called Bohr magneton.

To find the value of Bohr magneton:

We have, $\mu_L = \frac{eL}{2m_e}$

According to Bohr's postulates $L = \frac{nh}{2\pi}$

Where, n=1, 2, 3, ……, h=Planck's constant.

$$\mu_L = \frac{e}{2m_e} \times \frac{nh}{2\pi}$$

$$\mu_L = \frac{neh}{4\pi m_e}$$

If n=1, μ_L is minimum (i.e. for 1st orbit)

$$(\mu_L)_{min} = \frac{eh}{4\pi m_e}$$

$$(\mu_L)_{min} = \frac{1.6\times10^{-31}\times6.63\times10^{-34}}{4\times3.14\times9.11\times10^{-31}}$$

$$(\mu_L)_{min} = 9.27\times10^{-24}\ Am^2$$

[$(\mu_L)_{min}$ is called Bohr magneton (μ_B)]

i.e. $(\mu_L)_{min} = \mu_B = 9.27\times10^{-24}\ Am^2$

Moving coil galvanometer (MCG):

M.C.G. is a device used to measure very small current of the order of 10^{-10}A.

Principle of MCG: When a current carrying conductor is kept in an external magnetic field, it experiences a torque.

Construction:

M.C.G. consists of a coil. The coil is free to rotate about a fixed axis in a radial magnetic field. There is a cylindrical soft iron core which makes the field radial and strong. There is a scale and pointer to note down the deflection.

Working (Theory of MCG):

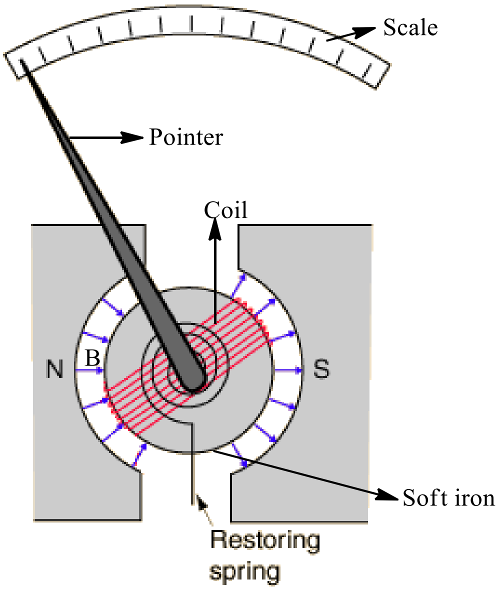
When a current flows through the coil, a torque acts on it. It is given by

$$\tau = NIAB\sin\theta$$

but $\theta=90^0$, $\therefore$ Magnetic field is radial

$\therefore$ $\sin 90^0=1$

$\therefore$ $\tau = NIAB$



N and S= Magnet,
S=Spring,
B=Uniform radial magnetic field

Where, τ= torque,

N=number of turns in the coil,

I=Current, A=Area of the coil,

B=Uniform radial magnetic field

Due to above magnetic torque, the coil is rotated.

As a result, the spring gets twisted.

This gives restoring torque.

$\therefore$ Restoring torque = $K\phi$

Where, K=restoring torque per unit twist

ϕ=deflection on the scale by the pointer

At equilibrium

Restoring torque = Magnetic torque

$$K\phi = NIAB$$

$$\therefore \quad \phi = \left(\frac{NAB}{K}\right)I$$

$\left(\frac{NAB}{K}\right)$ is constant for a given galvanometer.

Current sensitivity of the galvanometer:

It is defined as the deflection per unit current.

$$\therefore \text{ Current sensitivity}=\frac{\phi}{I}$$

S.I. unit is division / ampere (div/A)

Expression for current sensitivity:

We have, $\phi = \left(\frac{NAB}{K}\right)I$

$$\therefore \frac{\phi}{I} = \left(\frac{NAB}{K}\right)$$

$\therefore$ Current sensitivity can be increased by increasing the number of turns (N).

Voltage sensitivity of galvanometer:

It is defined as the deflection per unit voltage.

$$\text{Voltage sensitivity} = \frac{\text{Deflection}}{\text{Voltage}}$$

$$\text{Voltage sensitivity} = \frac{\phi}{V}$$

S.I. unit is division/volt.

Expression for voltage sensitivity:

We have, $\phi = \left(\frac{NAB}{K}\right) I$ $\qquad \frac{\phi}{V} = \left(\frac{NAB}{K}\right)\frac{I}{V}$

but $\frac{I}{V} = R$, R = Resistance of the coil (galvanometer)

$\therefore \quad \frac{\phi}{V} = \left(\frac{NAB}{K}\right)\frac{1}{R}$ $\qquad \therefore \quad \text{Voltage sensitivity} = \left(\frac{NAB}{K}\right)\frac{1}{R}$

Note:

1) We have $\frac{\phi}{I} = \left(\frac{NAB}{K}\right)$

   If $N \rightarrow 2N$, i.e. number of turns is doubled.

   Then, $\frac{\phi}{I} = \left(\frac{2NAB}{K}\right) = 2\left(\frac{NAB}{K}\right)$

   i.e. $\frac{\phi}{I} \longrightarrow 2\frac{\phi}{I}$ i.e. current sensitivity is doubled.

2) We have $\frac{\phi}{V} = \left(\frac{NAB}{K}\right)\frac{1}{R}$

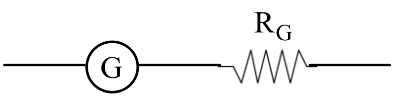
   If $N \rightarrow 2N$, then $R \rightarrow 2R$ i.e. resistance of galvanometer is doubled.

   $\therefore$ Then, $\frac{\phi}{V} = \left(\frac{2NAB}{K}\right)\frac{1}{2R} = \left(\frac{NAB}{K}\right)\frac{1}{R}$

   $$\frac{\phi}{V} \longrightarrow \frac{\phi}{V}$$

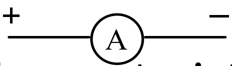
   i.e. voltage sensitivity remains unchanged.

3) The schematic representation of galvanometer is as shown



   G=Galvanometer, R_G=Resistance of the galvanometer.

Ammeter: It is device used measure electric current in the circuit.

Symbol of ammeter is 

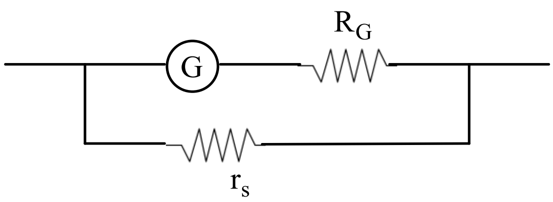
How do you convert galvanometer into ammeter?

A galvanometer is converted into a ammeter by connecting a small shunt resistance in parallel with it.

Let, r_s= Small resistance called shunt resistance

R_G= Resistance of galvanometer

The effective resistance of ammeter is



$$\frac{1}{R_{eff}} = \frac{1}{R_G} + \frac{1}{r_s}$$

$$\frac{1}{R_{eff}} = \frac{R_G + r_s}{R_G r_s}$$

$$\therefore \quad R_{eff} = \frac{R_G r_s}{R_G + r_s}$$

But $r_s << R_G$ $\quad \therefore R_G + r_s = R_G$

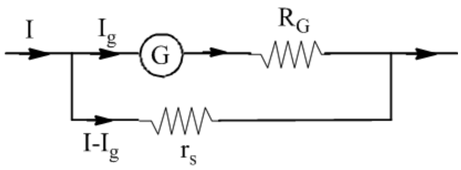
$$\therefore \quad R_{eff} = \frac{R_G r_s}{R_G}$$

$$R_{eff} = r_s$$

i.e. Resistance of ammeter is very small. Therefore it doesnot alter the current in the circuit.

Note: Resistance of an ideal ammeter is zero.

To find the expression for shunt resistance for galvanometer:



Let, I_g=current required for full scale deflection, I=Current to be measured.

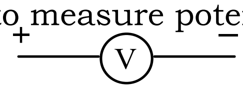
R_G and r_s are in parallel

$\therefore$ Potential difference across R_G = Potential difference across r_s

$$\therefore \quad I_g R_G = (I - I_g) r_s$$

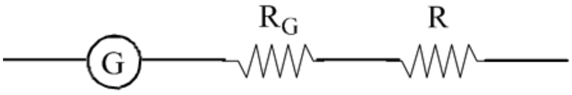
$$r_s = \frac{I_g R_G}{(I - I_g)}$$

This is the expression for shunt resistance (low resistance).

Voltmeter: It is a device used to measure potential difference in the circuit.
The symbol of voltmeter is 

How do your convert galvanometer into voltmeter:

A galvanometer is connected into voltmeter by connecting a high resistance in series with it.



Where, R= high resistance connected in series,

R_G=resistance of galvanometer

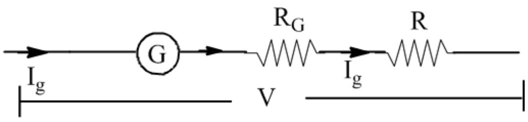
The effective resistance of voltmeter is

$R_{eff} = R_G + R$ but R is very larger than R_G

∴ Effective resistance of voltmeter is high.

Note: Resistance of an ideal voltmeter is infinity.

To find the expression for high resistance for galvanometer:



V=Voltage required to be measured, I_g= Current required for full scale deflection

Total potential difference = Potential difference across R_G + Potential difference across R

$$V = I_gR_G+I_gK$$

$$V = I_g(R_G+R)$$

$$\frac{V}{I_g} = R_G + R$$

$$R = \frac{V}{I_g} - R_G$$

This is the expression for high resistance.

One Marks Questions:

1. What is the nature of force between two parallel conductors carrying current in same direction? (March-2014) (July-2014)
2. State Ampere's circuital law.(march-2017)
3. A charged particle enters an electric field in the direction of electric field. What is the nature of the path traced by it? (July-2013)
4. What is a cyclotron? (March-2016)
5. When will the magnetic force on a moving charge be maximum in a magnetic field? (July-2016)

Two Marks Questions:

1. When the force on a charge moving in a magnetic field is a) Maximum b) Minimum
2. Mention the expression for Lorentz force and explain the terms.
3. What is the nature of the trajectory of a charged particle in a uniform magnetic field, if it enters with velocity at an angle. a) 0^0 b) 90^0
4. What is meant by velocity selector? Mention its use.
5. Write Biot-Savart's law in vector form and explain the terms
6. State and explain ampere circuit law. (July-2014)
7. What is a toroid? Mention an expression for magnetic field at a point inside a toroid. (March-2016)
8. What is solenoid? Mention an expression for magnetic field at a point inside a long current carrying solenoid.
9. Assuming the expression for magnetic dipole moment of a revolving electron in a hydrogen atom. Obtain the expression for Bohr magneton.
10. What is a moving coil galvanometer? Mention the principle behind it.
11. A galvanometer having a coil of resistance 12Ω gives full scale deflection for a current of 4mA. How can it be converted into a voltmeter of range 0 to 24V?

Three Marks Questions:

1. Describe Oersted's experiment for the discovery of magnetic effect of current.
2. Give an expression for force acting on a charge moving in magnetic field and explain the symbols. When does the force becomes maximum? (July-2014)
3. Obtain the expression for radius of a circular path described by a charge in uniform magnetic field.
4. Derive an expression for the angular frequency of a charged particle moving in a uniform magnetic field perpendicular to it.
5. Mention the uses of cyclotron. (March-2014)
6. State and explain Biot-Savart law.
7. Derive an expression for magnitude of magnetic field at a point around a straight conductor. (July-2015)
8. Drive an expression for magnetic field at a point inside a long current carrying Solenoid.
9. Derive an expression for the magnetic field at a point inside the toroid.
10. Derive the expression for the torque acting on a rectangular current loop in a uniform magnetic field.
11. Derive an expression for magnetic dipole moment of the electrons revolving in an hydrogen atom.
12. Explain how to convert galvanometer into ammeter.
13. Explain how to convert galvanometer into voltmeter. (March-2015, March-2017).
14. Write difference between ammeter and voltmeter.

Five Marks Questions:

1. Derive an expression for magnetic field at any point on the axis of a circular current loop by applying Biot-Savart's law. (March-2014, July-2014, March-2015, March-2017)
2. Explain the construction and working of a cyclotron.
3. Derive an expression for the force between two straight parallel current carrying wire and hence define ampere. (July-2015, July-2016)

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Chapter-5: MAGNETISM AND MATTER

Introduction:

Magnetic phenomena are universal in nature. The earth's magnetism predates human evolution.

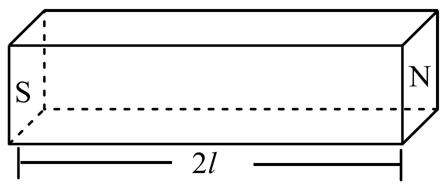
The word 'magnet' is derived from the name of an island called Magnesia in Greece. Magnetic ore deposits were found in Magnesia, as early as 600BC.

The directional property of magnet was also known since ancient times. When a thin long piece of a magnet is suspended freely, then it is pointed in the north-south direction. The technological exploitation of this property is generally credited to the Chinese (400BC).

The name lodestone (or load stone) is given to a naturally occurring ore of iron-magnetite.

Note: The earth behaves as a magnet with the magnetic field pointing approximately from the geographic south to the north.

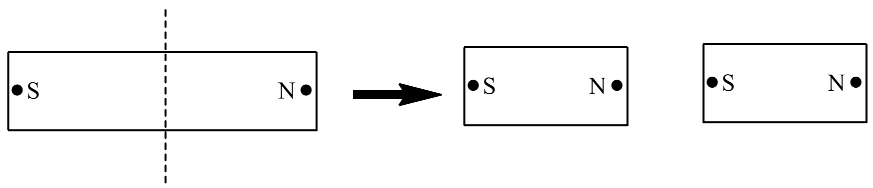
The bar magnet:



N=North pole
S= South Pole
$2l$=dipole length

Properties of bar magnet:

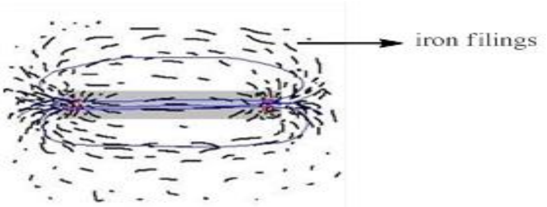
1. When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called north pole and the tip which points to the geographic south is called the south pole of magnet.
2. Like poles repel and unlike poles attract.
3. We cannot isolate the north pole or south pole of a magnet. i.e. magnetic monopoles do not exist. (if a bar magnet is broken into two halves, we get two smaller bar magnets).



4. It is possible to make magnets out of iron and its alloys.

Note:

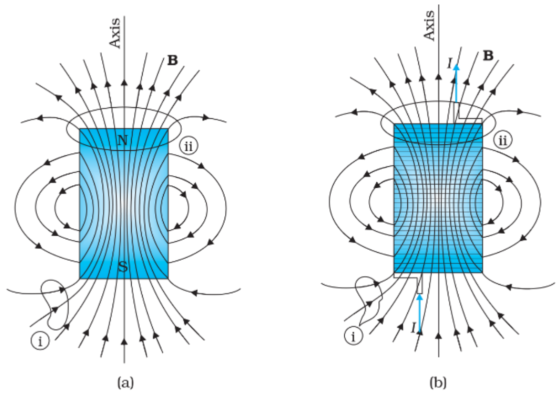
1. The arrangement of iron fillings around a bar magnet permits us to plot the magnetic field lines.



2. Magnetic field lines are a visual and intuitive realisation of the magnetic field.
3. Bar magnet and current carrying solenoid are similar in behaviour.

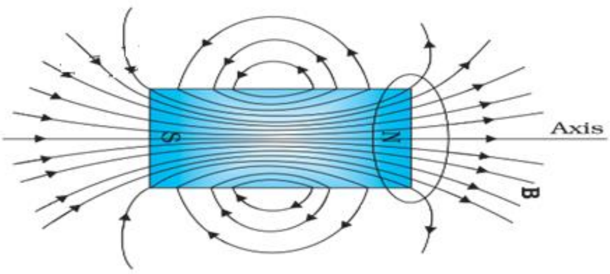
Properties of the magnetic field lines:

1. The magnetic field lines of a magnet or a solenoid form continuous closed loops.
2. The tangent to the field line at a given point represents the direction of the net magnetic field at that point.
3. The larger the number of field lines crossing per unit area, the stronger is the magnitude of magnetic field [in figure, $\vec{B}$ is larger around the region (ii) than in region (i)]
4. The magnetic field lines do not intersect.



The field lines of (a) a bar magnet, (b) a current-carrying finite solenoid

Draw a schematic diagram of magnetic field lines:



Note:

1. Electric field lines do not form a continuous closed path.
2. According to Ampere's hypothesis, all magnetic phenomena can be explained in terms of circulating currents.

Show that solenoid is equivalent to bar magnet (Show that bar magnet is equivalent to solenoid):

The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid.

Let, n=number of turns per unit length of a solenoid.

2l=length of solenoid, R=radius of solenoid

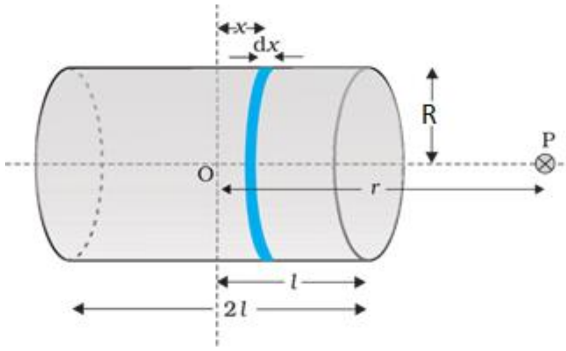
I=current flows through the solenoidO=centre of solenoid

P=Far point on the axis

r=distance between 'O' and 'P'

B=magnetic field at 'P' due to solenoid.

Consider a circular element of length dx at a distance of 'x' from 'O'.



$\therefore$ ndx=total number of turns in the circular element

Magnetic field at 'P' due to circular element is

$$dB = \frac{\mu_0 ndxIR^2}{2\left[(r-x)^2 + R^2\right]^{\frac{3}{2}}} \text{ --------------- (1)}$$

The total magnetic field due to entire solenoid is

$$\int_{-l}^{+l} dB = \int_{-l}^{+l} \frac{\mu_0 ndxIR^2}{2\left[(r-x)^2 + R^2\right]^{\frac{3}{2}}} \qquad \because \text{ x varies form } -l \text{ to } +l$$

$$\therefore \qquad B = \frac{\mu_0 nIR^2}{2} \int_{-l}^{+l} \frac{dx}{\left[(r-x)^2 + R^2\right]^{\frac{3}{2}}} \text{ --------------(2)}$$

but r>>R and r>>l (r>>x)

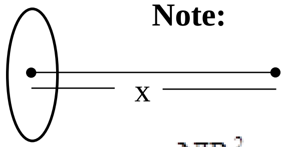
$$\therefore \qquad \left[(r-x)^2 + R^2\right]^{\frac{3}{2}} \longrightarrow \left[r^2\right]^{\frac{3}{2}} \longrightarrow r^3$$

$\therefore$ equation (2) becomes

$$B = \frac{\mu_0 nIR^2}{2} \int_{-l}^{+l} \frac{dx}{r^3}$$

$$B = \frac{\mu_0 nIR^2}{2r^3} \int_{-l}^{+l} dx$$

Note:



$$B = \frac{\mu_0 NIR^2}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Here, B=dB, N=ndx, x=(r-x), R=R

$$B=\frac{\mu_0 nIR^2}{2r^3}[x]_{-l}^{+l}$$

$$B=\frac{\mu_0 nIR^2}{2r^3}\left[+l-(-l)\right]$$

$$B=\frac{\mu_0 nIR^2}{2r^3}(2l)$$

But m=NIA, N=2*l*n,

∴ m=2*l*nIπR²

A=πR²

$$B=\frac{\mu_0 nI2l\times\pi R^2}{2\pi r^3}$$

$$B=\frac{\mu_0}{2\pi}\times\frac{m}{r^3}$$

Where, m=2*l*nIπR² = magnetic moment of solenoid

$$B=\frac{\mu_0}{4\pi}\times\frac{2m}{r^3}$$

This is the expression for far axial magnetic field of a solenoid. This is equal to far axial magnetic field of a bar magnet. Therefore the bar magnet is equivalent to solenoid.

Note:

1. Magnetic field due to bar magnet at a point on the axial line is

$$\vec{B}=\frac{\mu_0}{4\pi}\times\frac{2\vec{m}}{r^3} \qquad (r>>l)$$

2. Magnetic field due to bar magnet at a point along the equatorial line is

$$\vec{B}=-\frac{\mu_0}{4\pi}\times\frac{\vec{m}}{r^3} \qquad (r>>l)$$

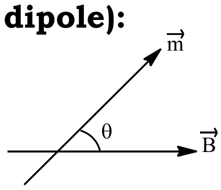
(Negative sign indicates that $\vec{B}$ is opposite to $\vec{m}$)

Expression for torque acting on magnetic needle (magnetic dipole):

Torque acting on a needle is given by

τ=mBsinθ

Or $\vec{\tau}=\vec{m}\times\vec{B}$

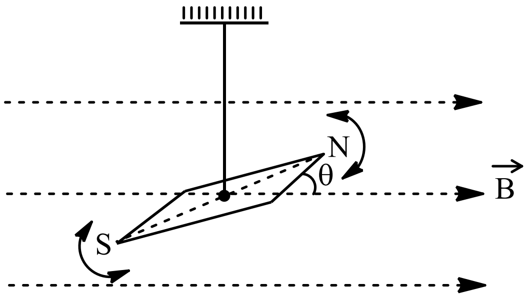


Where, $\vec{m}$ = magnetic moment of needle (direction of $\vec{m}$ is along S-N of needle)

$\vec{B}$ = applied magnetic field.

θ=angle between $\vec{m}$ and $\vec{B}$

Derive an Expression for time period and magnetic moment of Magnetic dipole in a uniform magnetic field:



Consider a small compass needle (small bar magnet). Let it be oscillated in the uniform magnetic field $\vec{B}$.

Let, $\vec{m}$ = magnetic moment of needle (direction of $\vec{m}$ is along N-S of needle)

I=moment of inertia of a needle, θ=angle between $\vec{m}$ and $\vec{B}$

Let, α=angular acceleration of needle

But, $\alpha = \frac{d^2\theta}{dt^2}$

Deflecting torque on a needle is given by

Deflecting torque=Iα

Deflecting torque $= I\frac{d^2\theta}{d\,t^2}$ -------------(1)

Restoring torque acting on a needle is given by

∴ τ=mBsinθ -------------- (2)

At equilibrium

Deflecting torque = –restoring torque

$$I\frac{d^2\theta}{dt^2} = -mB\sin\theta$$

$$\therefore \quad \frac{d^2\theta}{dt^2} = -\frac{mB\sin\theta}{I} \text{ ------------- (3)}$$

If θ is very small then sinθ=θ

Equation (3) becomes

$$\frac{d^2\theta}{dt^2} = -\frac{mB\theta}{I}$$

The above equation represents simple harmonic motion of needle.

but $\frac{mB}{I} = \omega^2$

Where,	ω=angular frequency
	$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$
Where, T=Period of oscillation of needle	

$$\therefore \quad \frac{mB}{I} = \left(\frac{2\pi}{T}\right)^2$$

$$\frac{mB}{I} = \frac{4\pi^2}{T^2}$$

$$T^2 = \frac{4\pi^2 I}{mB}$$

$$T = \sqrt{\frac{4\pi^2 I}{mB}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{mB}}}$$

This is the relation between period of oscillation and magnetic moment.

Magnetic potential energy stored in the needle suspended in the uniform magnetic field:

Magnetic potential energy is given by

$$U_B = \int \tau \, d\theta$$

$$U_B = \int mB \sin\theta \, d\theta$$

$$U_B = mB \int \sin\theta \, d\theta$$

$$U_B = mB(-\cos\theta) \qquad \left(\int \sin\theta \, d\theta = -\cos\theta\right)$$

$$U_B = -mB\cos\theta$$

Note:

1. In vector form, $U_B = -\vec{m}\cdot\vec{B}$ $\left(\because \vec{m}\cdot\vec{B} = mB\cos\theta = \text{dot product of } \vec{m} \text{ and } \vec{B}\right)$
2. We have, $U_B = -mB\cos\theta$

   If, $\theta=0^0$, $\cos 0^0=1$, $U_B = -mB$

   i.e. Potential energy is minimum and magnet is most stable position
3. If $\theta=180^0$, $\cos 180^0= -1$

   $\therefore \quad U_B = -mB(-1)$

   $U_B = +mB$

   i.e. potential energy is maximum and magnet is most unstable position.

The electrostatic analogy (Dipole analogy):

There is an analogy between electric dipole and magnetic dipole.

i.e. $\vec{E} \rightarrow \vec{B}$, $\vec{P} \rightarrow \vec{m}$, $\frac{1}{4\pi\varepsilon_0} \rightarrow \frac{\mu_0}{4\pi}$

Where, $\vec{E}$ = electric field, $\vec{B}$ = magnetic field

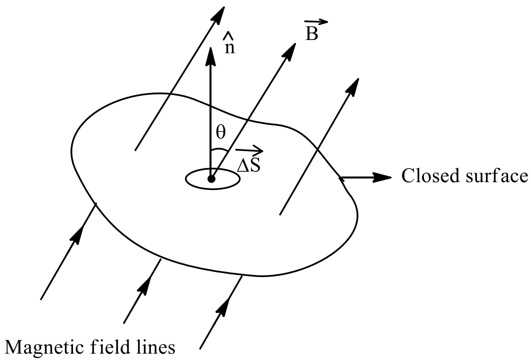
$\vec{P}$ = electric dipole moment, $\vec{m}$ = magnetic dipole moment

ε_0=Permittivity of free space, μ_0=permeability of free space.

	Electrostatic	Magnetism
	$\frac{1}{\varepsilon_0}$	μ_0
Dipole moment	$\vec{P}$	$\vec{m}$
Equatorial field for short dipole	$-\frac{1}{4\pi\varepsilon_0}\times\frac{\vec{P}}{r^3}$	$-\frac{\mu_0}{4\pi}\times\frac{m}{r^3}$
Axial field for short dipole	$\frac{1}{4\pi\varepsilon_0}\times\frac{2\vec{P}}{r^3}$	$\frac{\mu_0}{4\pi}\times\frac{2m}{r^3}$
Torque (τ)	$\vec{P}\times\vec{E}$	$\vec{m}\times\vec{B}$
Energy	$-\vec{P}\cdot\vec{E}$	$-\vec{m}\cdot\vec{B}$

Magnetic flux (ϕ_B):

The number of magnetic field lines flow through the area which is held perpendicular to flow is called magnetic flux.



$\hat{n}$ = normal to $\overrightarrow{\Delta S}$,

$\overrightarrow{\Delta S}$= small vector area element,

θ=angle between $\hat{n}$ and $\vec{B}$

Consider a small vector area element $\overrightarrow{\Delta S}$ in a closed surface.

The magnetic flux through the $\overrightarrow{\Delta S}$ is defined as

$$\Delta\phi_B = \vec{B}\cdot\overrightarrow{\Delta S} = B\Delta S\cos\theta$$

Where, $\vec{B} =$ magnetic field at $\overrightarrow{\Delta S}$

Divide the closed surface in to many small area elements $\overrightarrow{\Delta S}$.

$\therefore$ Total flux is given by

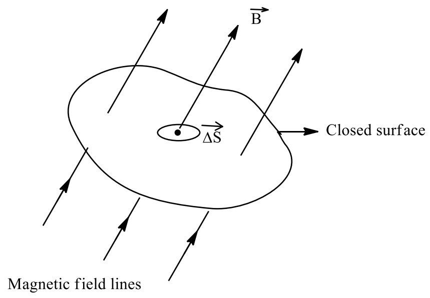
$$\phi_B = \sum_{\text{all}\,\Delta S} \Delta\phi_B = \sum \vec{B}\cdot\overrightarrow{\Delta S}$$

$$\phi_B = \sum_{\text{all}\,\Delta S} B\Delta S\cos\theta$$

Gauss's law of magnetism:

It states that "The net magnetic flux through any closed surface is zero".

i.e. $\phi_B = \sum \vec{B}\cdot\overrightarrow{\Delta S} = 0$



Where, $\overrightarrow{\Delta S} =$ small vector area element,

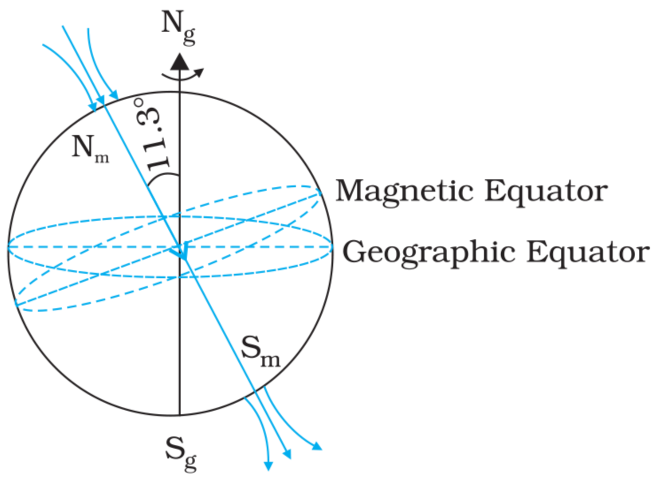
$\vec{B}$ = magnetic field at $\overrightarrow{\Delta S}$

According to Gauss law, number of field lines leaving the surface is balanced by the number of lines entering it. Therefore net magnetic flux is zero for a closed surface. i.e. monopoles donot exist in magnets.

The earth's magnetism:

The earth is a huge magnet. The strength of the earth's magnetic field varies from place to place on the earth's surface. Its value being of the order of 10^{-5}T.

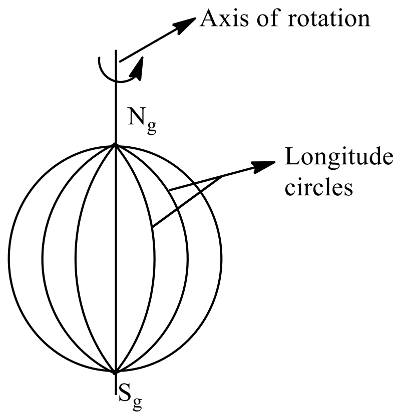
What causes the earth to have a magnetic field is not clear. However, now it is believed that the earth's magnetic field is arised due to electrical currents produced by convective motion of metallic fluids (consisting mostly of molten iron and nickel) in the outer core of the earth. This is known as the **dynamo effect**.



The Earth as a giant magnetic dipole

The axis of the dipole does not coincide with the axis of rotation of the earth. The angle between axis of dipole and axis of rotation is 11.3^0.

Note:

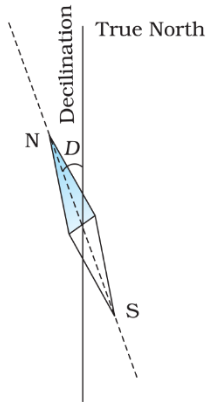


1. The pole near the geographic north pole (Ng) of the earth is called the north magnetic pole (N_m). The pole near the geographic south pole (S_g) is called south magnetic pole (S_m).
2. There is some confusion in the nomenclature of the poles. The filed lines go into the earth at the magnetic north pole (N_m) and come out from the magnetic south pole (S_m). The convention arose because the magnetic north was the direction to which the north pole of a magnetic needle pointed; Thus in reality, the north magnetic pole behaves like the south pole of a bar magnet inside the earth and vice versa.
3. Consider a point on the earth. At such a point the direction of the longitude circle determines the geographic north – south direction.
4. The location of the north magnetic pole is at a latitude of 79.74^0N and a longitude of 71.8^0W a place some where the North Canada. The magnetic south pole is at 79.74^0 and 108.22^0E in the Antarctica.

Geographic meridian: The vertical plane which passes through the longitude circle and axis of rotation of the earth is called the geographic meridian.

Magnetic meridian: The vertical plane which passes through the imaginary line joining the magnetic north and south poles is called magnetic meridian.

Magnetic declination (D):

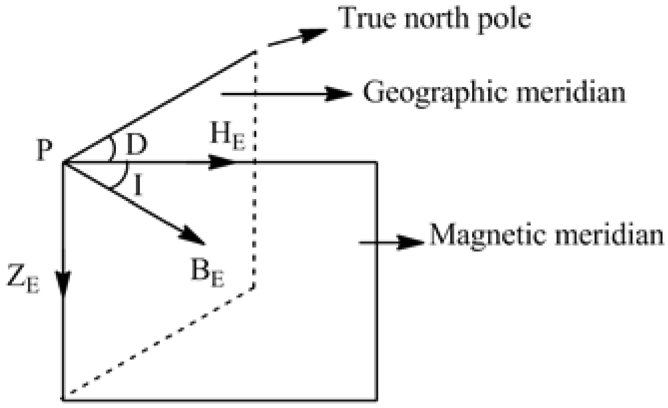


The declination at the place is the angle between the geographic meridian and the magnetic meridian.

Note:

1. The declination is greater at higher latitudes and smaller near the equator.
2. The declination(D) in India is small. It being $0^0\,41'$ E at Delhi and $0^0\,58'$ W at Mumbai.

Horizontal component of the earth's magnetic field (H_E):



Consider a point 'P' on the surface of earth.

Let, B_E=total magnetic field at 'P'.

B_E can be resolved into a horizontal component (H_E) and a vertical component (Z_E) in the magnetic meridian.

H_E= Horizontal component of the earth's magnetic field

Z_E= Vertical component of the earth's magnetic field

Define H_E:

The magnetic field of earth along the horizontal direction in a magnetic meridian is called H_E.

Angle of dip or Inclination (I):

The angle between total magnetic field of earth and horizontal component of earth's magnetic field is called dip.

Dip is zero at the magnetic equator and dip is 90^0 at the magnetic poles.

Mention the magnetic elements of Earth's magnetic field:

1. The declination (D),
2. Inclination (I) (angle of dip)
3. Horizontal component of the earth's magnetic field (H_E).

These are known as the elements of the Earth's magnetic field

Relation between I, H_E and Z_E:

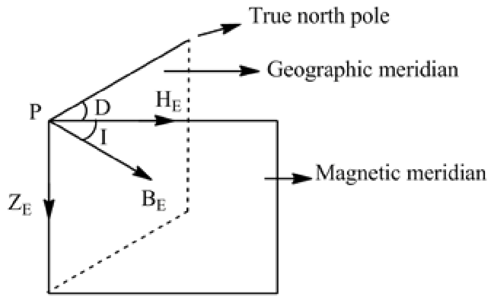
it is found that

$H_E = B_E \cos I$, $Z_E = B_E \sin I$

$$\therefore \quad \frac{Z_E}{H_E} = \frac{B_E \sin I}{B_E \cos I}$$

$$\frac{Z_E}{H_E} = \tan I \quad \text{or} \quad \tan I = \frac{Z_E}{H_E}$$

and $B_E^2 = H_E^2 + Z_E^2$



Magnetisation [M]: We have seen that a circulating electron in an atom has a magnetic moment. In a bulk material, these moments add up vectorially and they can give a net magnetic moment which is non-zero.

The net magnetic moment per unit volume is called magnetisation.

$$\therefore \quad \text{Magnetisation} = \frac{\text{net magnetic moment}}{\text{volume}}$$

$$M = \frac{m}{V}$$

Magnetisation is a vector.

SI unit is Am^{-1} (ampere/metre)

Note:

1. In vector form, $\vec{M} = \frac{\vec{m}}{V}$
2. Let $\vec{B} =$ magnetic field inside the material, $\vec{M} =$ magnetisation

   It is found that

   $\vec{B} \propto \vec{M}$

   $\vec{B} = \mu_0 \vec{M}$, Where, μ_0 = Permeability of free space

Define magnetic intensity (H):

It is the degree to which a magnetic field can magnetise a material.

SI unit of magnetic intensity is Am^{-1} (ampere/metre)

Expression for Magnetic intensity (H):

$$H = \frac{B}{\mu_0} - M$$

Where, H= magnetic intensity,

B=magnetic field inside the material, M=magnetisation

Note: It is a vector field which is defined as

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Expression for $\vec{B}$ in terms of $\vec{H}$ and $\vec{M}$:

We have $$H = \frac{B}{\mu_0} - M$$

$$H + M = \frac{B}{\mu_0}$$

$$\mu_0 (H + M) = B$$

or $$B = \mu_0 (H + M)$$

i.e. The total magnetic field inside the material is due to magnetic intensity (H) and magnetisation (M) of a material.

H depends on external factors and M depends on specific nature of material.

Note: **H = nI,**

n= number of turns per unit length of solenoid, I=Current in solenoid.

Magnetic susceptibility:

Magnetisation is directly proportional to magnetic intensity,

i.e. $$\vec{M} \propto \vec{H}$$

$$\vec{M} = \chi \vec{H}$$

Where, χ=magnetic susceptibility

$$\therefore \quad \chi = \frac{\vec{M}}{\vec{H}}$$

The ratio of magnetisation to the magnetic intensity is called magnetic susceptibility.

Magnetic susceptibility is a dimensionless quantity.

Magnetic susceptibility is a measure of how a magnetic material responds to an external magnetic field.

χ may be positive or negative.

Note:

1. We have $B = \mu_0 (H + M)$ -------------- (1)

   And $M = \chi H$ -------------(2)

   $\therefore$ equation (1) becomes

   $$B = \mu_0 (H + \chi H)$$

   $$B = \mu_0 H (1 + \chi)$$

   but $1+\chi=\mu_r$

   $\therefore$ $B = \mu_0 H (\mu_r)$

   $$B = \mu_0 \mu_r H$$

   But $\mu_0 \mu_r = \mu$ = permeability of material

   $\therefore$ $B = \mu H$

   Where, μ_r=relative magnetic permeability of material

2. $\mu=\mu_0\mu_r$

   but $\mu_r=1+\chi$

   $\therefore$ $\mu=\mu_0(1+\chi)$

Types of magnetic materials

Magnetic materials can be classified into three types

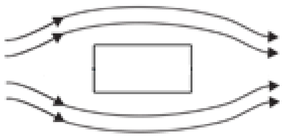
1. Diamagnetic material
2. Paramagnetic material
3. Ferro magnetic material

Diamagnetic materials:

The material which moves from stronger magnetic field to weaker field is called diamagnetic material.

Diamagnetic material is repelled by a magnet.

Ex: Copper, diamond, gold, silver, water, sodium chloride etc.



The above figure shows a diamagnetic material placed in an external magnetic field. The field lines are repelled or expelled and the field inside the material is reduced.

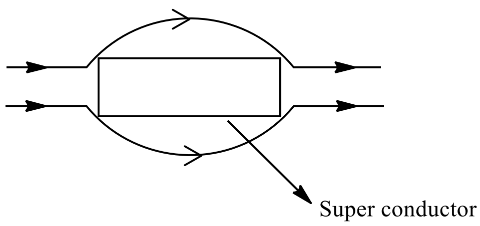
Theory of diamagnetism:

Electrons in an atom revolving (orbiting) around the nucleus possess orbital angular momentum. These orbiting electrons are equivalent to current carrying loop and therefore they possess orbital magnetic moment. In diamagnetic material, resultant magnetic moment in an atom is zero. When magnetic field is applied, the electrons having orbital magnetic moment in the direction of field slow down and the electrons having moment in opposite direction to the field speed up. This happens due to induced current in accordance with Lenz's law. As a result, the material develops a net magnetic moment in the direction opposite to that of applied field and hence the material is repelled by the applied field.

Note:

1) The most exotic diamagnetic materials are superconductors. They exhibit both perfect conductivity and perfect diamagnetism at very low temperature.

   In superconductor the field lines are completely expelled.



2) Super conducting magnets are used for running magnetically levitated trains.
3) The phenomenon of perfect diamagnetism in super conductor is called the **Maisner effect**.
4) For super conductors χ = -1 and μ_r = 0.
5) The conductor whose resistance is zero is called super conductor.

Paramagnetic material:

The material which has a tendency to move from a region of weak magnetic field to strong field is called paramagnetic material.

Paramagnetic material is weakly attracted by a magnet.

Ex: Aluminium, Calcium, Magnesium, Platinum, Tungsten.

The above figure shows a paramagnetic material placed in an external field. The field lines gets concentrated inside the material and field inside is enhanced.

Theory of paramagnetic:

The individual atoms of a paramagnetic material possess a permanent magnetic dipole moment. In the presence of strong external magnetic field $\vec{B}_0$ and at low temperature, the individual atomic dipole moment can be made to align in the direction of $\vec{B}_0$.

It is found that magnetisation (M) is inversely proportional to the absolute temperature (T) and directly proportional to $\vec{B}_0$.

i.e $M \alpha \frac{1}{T}$ and $M \alpha B_0$

$\therefore \quad M \alpha \frac{B_0}{T}$

$$M = C\frac{B_0}{T}$$

Where C = curie constant

It can be shown that

$$\chi = \frac{C\mu_0}{T}$$

The above equation is known as curie's law.

Note:

We have

$$M = \frac{CB_0}{T}$$

but $B_0 = \mu_0 H$

$$\therefore M = \frac{c\mu_0 H}{T}$$

$$\frac{M}{H} = \frac{c\mu_0}{T}$$

but $\frac{M}{H} = \chi$

$$\therefore \chi = \frac{c\mu_0}{T}$$

Curie's law:

Its states that susceptibility of a paramagnetic material is inversely proportional to absolute temperature.

i.e. $\chi \propto \frac{1}{T}$

Note:

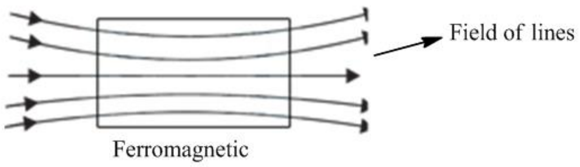
1. According to Curie's law χ and μ of paramagnetic material depend not only on the material, but also on the temperature of the material.
2. As the field increases or temperature is lowered, the magnetisation increases until it reaches the saturation value (M_S), at which point all the dipoles are perfectly aligned with the field. Behind this Curie's law is no longer valid.

Ferromagnetic material:

The material which has strong tendency to move from a region of weak magnetic field to strong field is called Ferromagnetic material.

Ferromagnetic materials are strongly attracted by magnet and strongly magnetised when placed in an external magnetic field.

Ex: Iron, Cobalt, Nickel, Fe_2O_3 etc.



Above figure shows that, in a ferromagnetic material the field lines are highly concentrated

Domains in Ferromagnetic material (Theory of Ferro magnetism):

The individual atoms in a Ferromagnetic materials possess a dipole moment as in paramagnetic material. Some atoms have magnetic moments in same direction due to mutual interaction. The space of ferromagnetic material in which large number of atoms have moments in same direction is called domain.

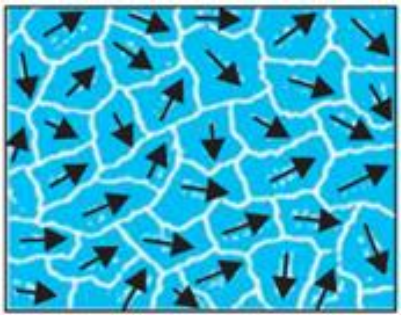


fig (1)

Typical domain size is 1mm and domain contains about 10^{11} atoms. Each domain has its own magnetisation. Direction of net magnetic moment (or magnetisation) varies from domain to domain as shown in figure (1).

When we apply an external magnetic field B_0, all the domains orient themselves in the direction of B_0 and all the domains are amalgamated to form a single giant domain as shown in figure (2).

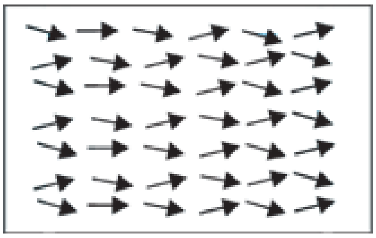


fig (2)

Mention the properties of diamagnetic material:

1. The material which moves from stronger magnetic field to weaker field is called diamagnetic material.
2. Diamagnetic material is repelled by a magnet. Ex: Copper, diamond, etc
3. Relative permeability is less than one ($\mu_r<1$).
4. Susceptibility is always negative.

Mention the properties of paramagnetic material:

1. The material which has a tendency to move from a region of weak magnetic field to strong field is called paramagnetic material.
2. Paramagnetic material is weakly attracted by a magnet. Ex: Aluminium, Calcium.
3. Relative permeability is slightly greater than one ($\mu_r>1$).
4. Susceptibility is always positive.

Mention the properties of ferromagnetic material:

1. The material which has strong tendency to move from a region of weak magnetic field to strong field is called Ferromagnetic material.
2. Ferromagnetic materials are strongly attracted by magnet and strongly magnetised when placed in an external magnetic field. Ex: Iron, Cobalt, Nickel, Fe_2O_3 etc
3. Relative magnetic permeability is much greater than one ($\mu_r>1000$).
4. Susceptibility is always positive.

Hard Ferromagnetics:

The Ferromagnetic materials in which magnetisation persists even when the external magnetic field is removed are called hard Ferromagnets.

Ex: Alnico (An alloy of aluminium, nickel, cobalt and copper) Lodestone

Uses: Hard Ferromagnets are used to prepare permanent magnets.

Soft Ferromagnetics:

The Ferromagnetic materials in which the magnetisation disappears on removal of external field are called soft ferromagnetics.

Ex: Soft iron

Uses: Soft Ferromagnetics are

1. used to prepare electromagnets
2. used in moving coil galvanometer
3. used in transformer.

Effect of temperature on Ferromagnetism:

The Ferromagnetic property depends on temperature. At high temperature, the domain structure disintegrates and magnetisation will disappear gradually with temperature. Then ferromagnet becomes paramagnet.

Curie temperature (T_c): The temperature of transition from ferromagnetic to paramagnetism is called curie temperature.

i.e, at and above curie temperature, ferromagnetic becomes paramagnetic.

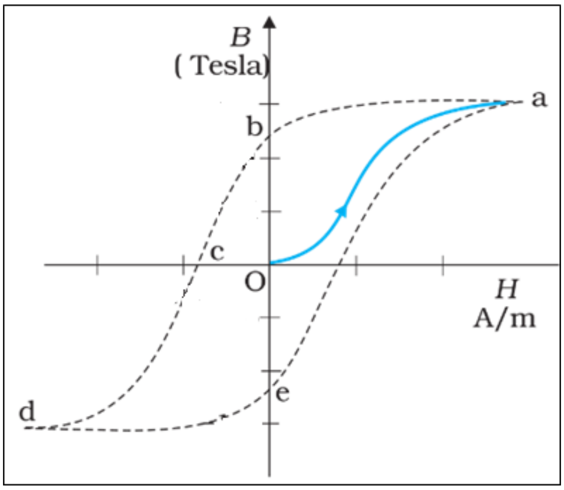
for, cobalt, $T_c = 1394K$, Iron $T_c = 1043K$

Note: The susceptibility of ferromagnetic above the curie temperature is (i.e in the paramagnetic phase of ferromagnetic)

$$\chi = \frac{C}{T - T_c} \qquad (T > T_c)$$

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1+\varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Magnetic hysteresis:



$\vec{B}$ =Magnetic field induced inside the ferromagnet

$\vec{H}$ =Magnetic intensity produced by the solenoid (applied field)

But H=nI

Where, n=number of turns per unit length

I= current in the solenoid

(As I increases H also increases and vice versa i.e. $H \propto I$)

Keep the ferromagnetic material inside the solenoid. As $\vec{H}$ increases then $\vec{B}$ in the ferromagnet also increases and reaches saturation as shown in the curve "Oa". At saturation point, all the domains are aligned and merged to form single domain.

Next, we decrease $\vec{H}$ then $\vec{B}$ also decreases as shown in the curve 'ab'.

at 'b', H=0, but $B \neq 0$. This is called retentivity.

Next, we increase the $\vec{H}$ in reverse direction (by reversing the current in solenoid) then $\vec{B}$ in the ferromagnet goes on decreases, as shown in the curve 'bc' because, alignment of domains is destroyed.

At 'c', H is negative ($H=-100\ Am^{-1}$) but B=0

This is called coercivity.

As reverse field $\vec{H}$ is increased $\vec{B}$ also increases in reverse direction and reaches saturation as shown in curve 'cd'. Similarly the curves 'de' and 'ea' showed the variation of $\vec{B}$ with $\vec{H}$.

From the above results, it is clear that $\vec{B}$ inside the ferromagnet follows $\vec{H}$ or $\vec{B}$ is lagging behind $\vec{H}$. Hence the name hysteresis curve. (Hysterisis means lagging behind).

Retentivity or Remanence:

The ferromagnetic material retain certain magnetisation even when an external magnetic field is removed. This property is called retentivity.

Coercivity: The ferromagnetic material gets demagnatised completely due to external reverse magnetic field. This property is called coercivity.

Coercive field: The reverse magnetic field require to demagnatise the material completely is called coercive field.

Permanent magnet:

The substance which retain their ferromagnetic properties for a long time at room temperature are called permanent magnets.

Example: Cobalt, steel, Alnico, Ticonal, etc.

Properties of permanent magnet:

The material for making permanent magnet should have

1. High permeability (μ_r)
2. High retentivity.
3. High coercivity.

Note: A ferromagnetic rod is kept inside the solenoid and current is passed, then the magnetic field of the solenoid magnetises the rod. This is the simple way to prepare permanent magnet.

Electromagnet: A Ferromagnetic rod kept inside the current carrying solenoid acts as an electromagnet.

Example: soft iron can be used as an electromagnet.

Properties of electromagnets: The materials for making electromagnet should have

1. High permeability (μ_r)
2. Low retentivity
3. Low coercivity.

Uses: Electromagents are used in

1. Electric bells
2. Loudspeakers.
3. Telephone diaphragms.
4. Giant electromagnets are used in cranes to lift machinery, and bulk quantities of iron and steel.

Note:

1. During magnetisation and demagnetisation of ferromagnetic material, energy will be lost in the form of heat. This loss of energy is called hysteresis loss.
2. Area of hysteresis loop gives the hysteresis loss.
3. For permanent magnet, area of hysteresis loop is large, therefore hysteresis loss is more.
4. For electromagnet, area of hysteresis loop is small. Therefore hysteresis loss is less.

One Marks Questions:

1. Draw the pattern of magnetic field lines for a bar magnet. (March-2014)
2. Mention the expression for torque acting on a magnetic needle (magnetic dipole).
3. Define H_E.
4. Define angle of dip or inclination.
5. What is magnetic declination? (March-2015)
6. Define geographic meridian.
7. Define magnetic meridian.
8. Mention the relation between I H_E and Z_E
9. What is curie temperature?
10. State Curie's law. (July-2015), (July-2016)
11. Define magnetisation of a sample. (March-2016)

Two Marks Questions:

1. Write any two properties of magnetic field lines. (July-2014)
2. Define magnetic flux and mention express for it.
3. Draw the variation of magnetic field (B) with magnetic intensity (H). When ferromagnetic material is subjected to a cycle of magnetisation. (March-2016)
4. State and explain Gauss's law of magnetism.(July-2016)
5. Define magnetic intensity and mention an expression for it.
6. What is magnetic susceptibility? For which material is it law and positive. (March-2014)
7. What are diamagnetic materials? Give one example
8. What are paramagnetic material? Give one example.
9. State and explain cusie's law
10. What are ferromagnetic material? Give one example.
11. What are Hard magnetics? Give an example.
12. What are soft ferromagnetic? Give an example.
13. What is permanent magnet? Give an example.
14. What is electromagnet? Give one example.

Three Marks Questions:

1. What are : i) Magnetic declination, ii) Magnetic dip
   iii) Horizontal component of earth's magnetic field at a place? (March-2014, March-2017)
2. Write three differences between diamagnetic and paramagnetic substances.(March-2015)

3. Define: a) Magnetic declination, b) Magnetic dip
   Mention the S.I unit of magnetisation. (July-2015)
4. Write three properties of ferromagnetic materials. (March-2016)
5. State and explain Gauss's law in magnetism. (Julu-2016)

Five Marks Questions:

1. Write any four properties of ferromagnetic materials and give an example for it. (July-2014, March-2017)

Chapter-6: ELECTRO – MAGNETIC INDUCTION

Introduction:

Electricity and magnetism were considered separate and unrelated phenomena for a long time. But in nineteenth century the scientist's oriested and Ampere established the fact that electricity and magnetism are interrelated. They found that moving electric charge or current produces the magnetic field. Michael Faraday and Joseph Henry have demonstrated the converse effect. That is variable magnetic fields produce the current.

The phenomena in which electric current is generated by varying magnetic fields is called electromagnetic induction.

Electromagnetic induction was discovered by Faraday and Joseph Henry.

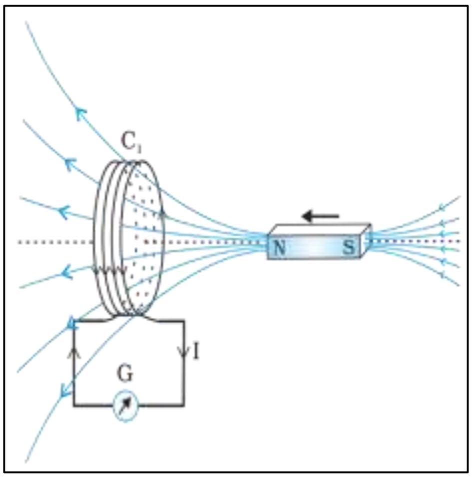
Note: Is not merely of theoretical or academic interest but also of practical utility. Imagine a world where there is no electricity - no electric lights, no trains, no telephones and no personal computers. The pioneering experiments of Faraday and Henry have led directly to the development of modern day generators and transformers. Today's civilization owes its progress to a great extent to the discovery of electromagnetic induction.

THE EXPERIMENTS OF FARADAY AND HENRY

The discovery and understanding of electromagnetic induction are based on a long series of experiments carried out by Faraday and Henry. We shall now describe some of these experiments.

Experiment 1: Coil-Magnet experiment:

The arrangement consists of a coil with a galvanometer and a bar magnet as shown in figure.



Observation: It was observed that.

1. When the bar magnet was at rest, the galvanometer showed no deflection.
2. When the north-pole of a bar magnet is moved towards the coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in the coil.
3. When north-pole of a bar magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal of the current's direction.
4. When the south-pole of the bar magnetic is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed with the north-pole for similar movements.
5. When the bar magnet is held fixed and the coil C_1 is moved towards or away from the magnet, the same effects are observed.

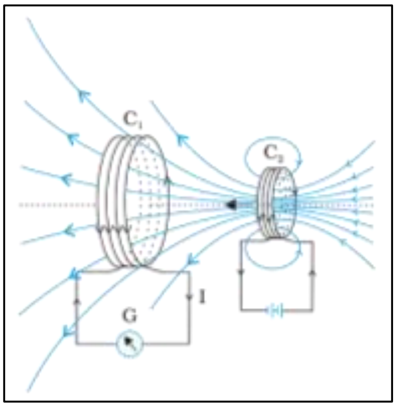
6. The deflection is found to be larger when the magnet is pushed towards or pulled away from the coil faster.
7. The deflection lasts as long as coil C_1 is in motion.

Conclusion: It shows that, 'The relative motion between the coil and magnet induces the electric current in the coil.'

Experiment 2: Coil-Coil experiment:

The coil C_1 is connected to galvanometer and second coil C_2 is connected to battery as shown.

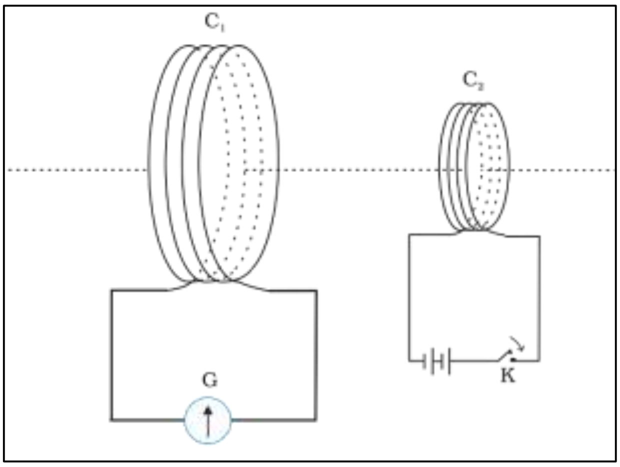
Observation: It was observed that,



1. As coil C_2 is moved towards the coils C_1 the galvanometer shows a deflection. This indicates that electric current is induced in coil C_1.
2. When the coil C_2 is moved away, the galvanometer shows a deflection again, but in the opposite direction.
3. The deflection lasts as long as coil C_2 is in motion.
4. When the coil C_2 is held fixed and C_1 is moved, the same effects are observed.

Conclusion: The relative motion between the coils that induces the electric current.

Experiment 3:



The coil C_1 is connected to galvanometer and second coil C_2 is connected to battery through a taping key (K) as shown.

Observation: It was observed that,

1. Galvanometer shows a momentary deflection when the tapping key k is pressed.
2. The pointer in the galvanometer returns to zero immediately, if the key is held pressed continuously.
3. When the key is released, galvanometer shows a momentary deflection again but in the opposite direction.
4. It is also observed that the deflection increases dramatically when an iron rod is inserted into the coils along their axis.

Conclusion: In this experiment, the coils have no relative motion yet changing current on primary coil induces emf, in the secondary coil.

Note: The motion of a magnet towards or away from coil C_1 in experiment (1) changes the magnetic flux linked with the coil C_1. The change in magnetic flux induces an emf in the coil C_1. Therefore induced emf produces the induced current to flow in coil C_1 and through galvanometer.

Faraday's law of Induction:

It states that "the magnitude of the induced emf in a circuit or coil is equal to the time rate of change of magnetic flux through the circuit".

i.e. induced emf= - rate of change of flux

i.e. $e = -\frac{d\phi_B}{dt}$

Where, e=induced emf

$d\phi_B$=change in magnetic flux in dt second.

Negative sign indicates the direction of emf.

Note:

1. Faradays law for a coil having 'N' number of turns is,

$$e = -N\frac{d\phi_B}{dt}.$$

   Where, N=number of turns in the coil

   $\therefore$ If N increases e also increases

2. We have $e = -\frac{d\phi_B}{dt}$

   But $d\phi_B = BA\cos\theta$ ($d\phi_B = BdS\cos\theta$)

   i.e. Magnetic flux can be changed by changing B, A or θ

   $\therefore$ Magnitude of e can be changed by changing B, A or θ

Lenz's law: It states that "The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it".

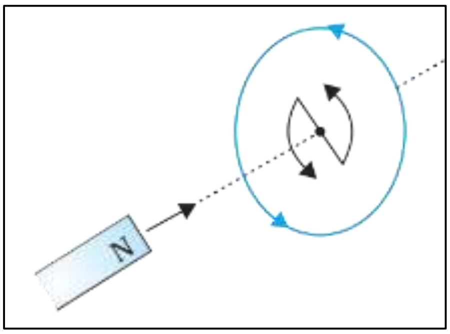
Verification of Lenz's law and conservation of energy (Qualitative study):

According to Faraday's law

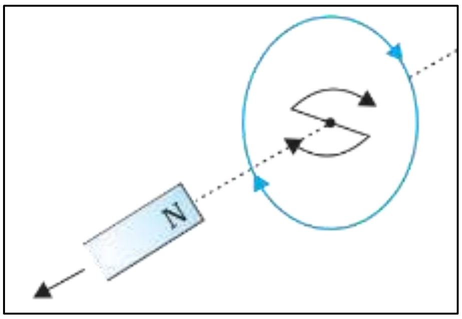
$$e = -\frac{d\phi_B}{dt}$$

Negative sign indicates that induced emf and induced current oppose the change in magnetic flux.

As the north pole of a magnet moves towards the coil, the magnetic flux through the coil increases. Hence current is induced in the coil in such a direction that it opposes the increase in flux. This is possible only if the current in the coil is in a anticlockwise direction. As a result the face of coil towards the north pole of magnet behaves as a north pole as shown in figure.



Similarly, if the north pole of a magnet is moving away from the coil, then magnetic flux through the coil decreases. To oppose this decrease in magnetic flux, the current in the coil is induced in clockwise direction. As a result the face of a coil towards the northpole of a magnet behaves as a south pole as show in figure.



In the above cases, the bar magnet experiences a repulsive force due to induced current therefore the person has to do work in moving the magnet. The work done by the person is converted into electrical energy. Hence Lenz's law follows law of conservation of energy.

Derive the expression for motional electromotive force:

Let us consider a rectangular conductor PQRS in which the conductor PQ is free to move.

Let, $\vec{B}$ = Magnetic field perpendicular to plane of PQRS,

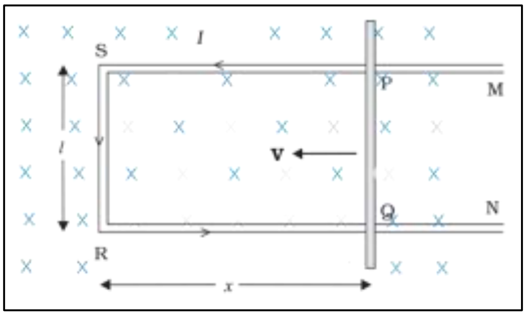
l=length of conductor PQ,

x=distance between RS and PQ

v=constant velocity of PQ

Area of loop PQRS is

$A = lx$



The magnetic flux enclosed by the loop PQRS is

$\phi_B = Blx$

According to Faraday's law

$$e = -\frac{d\phi_B}{dt}$$

Where, e=induced emf in straight conductor PQ

$$\therefore \quad e = -\frac{d}{dt}(Blx)$$

$$e = -Bl\frac{dx}{dt} \qquad [\because \text{ x changes with time}]$$

$$e = -Bl(-v)$$

$$e = Blv$$

This is the expression for motional emf.

Note:
$\phi_B = BA\cos\theta$
$\theta = 0^0$, $\cos 0^0 = 1$
$A = lx$
$\phi_B = Blx\cos 0^0$
$\phi_B = Blx$

$\because \frac{dx}{dt} = -v$, -ve sign indicates PQ moves from right to left (along –ve x-axis)

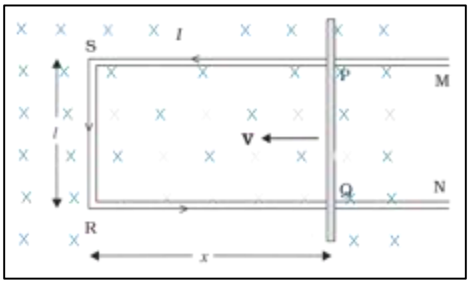
Note: The name motional emf is because we can produce induced emf by moving a conductor instead of varying the magnetic field.

Derive the expression for motional emf by using Lorentz force:

Let, q= any arbitrary charge in the conductor PQ

v=velocity of conductor PQ.

Therefore the charge will also be moving with velocity 'v' in the magnetic field B [$\vec{B}$ and $\vec{v}$ are perpendicular to each other]



The Lorentz force acting on 'q' is

$F = qvB$

Therefore all the charges experience the same force and they move from P to Q.

$\therefore$ workdone = force × displacement

$W = Fl$

Where, l=length of PQ

$W = qvBl$

$\frac{W}{q} = vBl$ ---------------- (1)

w.k.t. Lorentz force
$F = qE + qVB\sin\theta$
$E = 0$,
$\therefore F = qVB\sin\theta$
$\theta = 90^0$, $\sin 90^0 = 1$
$\therefore F = qVB$

But $e = \frac{W}{q}$ $\quad$ $\text{emf}(e) = \frac{\text{workdone}}{\text{charge}}$

$\therefore$ equation (1) becomes

$e = Blv$

A quantitative study of Lenz's law (Energy consideration):

Let, I=induced current in the loop PQRS, e=induced emf,

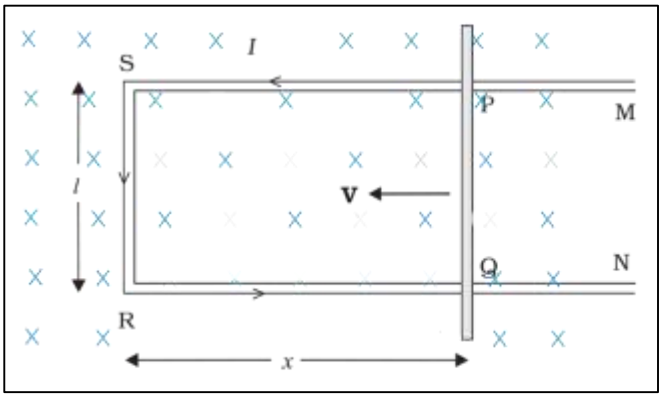
r=resistance of movable arm PQ

Assume that other arms QR, RS and SP have negligible resistance.

But $I = \frac{e}{r}$

But $e = Blv$

$\therefore$ $I = \frac{Blv}{r}$ -------------- (1)



Force acting on moving conductor PQ is

$F = BIl\sin\theta$

but $\theta = 90^0$, $\sin 90^0 = 1$

$\therefore$ $F = BIl$

$F = B\left(\frac{Blv}{r}\right)l$

(from equation (1))

$F = \frac{B^2 l^2 v}{r}$ -------------------(2)

To push the rod PQ, the power is required $\quad$ P=Fv from equation (2)

[Power=Force×Velocity]

$P = \frac{B^2 l^2 v^2}{r}$ ------------------ (3)

Equation (3) is the mechanical power (i.e. mechanical energy required to push the rod)

This mechanical energy is converted into electrical energy and finally electrical energy is converted into thermal energy (Joule heat).

Joule heat is given by

$P_J = I^2 r$ (Electrical power=current × resistance)

$P_J = \left[\frac{Blv}{r}\right]^2 r$ (from equation (1))

$P_J = \frac{B^2 l^2 v^2}{r}$ ---------------- (4)

i.e. equation (3) and (4) are identical

This indicates that, mechanical energy which was needed to move the arm PQ is converted into electrical energy and then thermal energy

i.e. Mechanical power → electrical power → thermal power.

Note: The relation between charge flow and change in magnetic flux.

According to Faraday's law

$e = -\frac{d\phi_B}{dt}$ $|e| = \frac{d\phi_B}{dt}$ ------------------ (1) $|e| =$ magnitude of emf

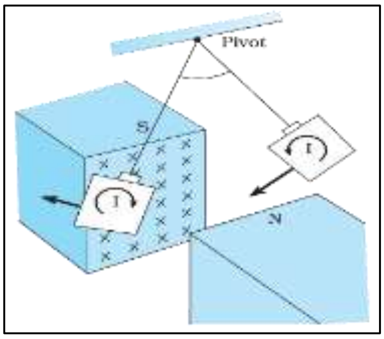
but $|e| = Ir$ $(\because V = IR)$

$\therefore$ $|e| = \frac{dq}{dt} r$ ---------------- (2) $\left(I = \frac{dq}{dt}\right)$

From equation (1) and (2) $\frac{dq}{dt} r = \frac{d\phi_B}{dt}$

$\therefore$ $dq \times r = d\phi_B$ $dq = \frac{d\phi_B}{r}$

Eddy currents:

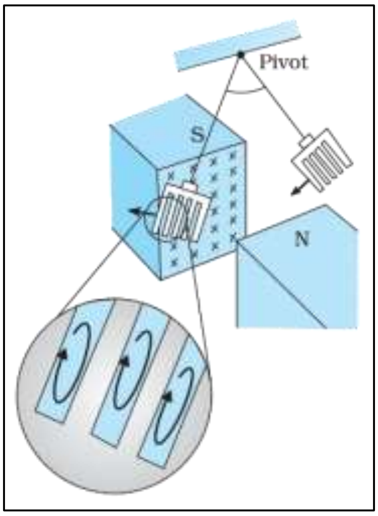


These are the circulating currents induced in a bulk piece of conductor due to change in magnetic flux linked with it.

When bulk piece of conductor is subjected to varying magnetic flux, then induced currents are produced in a conductor like swirling eddies in water. These currents are called eddy currents. Eddy current was discovered by Foucault.

A copper plate is allowed to swing between the pole pieces of strong magnet as shown in figure. Now a magnetic flux through the plate changes, as a results eddy currents are produced in the plate. According to Lenz's law eddy currents (induced currents) oppose the motion of a plate. Therefore oscillation of plate is damped and finally the plate comes to rest.

Note:



1. If rectangular slots are made in the copper plate as shown in figure area available to the flow of eddy currents is less. Thus oscillated plate with holes or slots reduces electromagnetic damping and plate swings more freely.
2. Eddy currents are minimised in transformer or electric motor by using laminations of metal to make core. The laminations are separated by an insulator.

Advantages (Applications) of Eddy currents:

Eddy currents are used to advantage in certain applications like,

1. ***Magnetic braking in trains*:** Strong electromagnets are situated above the rails in some electrically powered trains. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train. As there are no mechanical linkages, the braking effect is smooth.
2. ***Electromagnetic damping*:** Certain galvanometers have a fixed core made of nonmagnetic metallic material. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.
3. ***Induction furnace*:** Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.
4. ***Electric power meters*:** The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil. You can observe the rotating shiny disc in the power meter of your house.

Disadvantages of Eddy current:

1. The production of eddy currents in a metallic block leads to the loss of electric energy in the form of heat.
2. The heat produced due to eddy currents breaks the insulation used in the electrical machine or appliances.
3. Eddy currents may cause unwanted dampening effect.

Note 1) : If eddy current increases, the dissipation of electrical energy into heat also increases i.e. energy loss$\propto I^2$, I=eddy current.

Note 2) : We know that magnetic flux through a coil is directly proportional to the current.

i.e. $\phi_B \alpha I$, for N turns, $N\phi_B \alpha I$,

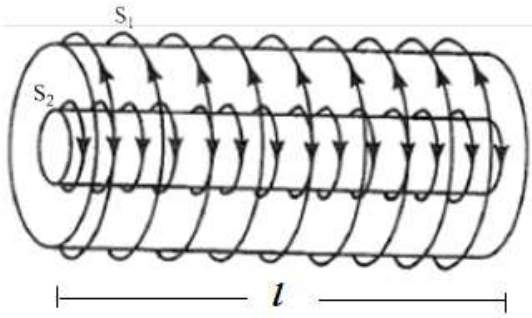
If flux vary with time, then

$$\frac{d\phi_B}{dt} = \frac{dI}{dt}$$

Mutual inductance: The phenomenon in which an emf is being induced in one coil due to varying current in neighbouring coil is called Mutual inductance.

Derive an expression for Mutual inductance [M]:

Consider two long co-axial solenoids S_1 and S_2 as shown in figure.



Let, r_1 = radius of s_1

r_2 = radius of s_2

l = Length of s_1 and s_2

N_1 = Number of turns of s_1

$n_1 = \frac{N_1}{l}$ =number of turns per unit length.

N_2 = Number of turns of s_2,

$n_2 = \frac{N_2}{l}$ = number of turns per unit length.

Let I_2 be the current in S_2. Therefore S_2 produces the magnetic field B_2.

Then magnetic flux linked with S_1 is

$N_1\phi_1 \alpha I_2$

$\therefore N_1\phi_1 = M_{12}I_2$

$\therefore M_{12} = \frac{N_1\phi_1}{I_2}$ ------- (1)

> $\phi = BA\cos\theta$
> $A = \pi r^2$
> $\theta = 0^0$, $\cos 0 = 1$
> $\phi_1 = B_2A_1$
> $\phi_1 = B_2(\pi r_1^2)$

Where M_{12} = Mutual inductance of S_1 w.r.t. S_2

Let, $\phi_1 = B_2\pi r_1^2$

but $B_2 = \mu_0 n_2 I_2$

$\therefore \phi_1 = \mu_0 n_2 I_2 \pi r_1^2$

$\therefore$ eqn (1) becomes

> For solenoid
> $B = \mu_0 nI$

> $n_1 = N_1 l$
> $n_2 = N_2 l$

$$M_{12} = \frac{n_1 l \times \mu_0 n_2 I_2 \pi r_1^2}{I_2}$$

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$$

Similarly it can be shown that

$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$, Where, M_{21}=Mutual inductance of S_2 w.r.t. S_1

$\therefore$ $M_{12} = M_{21} = M$

$$M = \mu_0 n_1 n_2 \pi r_1^2 l$$

This is the expression for mutual inductance or co-efficient of mutual inductance.

Note:

1. If air is present in the solenoid (coil), then $M = \mu_0 n_1 n_2 \pi r_1^2 l$
2. If a medium of relative permeability μ_r is present, then $M = \mu_0 \mu_r n_1 n_2 \pi r_1^2 l$

Derive the relation between induced e.m.f and mutual inductance:

We have $N_1\phi_1 = MI_2$ $(M_{12} = M)$

Or $\frac{d}{dt}(N_1\phi_1) = \frac{d}{dt}(MI_2)$

$$N_1\frac{d(\phi_1)}{dt} = M\frac{d}{dt}(I_2)$$

> According to Faradays law
> $e_1 = -N_1\frac{d\phi_1}{dt}$
> Where e_1 = induced e.m.f. in s_1

Multiplying negative sign on both side

$$-N_1\frac{d(\phi_1)}{dt}=-M\frac{d}{dt}(I_2)$$

$$e_1=-M\frac{dI_2}{dt}$$

It shows that varying current in s_2 can induce e.m.f. in s_1

Note: 1) Similarly $e_2=-M\frac{dI_1}{dt}$, Where e_2 = induced e.m.f in s_2

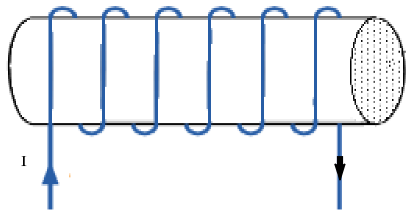
2) From the above equations, it is clear that varying current in one coil can produce induced e.m.f in another coil.

Self Inductance:

The phenomenon of an e.m.f. being induced in a coil due to a varying current in the same coil is called self induction.

Derive the relation between induced emf and self inductance:

Consider a single coil having 'N' number of turns. When current 'I' through the coil varies, then magnetic flux linked with the coil also varies. As a result an e.m.f is induced in that coil.



In this case

$$N\phi_B \propto I$$

Where ϕ_B = magnetic flux linked with the coil

$$\therefore N\phi_B = LI$$

Where, L = co-efficient of self inductance of the coil.

According to Faraday's law

$$e=-\frac{d(N\phi_B)}{dt}$$ where, e = induced e.m.f.

$$e=-\frac{d}{dt}(LI)$$

$$e=-L\frac{dI}{dt}$$

Derive an expression for self Inductance (L) of a coil:

Consider a coil like solenoid

Let N = total number of turns in the coil

l = length of coil (solenoid)

$n = \frac{N}{l} =$ number of turns per unit length

I = current flows

∴ Magnetic field produced by the coil (solenoid) is

$B = \mu_0 nI$

Magnetic flux, $\phi_B = BA$ $\left[\begin{array}{c}\phi_B = BA\cos\theta,\ \text{but}\ \theta = 0^0 \\ \therefore \phi_B = BA\end{array}\right]$

$\phi_B = \mu_0 nIA$

Where A = cross sectional area of coil

We have $N\phi_B = LI$ $\left(n = N/l\ \therefore N = nl\right)$

$\therefore (nl)(\mu_0 nIA) = LI$

$L = \mu_0 n^2 Al$

This is the expression for self inductance of a coil.

Note:

1) If we fill the inside of the solenoid (coil) with a material of relative permeability μ_r then $L = \mu_0\mu_r n^2 Al$
2) $L \propto A$ i.e. self inductance (L) depends in its geometry
   $L \propto \mu_r$ i.e L depends on permeability of medium.
3) The self induced e.m.f is also called the back e.m.f as it opposes any change in the current in a circuit.

Expression for the potential energy stored in a self inductance coil:

Consider a coil of self inductance 'L'. The induced e.m.f (back e.m.f) in a coil opposes any change in the current. So work is to be done in opposing the back e.m.f. This work done is stored as magnetic potential energy.

rate of work done = $\frac{dw}{dt}$

$\therefore \frac{dw}{dt} = eI$

But $e = L\frac{dI}{dt}$

$$\therefore \frac{dw}{dt} = L\,I\,\frac{dI}{dt}$$

$\therefore dw = L\,I\,d\,I$

Total work done is

$$\int dw = \int_0^I LIdI$$

$$W = L\int_o^I IdI$$

$$W = L\left[\frac{I^2}{2}\right]_o^I$$

$$W = \frac{L}{2}\left[I^2 - 0\right]$$

$$W = \frac{1}{2}L\,I^2$$ This total workdone is stored as magnetic energy.

$e = \frac{w}{q}$

W=eq, but q=It

W = e.I.t

$\frac{w}{t} = eI$

Or

$\frac{dw}{dt} = eI$

AC generator: It is a device which converts mechanical energy in to electrical energy.

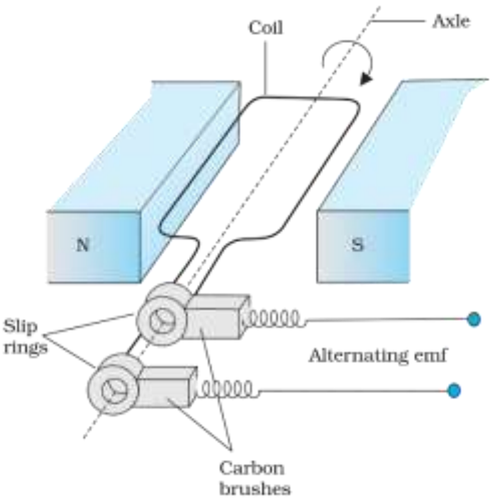
AC generator was developed by Nicola Tesla.

Principle of AC generator:

When a coil is rotated in a magnetic field, the effective area of a loop (coil) changes. As a result an e.m.f. is induced in the loop.

Construction of AC generator:

The basic elements of an AC generator are shown in figure. It consists of a coil mounted on a rotor shaft. The axis of rotation of the coil is perpendicular to the direction of the magnetic field. The coil [called armature] is mechanically rotated in the uniform magnetic field. When a coil is rotated, the magnetic flux through the coil changes, so an e.m.f is induced in the coil. The ends of a coil are connected to an external circuit by means of slip rings and brushes.



Theory of AC generator:

Let N = number of turns in the coil

A = Area of coil

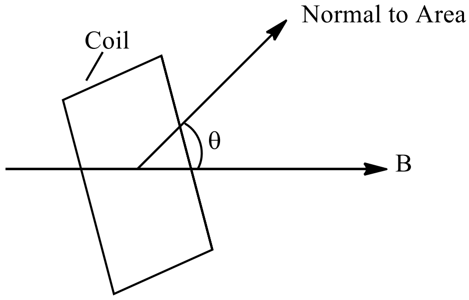
$\vec{B}$ =uniform magnetic field

θ=angle between $\vec{B}$ and normal to A

$\omega = \frac{\theta}{t}$ = angular velocity of coil

∴ θ=ωt

t=time



Effective area of coil = Acosθ

Flux through the coil is, ϕ_B=NBAcosθ

∴ ϕ_B=NBAcosωt

According to Faradays law, induced emf is

$$e = -\frac{d\phi_B}{dt}$$

$$e = -\frac{d[NBA\cos\omega t]}{dt}$$

$$e = -NBA\frac{d[\cos\omega t]}{dt} \qquad \frac{d(\cos\theta)}{dt} = -\sin\theta$$

$$e = -NBA[-\sin\omega t \times \omega]$$

e=NBAωsinωt

$e=e_m\sin\omega t$ --------------- (1)

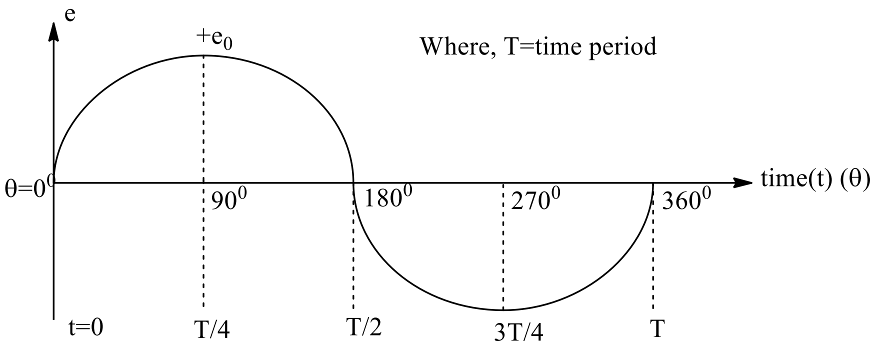
where, e_m=NBAω=Maximum value of emf (Peak value)

but ω=2πν, where γ=frequency of coil

∴ equation (1) becomes

$e=e_m \sin(2\pi\nu t)$

The induced emf varies from $+e_m$ to $-e_m$ with time ‘t’ as shown below.



Note:

1. In commercial generator, the mechanical energy required for rotation of the armature is provided by water falling from height. These are called hydro-electric generators.
2. In some generators, the steam at high pressure produces the rotation of the coil (armature). These are called thermal generators. In thermal generator, steam is produced by coal or other sources.
3. Instead of coal, if a nuclear fuel is used to produce steam, then generators are called nuclear power generator.
4. Modern day generators produce electric power as high as 500MW.
5. In most generators, the coil is held stationary and electromagnets are rotated. The frequency of rotation is 50Hz in India. In USA it is 60 Hz.

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One Mark Questions:

1. State Faraday's law of electromagnetic induction. (March-2016)
2. Mention the significance of Lenz's law (March-2015, 2016) (March 2017)
3. How can you minimize the loss due to eddy currents?
4. What is mutual inductance?
5. Mention the relation between induced emf and mutual inductance.
6. What is self induction? (July-2015)
7. What is AC generator?
8. On which principle AC generator works?
9. Give the expression for energy stored in a inductance coil carrying current (March– 2014)
10. State Faraday's law of electromagnetic induction (July-2014)

Two Mark Questions:

1. State and explain Faraday's law of induction. (March 2017)
2. What are eddy currents? Give one use of it (July-2014, 2015)
3. Mention any two advantage of eddy current in practical applications (March-2014)
4. What is meant by self inductance and mutual inductance? (July-2016)
5. Current in a coil falls 2.5A to 0.0A in 0.1 second inducing an emf of 200V. Calculate the value of self inductance (March-2015)
6. The current in a coil of self inductance. 5mH changes from 2.5A to 2A in 0.01 second. Calculate the value of self induced emf (March-2016).

Three Marks Questions:

1. Explain briefly the coil and magnet experiment to demonstrate electromagnetic induction (March-2016)
2. Explain briefly the coil-coil experiment.
3. Explain briefly coil-coil experiment without resistive motion to demonstrate electromagnetic induction.
4. State and explain Lenz's law for induced emf. (March-2014)
5. Derive an expression for electromotive force (motional emf) induced in a rod moving perpendicular to the uniform magnetic field (July-2014), (March-2015, July-2016, March 2017)
6. Derive an expression for mutual inductance or co – efficient of mutual inductance.
7. Derive the relation between induced emf and self.
8. Derive an expression for self inductance (L) of a coil.
9. Derive the expression for energy stored in a current carrying coil (July-2015)
10. Describe the working of Ac generator.

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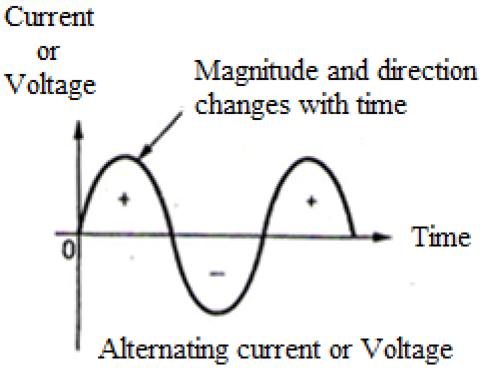
Chapter-07

ALTERNATING CURRENT

Alternating current (AC):

The current or voltage whose magnitude changes continuously with time and its direction reverses periodically is called Alternating current or Voltage (A.C.).

In alternating wave form there are two half cycles, one positive and other negative. These two half cycles make one cycle. Current increase in magnitude in one particular direction attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly as shown in figure.



Expression for alternating current:

$I = I_m \sin\omega t$

Where, I=instantaneous current, I_m = amplitude of current (peak current),

ω=angular frequency, t=time

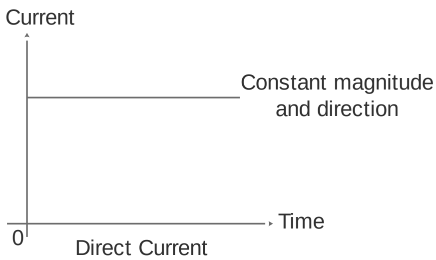
Expression for alternating voltage:

$V = V_m \sin\omega t$

Where, V=instantaneous voltage, V_m=amplitude of voltage (peak voltage)

Direct current (DC):

The current or voltage which does not change direction with time is called Direct current (DC).



Direct current, $I = \frac{q}{t}$ or $I = \frac{dq}{dt}$,

Where, q=charge flows in a time t.

Important terms used in the study of AC

Period [T]: It is the time taken by the A.C to complete one complete cycle of variation. SI unit is second.

Frequency [ν]: It is the number of complete cycles of variation of A.C produced in one second.

SI unit is hertz (Hz)

Instantaneous Value of V and I : It is the value of the alternating voltage or current induced at any instant of time.

Peak Value (V_m and I_m) ; It is the maximum value of the induced voltage or current. (OR) It is the amplitude of instantaneous voltage or current.

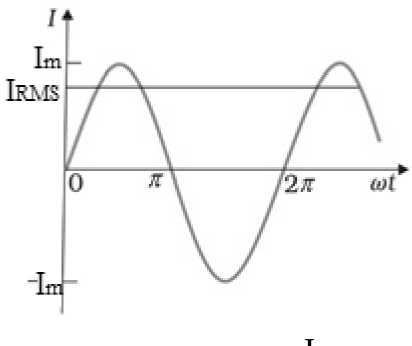
Phase: It is the fraction of time period which has elapsed since the current or voltage passed through zero value.

Note: We see that, the voltage and the current varies sinusoidally and have corresponding positive and negative values during each cycle. Thus the sum of the instantaneous current values over one complete cycle is zero and the average current is zero. To avoid this difficulty, rms value is taken.

R.M.S. Value (Root Mean Square Value) OR Effective value (V_{RMS} and I_{RMS}):

To express A.C. power in the same form as D.C. power $\left(P = I^2R\right)$, a special value of current is defined and used. It is called, root mean square (r.m.s) OR effective current.

The r.m.s current is defined as *"The equivalent D.C. current that would produce the same average power loss as the alternating current."*



The r.m.s current is defined by $I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 \times I_m$

The r.m.s voltage is defined by $V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 \times V_m$

Where, I_m= maximum current or peak current, V_m= maximum voltage or peak voltage.

Note: The RMS value is also defined as "the square root of the mean of the sum of the squares of all the instantaneous values of voltage or current taken over one full cycle."

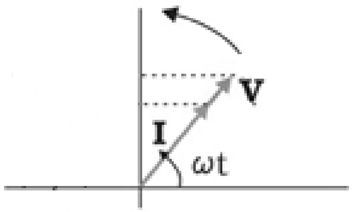
Advantages of A.C.

1) The voltages in A.C. system can be raised OR lowered with the help of a device called transformer. In D.C. system, raising and lowering of voltages is not so easy.
2) In a transmission line, we prefer A.C. transmission because copper loss is lesser and always economical and efficient.
3) Whenever it is necessary, A.C. supply can be easily converted to obtain D.C. supply.
4) Whenever we tune our radio to a favourite station, we are taking the advantage of a special property of A.C. circuits.

Note:

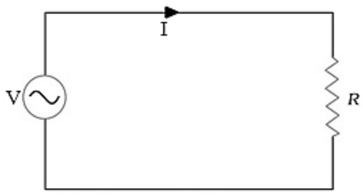
i) A.C. mains frequency is 50 Hertz OR 50 cycles/sec in India.
ii) D.C. frequency is '0' Hertz.
iii) The A.C. source is called oscillator. The symbol of AC source in a circuit is ∿

Representation of A.C. Current and voltage by Rotating vectors – Phasors:



In order to show the phase relationship between voltage and current in an A.C. circuit, we use the notion of phasors. A phasor is a rotating vector which rotates about the origin with angular speed 'ω' as shown in figure. Rotating vector rotates in anticlockwise direction.

A.C. voltage applied to resistor:



The resistor is connected to A.C source as shown in figure. A.C. voltage of a source is given by

$$V = V_m \sin\omega t \longrightarrow (1)$$

Where, V_m = amplitude of voltage or peak voltage and

ω= angular frequency.

Let, I=alternating current, R=resistance of resistor,

V_R=IR=Potential difference across R.

According to Kirchhoff's loop rule,

$$V = V_R$$

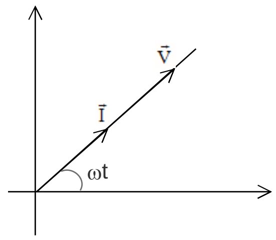
$$V_m \sin\omega t = IR$$

$$I = \frac{V_m}{R}\sin\omega t$$

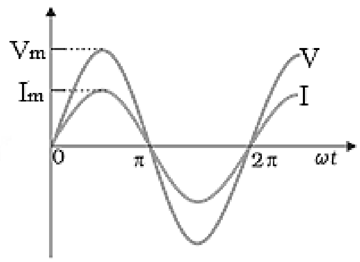
$$I = I_m \sin\omega t \longrightarrow (2)$$

Where, $I_m = \frac{V_m}{R}$ =amplitude of current (peak current)

Comparing equation (1) and (2), it is observed that the voltage (V) and current (I) are similarly varying and are in phase. The phase relationship between V and I is shown vectorically (Phasor diagram**)** as shown in the figure.



Phasor diagram



Graph of V and I verses ωt

Note: In an A.C. circuit containing R, both V and I reach zero, maximum and minimum values at the same time, clearly voltage and current are in phase each other.

Phase difference between V & I = ϕ = zero

Derive an expression for instantaneous power dissipated in the resistor:

We know that, $P = I^2R$

$$P = (I_m \sin\omega t)^2 R \quad (\because I = I_m \sin\omega t)$$

$$P = I_m^2 \sin^2\omega t\, R$$

$$P = I_m^2 R \sin^2\omega t$$

The average value of 'P' over a cycle is

$$\bar{P} = \langle I_m^2 R \ \sin^2\omega t \rangle$$

Since $I_m^2 R$ are constant,

$$\bar{P} = I_m^2 R \ \langle \sin^2\omega t \rangle \text{ ------------ } (1)$$

Note:

$$\langle \cos 2\omega t \rangle = \frac{1}{T}\int_0^T \cos 2\omega t \ dt$$

$$= \frac{1}{T}\left[\frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{1}{2\omega T}\left[\sin 2\omega T - 0\right] = 0$$

$\langle \ \rangle$ = Symbol of average

We have, $\langle \sin^2 \omega t \rangle = \left\langle \frac{1}{2}\left(1-\left(\cos 2\omega t\right)\right)\right\rangle$

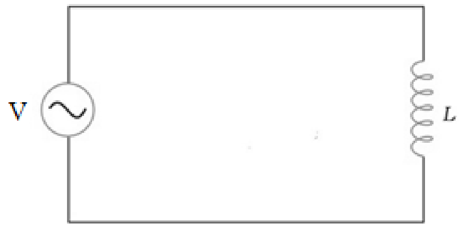
$\left(\text{But, } \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\right)$ and since, $\langle \cos 2\omega t \rangle = 0$

$\therefore \quad \langle \sin^2 \omega t \rangle = \frac{1}{2}[1-0] = \frac{1}{2}$

Equation (1) becomes,

Thus, $\boxed{\bar{P} = \frac{1}{2} I_m^2 R}$

A.C. Voltage applied to an inductor



The inductor of negligible resistance is connected to A.C. source as shown in figure.

A.C voltage of the source is given by

$$V = V_m \sin \omega t \text{ . -----------(1)}$$

Let, L= self inductance of an inductor (coil), I= Alternating current.

emf induced across L is, $V_L = -L\frac{dI}{dt}$. $\left(\frac{dI}{dt} = \text{Rate of change of current}\right)$

According to Kirchhoff's loop rule,

$$V = -V_L$$

$$V_m \sin \omega t = -\left(-L\frac{dI}{dt}\right)$$

$$V_m \sin \omega t = L\frac{dI}{dt}$$

Then, $\frac{dI}{dt} = \frac{V_m}{L}\sin \omega t$

Integrating on both sides with respect to time,

$\int \frac{dI}{dt}dt = \frac{V_m}{L}\int \sin(\omega t)dt$ $\left[\int \sin \omega t = -\frac{\cos \omega t}{\omega}, \ \int \frac{dI}{dt}dt = \int dI = I\right]$

And get,

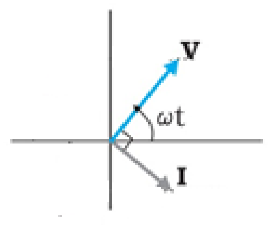
$$I = -\frac{V_m}{\omega L}\cos(\omega t) + \text{constant}$$

$$I = -\frac{V_m}{\omega L}\cos(\omega t) \qquad [\text{constant} = 0]$$

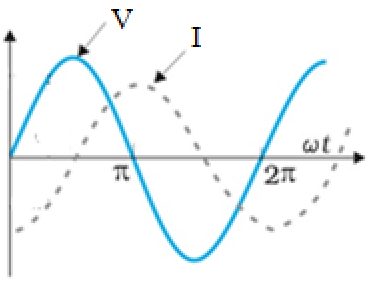
$$I = I_m \sin\left(\omega t - \pi/2\right) \text{ ----------(2)} \qquad \left[-\cos\omega t = \sin\left(\omega t - \pi/2\right)\right]$$

Where, $I_m = \frac{V_m}{\omega L}$ = peak current

Comparing equations (1) and (2) it is observed that the current lags (behind) the applied voltage by $\frac{\pi}{2}$ as shown.



Phasor diagram



Graphical representation

Inductive reactance (X_L) : It is the opposition for the flow of A.C in an inductance coil.
SI unit of Inductive reactance is Ohm (Ω).

We have, $I_m = \frac{V_m}{\omega L}$

But, $I_m = \frac{V_m}{X_L}$

$\therefore \quad X_L = \omega L \quad$ or $\quad X_L = 2\pi\nu L \quad$ (But, $\omega = 2\pi\nu$)

Note: The quantity 'ωL' is analogous to the resistance and is called inductive reactance denoted by 'X_L'. Inductive reactant is directly proportional to frequency.

Prove that instantaneous power in an inductive circuit is zero:

The instantaneous power supplied to the inductor is

$$P = IV$$

$$= I_m \sin\left(\omega t - \pi/2\right) \times V_m \sin(\omega t)$$

$$= I_m V_m \left(-\cos(\omega t)\right)\sin(\omega t)$$

$$\left[\sin\left(\omega t - \frac{\pi}{2}\right) = -\cos\omega t\right]$$

$$= -I_m V_m \cos(\omega t)\sin(\omega t)$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$P = -\frac{I_m V_m}{2} \sin(2\omega t)$$

So, the average power over a complete cycle is

$$\bar{P} = \left\langle -\frac{I_m V_m}{2} \sin(2\omega t)\right\rangle$$

$$= -\frac{I_m V_m}{2}\left\langle \sin(2\omega t)\right\rangle$$

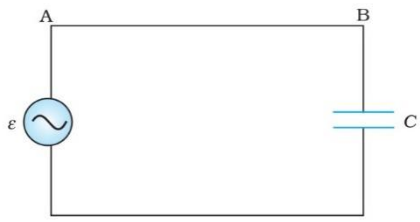
But the average of sin(2ωt) over a complete cycle =0

$$\therefore \quad \bar{P} = -\frac{I_m V_m}{2} \times 0$$

$$\bar{P} = 0,$$

Thus, the average power supplied to an inductor over complete cycle is zero.

A.C. Voltage applied to a capacitor:



The capacitor is connected to A.C source as shown in figure. A.C voltage of a source is given by $V = V_m \sin \omega t$ ------------(1)

Let, C=capacitance of a capacitor, q=the charge on the capacitor at any time 't'.

The instantaneous voltage 'V_C' across the capacitor is

$$V_C = \frac{q}{C} \longrightarrow (2)$$

According to Kirchhoff's loop rule,

$V = V_C$

$$V_m \sin \omega t = \frac{q}{C}$$

$\therefore \quad q = CV_m$

Instantaneous flow of current = Rate of flow of charges

Then, $$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt}(CV_m \sin \omega t)$$

$$I = (CV_m)\frac{d}{dt}(\sin \omega t)$$

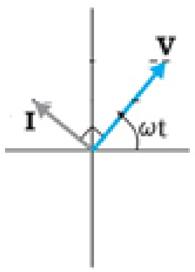
$$I = \omega CV_m \cos \omega t$$

$$\boxed{I = I_m \sin\left(\omega t + \pi/2\right)} \longrightarrow (3)$$

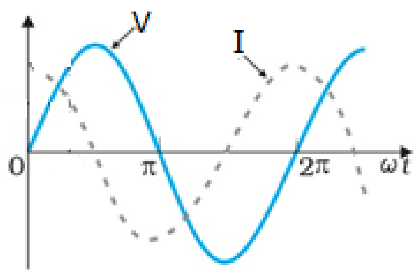
$$\frac{d(\sin \omega t)}{dt} = \omega \cos \omega t$$
$$\cos\theta = \sin\left(\theta + 90^0\right)$$
$$\cos \omega t = \sin\left(\omega t + 90^0\right)$$

Where, $I_m = \omega C\, V_m$, I_m = amplitude of the alternating current.

From Equations (1) and (3), it is observed that the current leads (ahead) the voltage by $\frac{\pi}{2}$ as shown



Phasor diagram



Graphical representation

Capacity reactance (X_C) : It is opposition for the flow of A.C. in capacitor. SI unit is ohm (Ω).

We have, $I_m = \omega CV_m$

$\therefore \quad V_m = \frac{I_m}{\omega C}$

$\therefore \quad V_m = I_m X_C$

Where, $X_c = \frac{1}{\omega c} = \frac{1}{2\pi fc}$

Capacitive Reactance is inversely proportional to frequency and the capacitance.

Prove that Instantaneous power in a Capacitive Circuit is zero:

The instantaneous power supplied to the capacitor is

$$P = IV$$

$$P = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \times V_m \sin \omega t$$

$$= I_m \cos(\omega t)\ V_m \sin(\omega t) \qquad \left[I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)\right]$$

$$= I_m V_m \cos(\omega t) \sin(\omega t) \qquad [I = I_m \cos \omega t]$$

$$P = \frac{I_m V_m}{2} \sin 2\omega t$$

Average power is

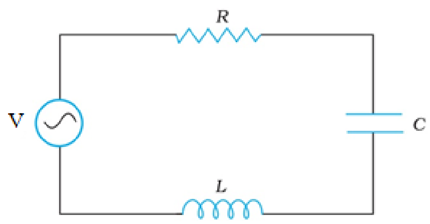
$$\bar{P} = \frac{I_m V_m}{2} \langle \sin(2\omega t) \rangle$$

$$\bar{P} = \frac{I_m V_m}{2} \langle \sin(2\omega t) \rangle$$

but, $\langle \sin(2\omega t) \rangle = 0$ over a complete cycle

$\therefore \quad \bar{P} = 0$

A.C. Voltage applied to a series LCR circuit (Derive an expression for impedance and maximum current of LCR circuit using phasor diagram:



Inductor, capacitor and resistor are connect in series with A.C source as shown in figure.

A.C Voltage of the source is given by

$$V = V_m \sin \omega t \text{ ---------------(1)}$$

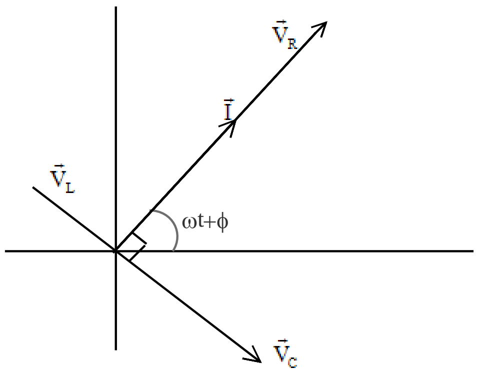
Where, V_m=amplitude of alternating voltage.

Let, q= the charge on the capacitor, L=self inductance of a coil,
C=capacitance of capacitor, R=Resistance of resistor,
I= current in the circuit, t=time

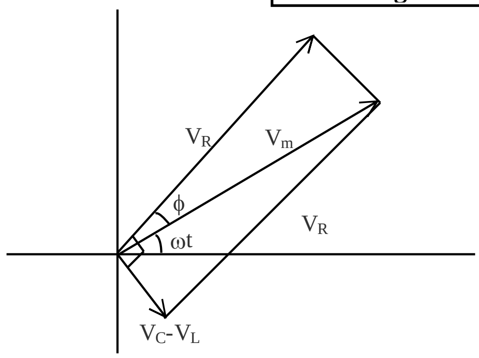
The A.C. current in the circuit is same in each element at any time. Let it be

$$I = I_m \sin(\omega t + \phi) \text{ ------------ (2)}$$

Where, 'ϕ' is the phase difference between the voltage across the source and the current in the circuit.



Phasor diagram (a)



Phasor diagram (b)

`From the Phasor diagram we came to know that $\vec{V}_L, \vec{V}_R, \vec{V}_C$ and '$\vec{V}$' represent the voltage phasor across inductor, resistor, capacitor and the source.

$\vec{V}_R$ Is parallel to I, $\vec{V}_C$ is $\pi/2$ behind I and $\vec{V}_L$ is $\pi/2$ ahead of 'I'.

From phasor diagram (a), $\vec{V}_C$ and $\vec{V}_L$ are always along the same line and in opposite directions.

$\therefore$ Net magnitude of voltage is $V_C - V_L$.

From phasor diagram(b), According to Pythagoras theorem,

$$V_m^2 = V_R^2 + (V_C - V_L)^2 \quad \text{----------} (3)$$

Where, $V_R = I_m R$ =magnitude of voltage across R,

$V_C = I_m X_C$ =magnitude of voltage across C, X_C=capacitive reactance,

$V_L = I_m X_L$ =magnitude of voltage across L, X_L=inductive reactance,

Equation (3) becomes,

$$V_m^2 = (I_m R)^2 + (I_m X_C - I_m X_L)^2$$

$$V_m^2 = I_m^2 \left[R^2 + (X_C - X_L)^2 \right]$$

$$V_m = I_m \sqrt{R^2 + (X_C - X_L)^2}$$

$$\therefore \qquad I_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} \quad \text{------------} (4)$$

This is the expression for amplitude of current.

In an A.C. circuit, $I_m = \dfrac{V_m}{Z}$ ---------------(5)

Where, Z=impedance

From equations (4) and (5)

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

This is the expression for impedance of LCR circuit.

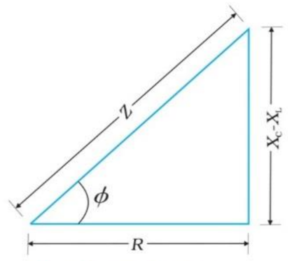
Note:

1. The opposition for the flow of A.C in LCR circuit is called impedance (Z). SI unit is ohm (Ω).

2. From the impedance diagram

$$\tan\phi = \frac{Opp}{Adj} = \frac{V_C - V_L}{V_R} = \frac{I_m X_C - I_m X_L}{I_m R}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$



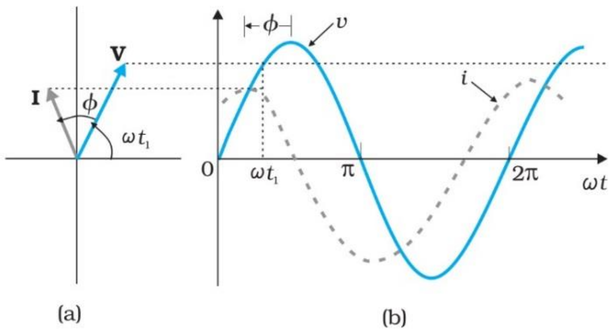
Impedance diagram.

3. If $X_L > X_C$ then $Z = \sqrt{R^2 + (X_L - X_C)^2}$

But, $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

$$\therefore \qquad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Phase Relationship w.r.t. LCR circuit:



(a) Phasor diagram of **V** and **I**.
(b) Graphs of v and i versus ωt for a series *LCR* circuit where $X_C > X_L$.

Case (i): When $X_C > X_L$, 'ϕ' is positive and the circuit is predominantly Capacitive Circuit

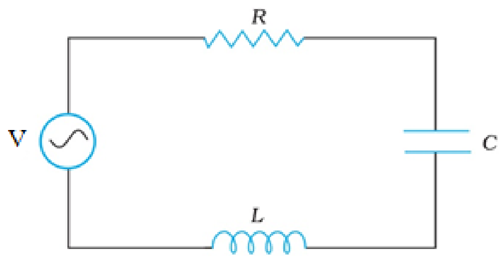
$$I = I_m \sin(\omega t + \phi)$$

Consequently, the current in the circuit leads the source voltage as shown.

Case (ii): $X_C < X_L$, ϕ is negative and the circuit is predominantly Inductive Circuit. Consequently current in the circuit lags the source voltage.

Then $I = I_m \sin(\omega t - \phi)$

Derive an expression for maximum current in LCR circuit by analytical solution:



According to Kirchhoff's loop rule voltage equation for LCR circuit is

$$V_L + V_R + V_C = V$$

$$\therefore \quad L\frac{dI}{dt} + IR + \frac{q}{C} = V_m \sin\omega t \text{ -------- (1)}$$

Where, L=self inductance of coil, R=Resistance or resistor, C=capacitance of capacitor

q=charge, I=current in LCR circuit

But, $I = \frac{dq}{dt}$, $\therefore \frac{dI}{dt} = \frac{d^2q}{dt^2}$

$\therefore$ equation (1) becomes

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin\omega t \text{ ---------(2)}$$

The above equation is like equation for forced damped oscillation.

Let us assume a solution

$q = q_m \sin(\omega t + \theta)$, Where, q_m=maximum charge

$$\therefore \quad \frac{dq}{dt} = \frac{d}{dt}\left[q_m \sin(\omega t + \theta)\right]$$

$$\frac{dq}{dt} = q_m \frac{d}{dt}\left[\sin(\omega t + \theta)\right]$$

$$\frac{dq}{dt} = q_m \cos(\omega t + \theta) \times \omega$$

$$\frac{dq}{dt} = \omega q_m \cos(\omega t + \theta) \text{ ------------ (4)}$$

$$\frac{d^2q}{dt^2} = \omega q_m \frac{d}{dt}\left[\cos(\omega t + \theta)\right]$$

$$\frac{d^2q}{dt^2} = \omega q_m \times \left[-\sin(\omega t + \theta)\right]\omega$$

$$\frac{d^2q}{dt^2} = -\omega^2 q_m \sin(\omega t + \theta) \text{ ----------- (5)}$$

$$\frac{d^2q}{dt^2} = -\omega^2 q \qquad \left(\because,\ q = q_m \sin(\omega t + \theta)\right)$$

Substituting equations (3), (4) and (5) in (2)

$$-L\omega^2 q_m \sin(\omega t+\theta)+R\omega q_m \cos(\omega t+\theta)+\frac{q_m \sin(\omega t+\theta)}{C}=V_m \sin\omega t$$

$$q_m\omega\left[R\cos(\omega t+\theta)\right]+\frac{\sin(\omega t+\theta)}{\omega C}-\omega L\sin(\omega t+\theta)=V_m\sin\omega t$$

$$q_m\omega\left[R\cos(\omega t+\theta)+X_C\sin(\omega t+\theta)-X_L\sin(\omega t+\theta)\right]=V_m\sin\omega t$$

$$q_m\omega\left[R\cos(\omega t+\theta)+(X_C-X_L)\sin(\omega t+\theta)\right]=V_m\sin\omega t$$

Multiplying and dividing above equation by Z

$$q_m\omega Z\left[\frac{R}{Z}\cos(\omega t+\theta)+\frac{X_C-X_L}{Z}\sin(\omega t+\theta)\right]=V_m\sin\omega t \text{ -------------- } (6)$$

Let, $\frac{R}{Z}=\cos\phi$ & $\frac{X_C-X_L}{Z}=\sin\phi$

Equation (6) becomes,

$$q_m\,\omega Z\left[\cos\phi\cos(\omega t+\theta)+\sin\phi\sin(\omega t+\theta)\right]=V_m\sin\omega t$$

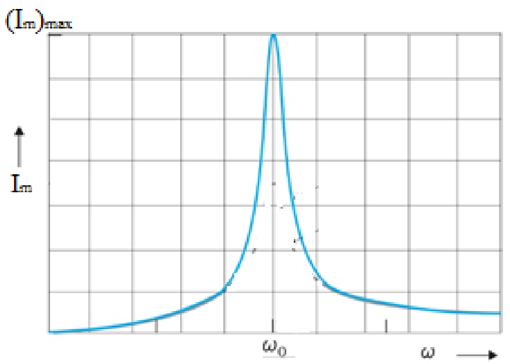
After solving, it is found that

$$I=I_m\sin(\omega t+\phi)$$

Where, $I_m=\dfrac{V_m}{\sqrt{R^2+(X_C-X_L)^2}}$

Resonance:

An interesting characteristic of the series LCR circuit. The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency. This frequency is called the system's natural frequency.



Variation of I_m with ω

Define resonance: The phenomenon at which inductive reactance is equal to capacitive reactance and impedance of LCR circuit is minimum is called resonance.

Define resonant frequency:

It is particular frequency at which inductive reactant is equal to capacitive reactance and impedance of LCR circuit is minimum.

Conditions for Resonance:

From the LCR series circuit, we found that the current amplitude is given by

$$I_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

Where, $X_C = 1/\omega C$ and $X_L = \omega L$. So, if 'ω' is varied, then at a particular frequency ω_0,

$X_C = X_L$ and the impedance is minimum.

Then, $Z = \sqrt{R^2 + O^2} = R$

This condition is called Resonant condition. The frequency occur is called the resonant frequency 'ω_0'.

Derive an expression for resonant frequency:

At resonance, $X_C = X_L$, Where, X_C= capacitive reactance,

X_L= inductive reactance.

$$\frac{1}{\omega_0 C} = \omega_0 L$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

But, $\omega_0 = 2\pi\nu_0$

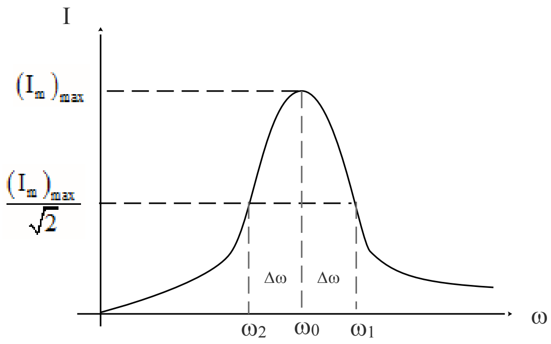
$\therefore$ $\nu_0 = \dfrac{1}{2\pi\sqrt{LC}}$

This is the expression for resonant frequency.

Note: At resonance,

1. Current is maximum i.e., $I_m = \dfrac{V_m}{R}$
2. Impedance is minimum i.e., Z = R.
3. $X_L = X_C$.
4. $V_L = V_C$
5. It is important to note that resonance phenomenon is exhibited by a circuit only if both 'L' and 'C' are present in the circuit. Only then do the voltages across 'L' and 'C' cancel each other (both being out of Phase) and the current amplitude is $I_m = V_m/R$, the total source voltage appearing across 'R'. This means that we cannot have resonance in a RL (or) RC circuit.

Band width of LCR circuit:



The difference between two frequencies for which the current is $\left(\frac{1}{\sqrt{2}}\right)$ times its maximum current is called band width.

Band width=$\omega_1 - \omega_2$

Let, ω_0=resonant frequency

ω_1 and ω_2 = two frequencies for which current is $\frac{(I_m)_{max}}{\sqrt{2}}$

But, $\omega_1 > \omega_0$ and $\omega_2 < \omega$

$\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$

But, bandwidth $= \omega_1 - \omega_2$

$= \omega_0 + \Delta\omega - (\omega_0 - \Delta\omega)$

Bandwidth=$2\Delta\omega$

SI unit is radian/sec (or) hertz.

Note: At ω_1 and ω_2, the power dissipated by the LCR circuit becomes half.

Define sharpness or quality factor:

It is defined as the ratio of resonant frequency to the bandwidth

$$\text{sharpness} = \frac{\text{Resonant frequency}}{\text{Band width}}$$

$$\text{sharpness} = \frac{\omega_0}{2\Delta\omega}$$

Note: Smaller the value of $\Delta\omega$, the sharpner (or) narrower is the resonance.

Derive an expression for sharpness of Resonance on the basis of quality factor 'Q':

The amplitude of the current in the series LCR circuit is given by

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \text{ ------------ (1)}$$

At a particular frequency, say $\omega=\omega_1$ and $\omega=\omega_2$, then, I_m becomes $\frac{(I_m)_{max}}{\sqrt{2}}$.

From the resonant curve shown, we see that there are two such values of 'ω' say ω_1 and ω_2, one greater and the other smaller than ω_0 and symmetrical about ω_0. We may write

$$\omega_1 = \omega_0 + \Delta\omega$$

$$\omega_2 = \omega_0 - \Delta\omega$$

$$\frac{(I_m)_{max}}{\sqrt{2}} = \frac{V_m}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

$$\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = \frac{\sqrt{2}V_m}{(I_m)_{max}}$$

$$\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = \sqrt{2}R$$

According to ohm's law

$$R = \frac{V_m}{(I_m)_{max}}$$

Squaring on both side,

$$R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2$$

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2 - R^2$$

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = R^2$$

$$\therefore \quad \omega_1 L - \frac{1}{\omega_1 C} = R \text{ -------------- (2)}$$

But, $\omega_1=\omega_0+\Delta\omega$, equation (2) becomes

$$(\omega_0 + \Delta\omega)L - \frac{1}{(\omega_0 + \Delta\omega)C} = R$$

$$\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\frac{1}{\omega_0 C\left(1+\frac{\Delta\omega}{\omega_0}\right)}=R \text{ ----------- (3)}$$

At resonance $X_L=X_C$

$$\omega_0 L=\frac{1}{\omega_0 C}$$

$$\omega_0^2=\frac{1}{LC}$$

$$\omega_0 C=\frac{1}{\omega_0 L}$$

Equation (3) becomes,

$$\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\frac{\omega_0 L}{\left(1+\frac{\Delta\omega}{\omega_0}\right)}=R$$

$$\omega_0 L\left[\left(1+\frac{\Delta\omega}{\omega_0}\right)-\frac{1}{\left(1+\frac{\Delta\omega}{\omega_0}\right)}\right]=R$$

$$\omega_0 L\left[\left(1+\frac{\Delta\omega}{\omega_0}\right)-\left(1+\frac{\Delta\omega}{\omega_0}\right)^{-1}\right]=R \text{ ---------- (4)}$$

We can approximate, $\left(1+\frac{\Delta\omega}{\omega_0}\right)^{-1}$ as $\left(1-\frac{\Delta\omega}{\omega_0}\right)$ Since, $\frac{\Delta\omega}{\omega_0}<<1$.

Equation (4) becomes,

$$\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\omega_0 L\left(1-\frac{\Delta\omega}{\omega_0}\right)=R$$

$$\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}-1+\frac{\Delta\omega}{\omega_0}\right)=R$$

$$\omega_0 L\frac{2\Delta\omega}{\omega_0}=R$$

$$\frac{2\Delta\omega}{\omega_0}=\frac{R}{\omega_0 L}$$

Or $\frac{\omega_0}{2\Delta\omega}=\frac{\omega_0 L}{R}$

But, $\frac{\omega_0}{2\Delta\omega}=\text{sharpness}$

$\therefore$ Sharpness= $\frac{\omega_0 L}{R}$

The ratio $\frac{\omega_0 L}{R}$ is also called quality factor, (Q) of the circuit $Q=\frac{\omega_0 L}{R}$

Note1: Band width $(2\Delta\omega)=\frac{\omega_0}{Q}$, So, larger the value of 'Q', the smaller is the value of $2\Delta\omega$ or the bandwidth and sharper is the resonance.

Note2: We can also prove that : $Q = \frac{1}{\omega_0 RC}$

Sharpness = Q-factor = $\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$.

Applications: Resonant circuits have a variety of applications.
i) In tuning of Radio and TV receivers.
ii) These circuits helps in accepts signals from many broadcasting stations.

Derive an expression for Average Power in A.C. circuit (Power factor):
Consider series LCR circuit connected to A.C source.

A.C voltage is, $V=V_m \sin\omega t$

and current in the LCR circuit is, $I=I_n \sin(\omega t+\phi)$

Where, $I_m = \frac{V_m}{Z}$ and $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

There fore, the instantaneous power 'P' supplied by the source is

$$P = VI$$
$$= (V_m \sin\omega t)\times\left[I_m \sin(\omega t+\phi)\right]$$
$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t+\phi)\right]$$
$$P=\left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t+\phi)\right] \longrightarrow (1)$$

The average power over a cycle is given by the average of the two terms in R.H.S. of equation (1). It is only the second term which is time dependant. Its average is zero (the positive half of the cosine cancels the negative half). i.e. $\frac{V_m I_m}{2}\cos(2\omega t+\phi) = 0$.

$$\therefore \quad P = \frac{V_m I_m}{2}\cos\phi$$

$$\boxed{V = \frac{V_m}{\sqrt{2}} \text{ and } I = \frac{i_m}{\sqrt{2}}}$$

$$= \frac{V_m}{\sqrt{2}}\times\frac{I_m}{\sqrt{2}}\cos\phi$$
$$\boxed{P = VI\cos\phi}$$

This can also be written as

$$P = I^2 Z\cos\phi \qquad (\text{where, } V = IZ,\ \cos\phi = \text{Power factor})$$

Define the term power factor (cos ϕ):

Power factor is defined as cosine of phase difference between current and voltage.

OR

It is also defined as the Ratio of circuit resistance to impedance

i.e., $\cos\phi = \frac{R}{Z}$

Note:

1. w.r.t A.C. $\cos\phi$ varies from 0 to 1,
   w.r.t D.C. $\cos\phi = 1$.
2. The power factor in a LCR circuit is measure of how close the circuit is to expending the maximum power.

What are the variations in cosϕ (power factor) w.r.t following circuits?

Case (i): Resistive circuit: If the circuit contains only pure 'R', it is called resistive circuit. In that case $\phi=0$, $\cos\phi = 1$. There is maximum power dissipation.

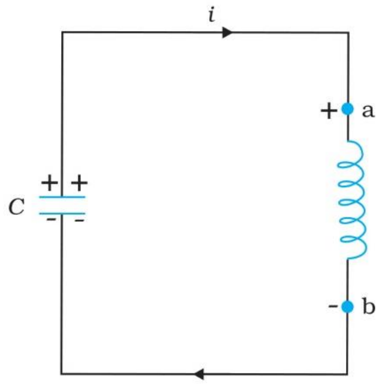
Case (ii): Purely Inductive (or) Capacitive Circuit: If the circuit contains only an inductor (OR) capacitor, we know that the Phase difference between voltage and current is $\pi/2$ OR 90^0

i.e., $\phi = 90^0$. Therefore $\cos\phi=0$, and no power is dissipated even though a current flowing in the circuit. This current is sometimes referred to as wattles current.

Case (iii): LCR Series circuit: $\phi = \tan^{-1}\frac{(X_C - X_L)}{R}$, So, '$\phi$' may be non-zero in RL, RC and RLC circuit. Even in such cases, power is dissipated only in the resistor.

Case (iv): Power Dissipated at resonance in LCR circuit: At resonance $X_C - X_L = 0$ and $\phi = 0$. Therefore, $\cos\phi=1$ and $P = I^2Z = I^2R$, That is maximum power is dissipated in a circuit (through R) at resonance

Derive an expression for frequency of LC – Oscillations and energy stored in it:



A circuit containing an inductor 'L' and a capacitor 'C' (initially charged) with no A.C. source and no resistor exhibits free oscillations.

Let a capacitor be charged 'q_m' (at t = 0) and connected an inductor as shown in figure.

The moment the circuit is completed, the charge on the capacitor starts decreasing, giving rise to current in the circuit. Let 'q' and 'i' be the charge and current in the circuit at time 't'. Since $\frac{dI}{dt}$ is positive, the induced e.m.f in 'L' will have a polarity as shown, i.e., $V_b < V_a$

Then $\frac{q}{c} - L\frac{dI}{dt} = 0$

Dividing by 'L',

$\frac{q}{Lc} - \frac{dI}{dt} = 0$ ----------- (1)

But, $I = -\left(\frac{dq}{dt}\right)$ (in the present case as 'q' decreases, 'I' increases)

Therefore equation (1) becomes

$$\frac{q}{LC} + \frac{d^2q}{dt^2} = 0$$

Or $$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

This equation has the form $\frac{d^2x}{dt^2} + \omega_0^2 x = 0$ for a simple harmonic oscillator.

The charge, therefore, oscillates with natural frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

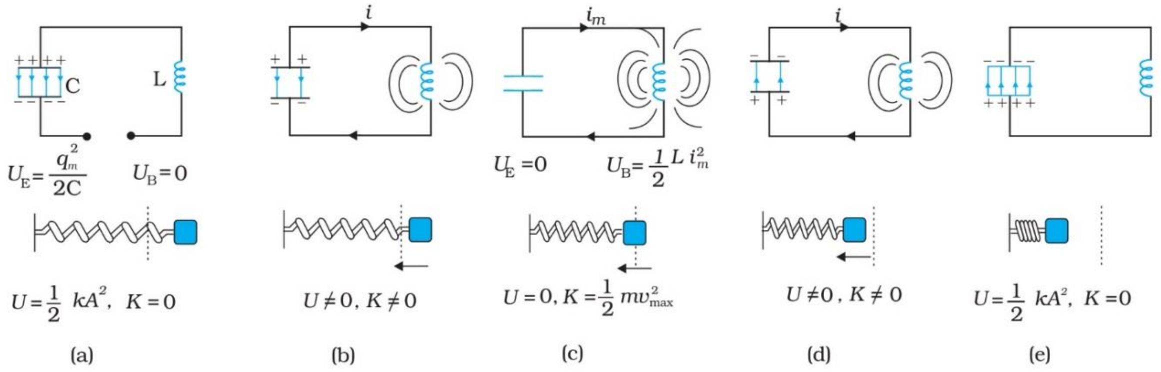
'q' varies as $q=q_m \cos(\omega_0 t+\phi)$ and 'I' varies as $I=I_m \sin(\omega_0 t+\phi)$

Where, $I_m=\omega q_m$.

Energy stored in charged capacitor is, $u_E = \frac{1}{2}\frac{q_m^2}{C}$. At this time no current flows through L.

$\therefore$ Total energy of LC circuit is $u = \frac{1}{2}\frac{q_m^2}{C}$.

Explain with circuit diagram how the LC elements sustains the oscillation



The above figure (a) shows a capacitor with initial charge 'q_m' connected to an ideal inductor. The electrical energy stored in the charged capacitor is $U_E = \frac{1}{2}\frac{q_m^2}{C}$. Since, there is no current in the circuit, energy in the inductor is zero. Thus the total energy of LC circuit

$$u = u_E = \frac{1}{2}\frac{q_m^2}{C}$$

At t = 0, the switch is closed and the capacitor starts to discharge as shown in fig (b). As the current increases, it sets up a magnetic field in the inductor and thereby, some energy stored in the inductor in the form of magnetic energy i.e., $U_B = \frac{1}{2}LI^2$. As the current reaches its maximum value $i_m\left(\text{at } t = \frac{T}{4}\right)$ as shown in figure (c), all the energy is stored in the magnetic field $U_B = \frac{1}{2}LI_m^2$ The capacitor now has no charge and hence no energy. The current now starts charging the capacitor as shown in figure (d). This process continues till the capacitor is fully charged $\left(\text{at } t = \frac{T}{2}\right)$. From the figure (e) the capacitor is charged with opposite polarity to it's initial state w.r.t figure (a). The whole process will now repeat itself till the system reverts to its original state. Thus, the energy in the system oscillates between the capacitor and the inductor.

Note:

1. An oscillator merely as an energy converter. It receives D.C. energy (power) and changes it into A.C. energy power of desired frequency.
2. An oscillator employs positive feed back.
3. In an LC-Oscillator, the frequency of oscillation is inversely proportional to square root of L and C. $\left\langle f_r = \frac{1}{2\pi\sqrt{LC}} \text{Hz} \right\rangle$
4. LC oscillation will be damped by two reasons:
   a) Every Inductor has some resistance the effect of this resistance is to introduce a damping effect on the charge and current in the circuit and oscillation and finally die away.
   b) Even if resistance is zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves.

Note-5: Analogies between mechanical and electrical quantities:

Mechanical system	Electrical system
Mass m	Inductance L
Force constant K	Reciprocal capacitance $\frac{1}{C}$
Displacement x	Charge q
Velocity $v = \frac{dx}{dt}$	Current $i = \frac{dq}{dt}$
Mechanical energy $E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	Electromagnetic energy $U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$

Transformer:

It is a device used to vary the alternating voltage from few volts to several million volts.

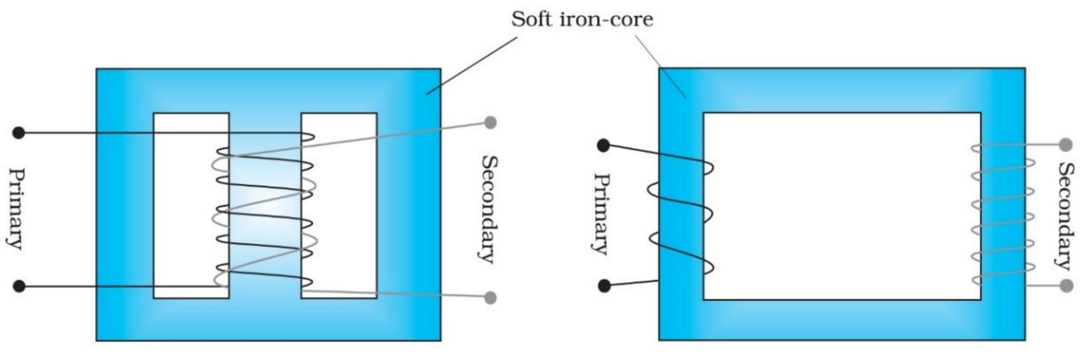
Principle of Transformer:

The transformer works on the principle of mutual induction.

Note: It is necessary to change OR transform an alternating voltage from one to another of greater OR smaller value. This is done with a device called transformer using the principle mutual induction without change in frequency but there is a change in voltage level.

Construction:

A transformer consists of two sets of coils, insulated from each other. They are wound on a soft iron core, either one on top of the other a shown in fig (a) OR on separate limbs of the core as shown in figure (b). One of the coil called the primary coil. The other coil is called the secondary coil. Often the primary coil is the input coil and the secondary coil is the output coil of the transformer.



(a) (b)

Working Principle: When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an e.m.f in it. The value of this e.m.f depends on the number of turns in the secondary. We consider an ideal transformer in which the primary has negligible resistance and all the flux in the core links both primary and secondary windings. Let 'ϕ' be the flux in each turn in the core at time 't' due to current in the primary, when a voltage 'V_p' applied to it. Then the induced e.m.f OR voltage 'E_s', in the secondary with 'N_s' turns is

$$E_s = -N_s \frac{d\phi}{dt}$$

The alternating flux 'ϕ' also induces an e.m.f called back e.m.f in the primary. This is

$$E_p = -N_p \frac{d\phi}{dt}$$

But, $E_p = V_p$

$$E_s = V_s$$

$$V_s = -N_s \frac{d\phi}{dt} \longrightarrow (1)$$

$$V_p = -N_p \frac{d\phi}{dt} \longrightarrow (2)$$

From equations (1) and equation (2)

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \longrightarrow (3)$$

If the transformer is assumed to be 100% efficient (no energy losses) w.k.t

Input power = Out Put Power and P = VI

$$I_p V_p = I_s V_s \longrightarrow (4)$$

Then combining equation (3) and (4) we have

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} \longrightarrow (5)$$

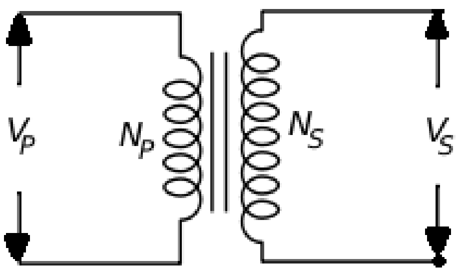
Since 'I' and 'V' both oscillate with same frequency as the A.C source. Now, we can see how a transformer affects the voltage and current we have

$$V_s = \left(\frac{N_s}{N_p}\right)V_p \quad \text{and} \quad I_s = \left(\frac{N_p}{N_s}\right)I_p$$

Step-Up transformer: If the secondary coil has a greater number of turns than the primary $\left(N_s > N_p\right)$, the voltage is stepped up $\left(V_s > V_p\right)$. This type of arrangement is called a step-up transformer.

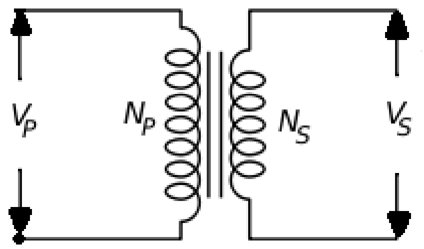
However in this arrangement there is less current in the secondary than in the primary (i.e., $I_s < I_p$).

Step-Down transformer: If the secondary coil has less turns than the primary $\left(N_s < N_p\right)$. In this case, $V_s < V_p$ and $I_s > I_p$. That is the voltage is stepped down, OR reduced and the current is increased.



Step – Up transformer

$N_s > N_p$
$V_s > V_p$
$I_s < I_p$



Step – Down Transformer

$N_p > N_s$
$V_p > V_s$
$I_s > I_p$

Explain briefly the power losses in a transformer?

In actual transformers, smaller energy losses do occur due to the following reasons;

i) Flux leak loss, ii) Resistance of the windings loss (Copper loss), iii) Eddy current loss and

iv) Hysteresis loss

i) **Flux leakage:** There is always some flux leakage. That is not all the flux due to primary passes through the secondary due to poor design of the core OR the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

ii) **Resistance of the windings:** The wire is used for the windings has some resistance and so, energy is lost due to heat produced in the core winding (i.e., I^2R loss). In high current, low voltage windings, these are minimised by using thick wire.

iii) **Eddy current loss:** The alternating magnetic flux induced eddy currents in the core and causes heating. The effect is reduced by having a laminated core.

iv) **Hysteresis loss:** The magnetisation of the core is repeatedly reversed by the alternatively magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material (i.e. mild steel is used) which has a low hysteresis loss.

Note: Rating of transformers is expression in KVA (Kilo–Volt Amperes)

Applications of transformers:

1) It is used in power supplies.
2) Used in modern computers
3) Used in motor control devices
4) Used in stabiliser
5) Using in rectifier and filter circuits
6) Used in wave analysers etc.

One Marks Questions:

1. Define alternating current.
2. Define peak value of voltage and current.
3. Define r.m.s value.
4. How is r.m.s voltage of ac related to peak value of ac voltage? (March-2014)
5. What is resonance?
6. Define resonant frequency.
7. What is quality factor (sharpness)?
8. Define power factor.
9. Define stepup transformer.

Two marks questions:

1. Write the condition for resonance.
2. Derive the expression for resonant frequency.
3. Distinguish between inductive reactance and capacitive reactant.

Three Marks Questions:

1. Explain the construction of transformer. Mention its principle. (March-2014)
2. Derive an expression for resonant frequency of series circuit containing inductor, capacitor and resistor (July-2014)
3. Show that voltage leads currents when AC voltage applied to pure inductance. (March-2015)
4. Show that voltage leads currents when AC voltage applied to pure capacitance.
5. Show that voltage leads currents when AC voltage applied to pure resistance.
6. What is the principle behind the working of a transformer? Mention any two sources of energy loss in transformer. (July-2015)
7. What is a transformer? Mention two sources of energy loss in a transformer. (March-2016)
8. With a diagram, Explain the working of a transformer. (July-2016)

Five Marks Questions:

1. Derive an expression for impedance and current of LCR series circuit.
2. Derive an expression for frequency of LC-Oscillations and energy stored in it.
3. Derive an expression for sharpness of Resonance on the basis of quality factor 'Q'.

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Chapter – 08

ELECTROMAGNETIC WAVES

Inconsistency of Ampere's circuital law (Failure of Ampere's law):

The ampere's circuital law fails to find magnetic field at a point outside a capacitor connected to a time-varying current, Maxwell noticed on inconsistency in the ampere's circuital law. He suggested the existence of an additional current, called by him, the displacement current to remove this inconsistency.

What is conduction current?

The current carried by conductors due to flow of charges is called conduction current (I_C).

What is displacement current?

The current carried by the conductor due to changing electric field with time is called displacement current (I_D).

Expression for displacement current:

$$I_d = \varepsilon_0 \left(\frac{d\phi_E}{dt} \right)$$

Where, I_d=displacement current, ε_0=permittivity of free space,

$\frac{d\phi_E}{dt}$=time rate of change of electric flux

State Ampere's-Maxwell law: It states that "line integral of magnetic field around the closed path is equal to μ_0 times the sum of conduction and displacement currents passing through that surface."

$$\oint B \cdot dl = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$ is known as Ampere – Maxwell Law.

Electromagnetic waves: Electromagnetic waves are time varying electric and magnetic fields that propagate in space.

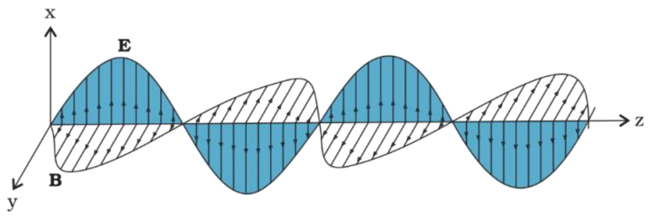
Explain the sources of Electromagnet waves.

It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves. Consider a charge oscillating with some frequency, this produces an Oscillating electric field in space, which produces an oscillating magnetic field. The oscillating electric and magnetic fields thus regenerate each other.

Note: Hertz's successful experimental test succeeded in producing and observing electromagnetic waves of much shorter wavelength (25mm to 5mm) at around the same time, succeeded in transmitting EM –waves (radio waves) over distances of many kilometres in the field of communication.

Explain the nature of electromagnetic waves.

It can be shown from Maxwell's equations that electric and magnetic fields in an electromagnetic wave are perpendicular to each other, and to the direction of propagation.



Note: It is also seen from Maxwell's equations that the magnitude of the electric and the magnetic fields in EM – wave are related as

$$C = E_0 / B_0 \quad \text{or} \quad B_0 = E_0 / C$$

Expression for speed of electromagnetic waves in any medium:

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

Where, $\mu=\mu_0\mu_r$= absolute permeability of medium,

$\varepsilon=\varepsilon_0\varepsilon_r$= absolute permittivity of medium.

Expression for speed of electromagnetic waves in vaccum:

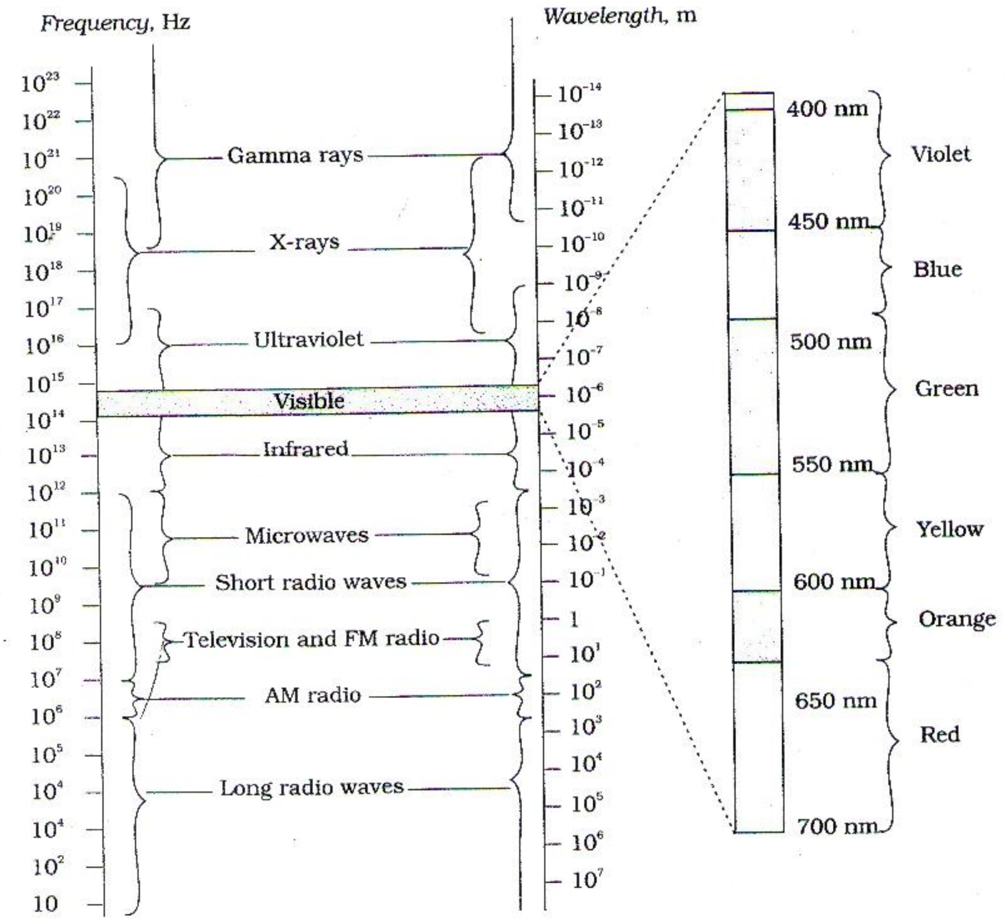
$$C = \frac{1}{\sqrt{\mu_0 \in_0}}$$

Where, $\mu_0 = 4\pi\times10^{-7}\,\text{Henry / mt}$ $\varepsilon_0 = 8.854\times10^{-12}\,\text{farad / mt}$

$C = 2.9979246\times10^{8}\,\text{m / s}$ $C \approx 3\times10^{8}\,\text{m / s}$

Explain with a sketch of an Electromagnetic spectrum.

At the time Maxwell predicted the existence of electromagnetic waves, the only familiar electromagnetic waves were the visible light waves. The existence of ultraviolet and infrared waves was barely established. By the end of the nineteenth century, X-rays and gamma rays had also been discovered. We now know that, electromagnetic waves include visible light waves, X-rays, gamma rays, radio waves, microwaves, ultraviolet and infrared waves. The classification of electromagnetic waves according to frequency is the electromagnetic spectrum. There is no sharp division between one kind of wave and the next. The classification is based roughly on how the waves are produced and detected.



We briefly describe these different types of electromagnetic waves, in order of decreasing wavelengths.

Radio waves:

1. Radio waves are produced by the accelerated motion of charges in conducting wires.
2. Wavelength of radio wave is greater than 0.1m.
3. Frequency of the radio wave is 5×10^5 to 10^9 Hz.

Classification of radio waves and their uses:

1. Frequency band 530 kHZ to 1710 kHz used as medium wave AM broadcasting.
2. Frequency band 3MHz to 54MHz are used in telecast TV programs.
3. Frequency band 88MHz to 108MHz used for transmission of FM broadcasting.
4. Frequency band 840MHz to 935MHz is known as ultrahigh frequency (UHF) band used in cell phones or Mobile communication.

Uses of radio waves:

Radio waves are used in

1. Radio and TV broadcasting.
2. Mobile communication systems.

Microwaves:

1. **Production:** Microwaves are produced in special vacuum tubes: Klystron, Magnetrons, Gun diode etc.
2. Wavelength of radiowaves is from 0.1m to 1mm.
3. Frequency of microwaves is from 10^9 to 10^{12}Hz.

Uses: Microwaves are used in

1. Radar system for air craft navigation.
2. Atomic and molecular research.
3. Microwave ovens for cooking and warming food (set frequency around 3GHz).
4. Weather RADAR (Radio Detector and Ranging)
5. A radar using microwave can help in detecting the speed of tennis ball, cricket ball.

Infrared waves:

1. Production: Infrared waves are produced by the excitation of atoms and molecules. These are also called as heat waves.
2. The wavelength range is from 1mm to 700nm.
3. The frequency range is from 3×10^{11} to 4×10^{14} Hz.

Uses: Infrared rays are used in

1. Physical therapy to treat muscular strains.
2. To operate electronic devices like TV, VCD, Music, High-fi systems etc.
3. The IR waves from the sun keep the earth warm.
4. For taking photographs of earth under foggy conditions.
5. These are used in solar water heaters and cookers.

Visible light:

1. **Production:** These are produced when electrons jump from higher energy level to lower energy level.
2. **Wavelength:** Wavelength rage is from 700 nm to 400 nm
3. **Frequency:** Frequency range is from 4×10^{14} Hz to 7×10^{14}Hz.

Uses: Visible light are used to

1. Stimulate the sense of sight in human beings.
2. Visible light is useful in photography.
3. It is useful in astronomy.
4. It is a greater source of energy for human life.

Ultraviolet rays (UV-rays):

1. **Production:** The sun is the most important source of UV-waves. These are radiations produced by electric arc lamps or very hot bodies.
2. **Wavelength:** Wavelength range is from 400nm to 1nm.
3. **Frequency:** Frequency rage is from 7.5×10^{14} Hz to 5×10^{15}Hz.

Uses: UV-rays are used to

1. Sterilize food stuff, milk and water.
2. Used in detecting forged documents and finger prints.
3. Used for sterilising the surgical instruments.
4. Used in LASIK eye surgery. (Laser Assisted In Situ Keratom) (LEUSIS)

X-ray:

1. Production: X-rays can be produced by bombarding a metal target of high atomic number with a beam of fast moving electrons.
2. Wavelength: Wavelength range is from 1nm to 10^{-3} nm.
3. Frequency: Frequency range is from 1×10^{16}Hz to 3×10^{20}Hz.

Uses: X-rays are used in

1. Medical diagnosis like locating the fracture in bones.
2. These are used in radio therapy to cure skin diseases.
3. These are used in engineering for locating faults, cracks and flaws in the finished metallic materials.
4. Detective agencies to detect Gold, Silver and Diamonds etc. concealed in bags or the body of a person.

Gamma rays (γ-rays):

1. **Production**: These are produced by nuclear phenomenon. i.e. Radio active substances are the sources of γ-rays. The penetration power of these rays is extremely high.
2. **Wavelength** : Less than 10^{-3} nm (10^{-10}m to 10^{-14}m)
3. **Frequency** : Frequency range is from 2×10^{18}Hz to 5×10^{22}Hz.

Uses:

1. These are used in radiotherapy for the treatment of cancers and tumor.
2. These rays provide useful information about the structure of atom.
3. These are used for food preservation.

****

One Mark Questions:

1. What is conduction current?
2. Define displacement current? (March – 2017)
3. Write the expression for displacement current.
4. What are electromagnetic waves?
5. What is the source of an electromagnetic wave?
6. What is the nature of electromagnetic waves?
7. What is the expression for velocity of electromagnetic waves in vacuum?
8. Name the electromagnetic radiation used for viewing objects through haze and fog.
9. What part of electromagnetic spectrum is used in operating a radar?
10. Mention the role of ozone on the atmosphere?
11. Name the part of the electromagnetic spectrum which is used in green house to keep plants warm.
12. What was the range of wavelength of electromagnetic wave produced by hertz?
13. What is the velocity of light according to Michelson experiment?
14. Give the wavelength of X rays? (March 2016)
15. Mention two applications of infrared radiation. (March 2015)
16. What is the wavelength range of radio wave?
17. What is the wavelength range of micro wave?
18. What is the wavelength range of infrared wave?
19. What is the wavelength range of light wave?
20. What is the wavelength range of UV rays?
21. What is the wavelength range of gamma rays?

22. Write the frequency limit of visible range of gamma rays.
23. Write the frequency limit of visible range of X rays.
24. Write the frequency limit of visible range of UV rays.
25. Write the frequency limit of visible range of visible rays.
26. Write the frequency limit of visible range of IR rays.
27. Write the frequency limit of visible range of micro waves.
28. Write the frequency limit of visible range of radio waves.

Two Marks Question:

1. Who predicted the existence of electromagnetic waves? Give the wavelength range of electromagnetic spectrum. (March-2014)
2. Give any two uses of microwaves. (July-2014, March-2017)
3. What are micro waves? Give their one use.
4. Write an expression for velocity of light in terms of permittivity and permeability.
5. Write the expression for velocity of light in medium in terms of absolute of permittivity and absolute permeability.
6. Write any four properties of electromagnetic wave.
7. Give any two uses of IR waves.
8. Give any two uses of UV waves.

Chapter 9:

RAY OPTICS AND OPTICAL INSTRUMENTS

Optics is a branch of physics which deals about light. Optics can be divided into three branches.

1. Geometrical Optics or Ray optics: It is a branch of optics which deals about light on the basis of ray nature.
2. Physical optics or Wave optics: It is a branch of optics which deals about light on the basis of wave nature.
3. Quantum Optics: It is a branch of optics which deals about light on the basis of particle nature.

Light: The electro-magnetic radiation of wavelength about 400nm to 750nm is called light.

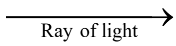
Light produces the sensation of vision when it falls on retina of our eye. So we can see the world around us.

Speed of light: Light travels with enormous speed. Speed of light in vacuum is

$C=2.9979\times10^{8}ms^{-1}$, it is taken as $C=3\times10^{8}ms^{-1}$

Rectilinear propagation of light: In the homogeneous medium, the light travels in a straight line. This is called rectilinear propagation of light.

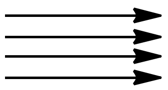
Ray of light: The path along which the light travels is called ray of light.

It is represented as 

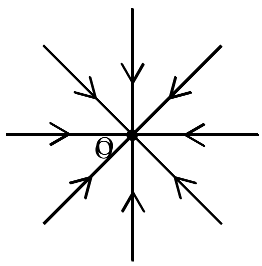
Beam of light: A bundle of rays of light is called beam of light.

Types of beam of light:

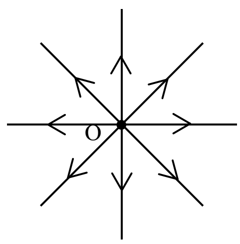
1) Parallel beam



2) Converging beam



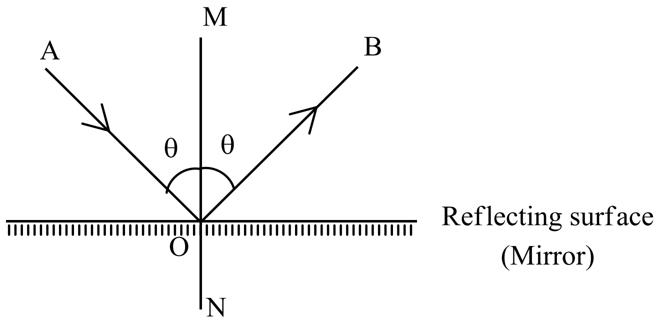
3) Diverging beam



What is reflection of light?

The phenomenon in which light travelling in one medium incident on the surface of another returns to first medium is called reflection of light.

Ray diagram to show reflection of light:

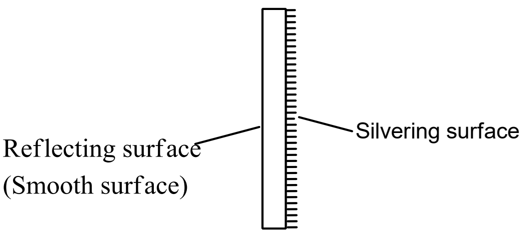


AO=Incident ray, OB=reflected ray,
O=point of incidence, MN=Normal to 'O'
$\angle AOM$ =θ=angle of incidence (angle between normal and incident ray)
$\angle MOB$ =θ=angle of reflection (angle between normal and reflected ray)

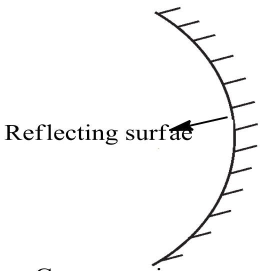
Laws of reflection: There are two laws.
I Law of reflection: The incident ray, reflected ray and normal to the reflecting surface at the point of incidence lie in the same plane.
II Law of reflection: The angle of reflection is equal to angle of incidence.
i.e. angle of reflection = angle of incidence

Mirror: Mirror is a device which reflects the light.
Types of mirror:
1) Plane mirror: Reflecting surface is plane in plane mirror

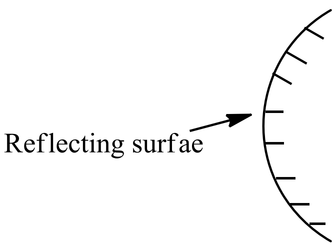
2) Spherical mirrors: A spherical mirror is one whose surface forms part of sphere.
a) Concave mirror **:** In concave mirror inner part is reflecting surface.
b) Convex mirror: In concave mirror outer part is reflecting surface.



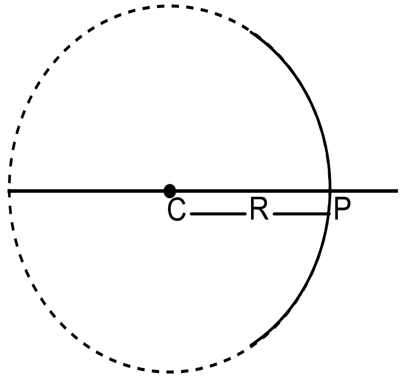
Plane mirror



Concave mirror



Convex mirror



Pole(P): It is the geometrical midpoint of spherical mirror.

Centre of curvature (C): It is centre of sphere of which the mirror is a part.

Radius of curvature (R): It is the distance between pole and centre of curvature.

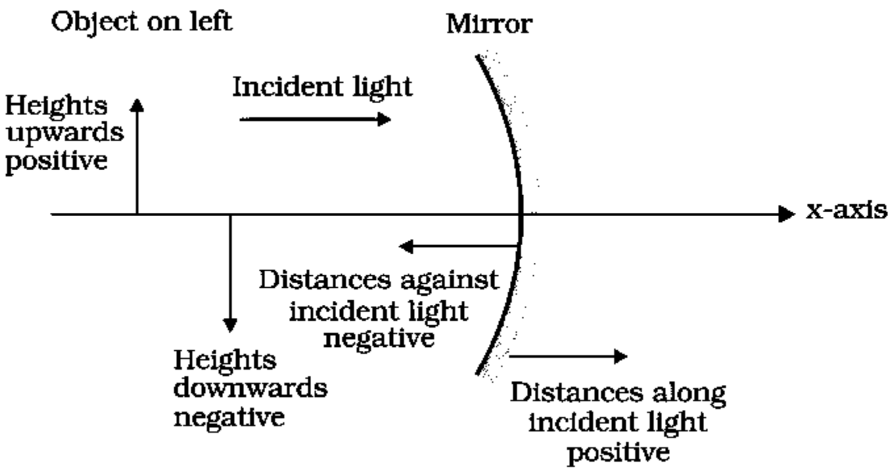
i.e. R=PC=radius of curvature

Principal axis: It is straight line joining the pole and centre of curvature.

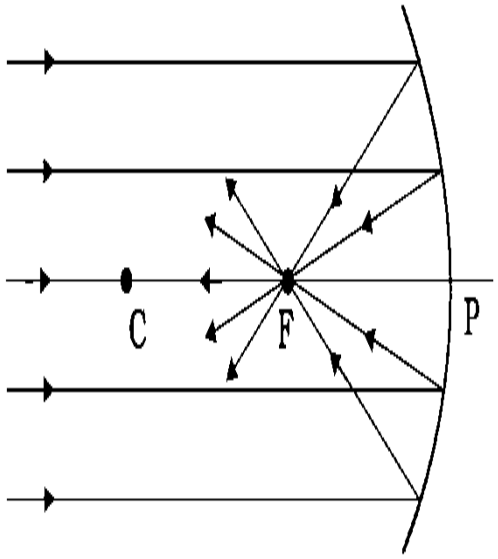
Sign convention for spherical mirrors and lenses (Cartesian rule):

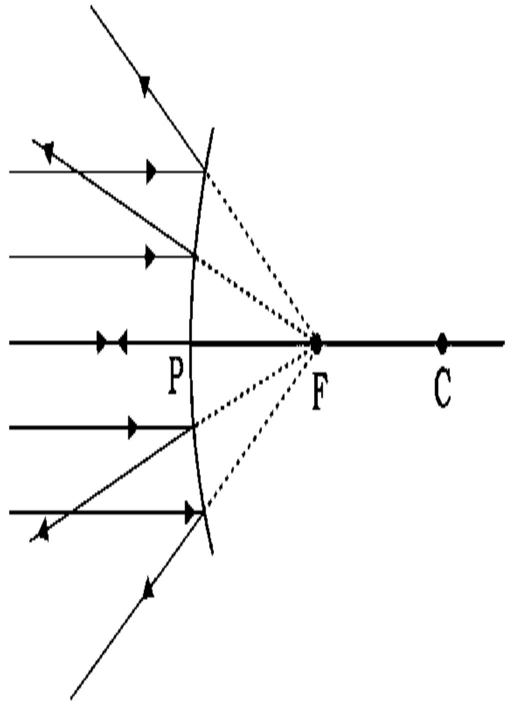
According to Cartesian sign convention

1. All distances are measured from the pole of the mirror (or, from the optical centre of lens).
2. The distances measured along the direction of incident ray are taken as positive.
3. The distances measured opposite to the direction of incident ray are taken as negative.
4. The heights measured upwards with respect to principal axis are taken positive.
5. The heights measured downwards w.r.t. principal axis are taken as negative.



Principal focus of spherical mirror:





When a parallel beam of light incident on a spherical mirror, then reflected beam converges at a point or appear to diverge from a point on the principal axis. That point is called principal focus.
In case of concave mirror, the principal focus is **converging point (real point).**
In case of convex mirror, the principal focus is **diverging point (Virtual point).**

Note:

1) Paraxial rays: The rays which are parallel and close to the principal axis are called paraxial rays.

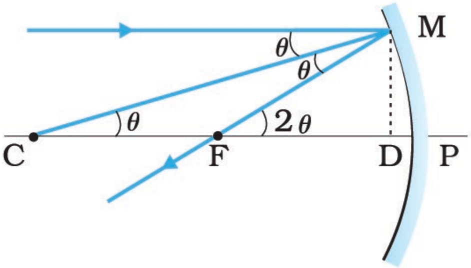
2) Marginal rays: The rays which are parallel and far away from the principal axis are called marginal rays.

3) Focal plane: It is a plane passing through principal focus but perpendicular to the principal axis.

Focal length of spherical mirror:

The distance between pole and principal focus is called focal length.

Derive the relation between focal length and radius of curvature of spherical mirror:

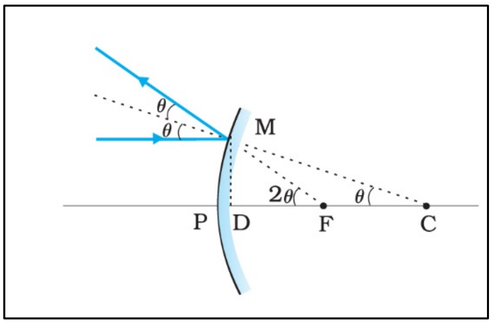


The geometry of reflection of an incident ray is shown in figure.

Let, C=centre of curvature, P=Pole,
F=Principal focus, R=radius of curvature,
f=focal length,
MD=Perpendicular from M on the principal axis.

But, R = PC and f = PF

$\angle MCD = \theta$ and $\angle MFD = 2\theta$



From right angled triangle MDC

$$\tan\theta = \frac{MD}{DC}$$

For small angle θ, tanθ = θ

$$\therefore \quad \theta = \frac{MD}{DC} \quad \text{-------------} \ (1)$$

From right angled triangle MDF

$$\tan 2\theta = \frac{MD}{DF} \qquad \therefore 2\theta = \frac{MD}{DF} \quad \text{---------------} \ (2)$$

From equation (1) and (2)

$$2 \times \frac{MD}{DC} = \frac{MD}{DF} \ ,$$

$$\therefore \quad DF = \frac{DC}{2}$$

For small angle θ,

D is very close to P,

i.e. D≈P,

∴ DF=PF= -f,

DC=PC= -R

$$\therefore \quad \boxed{f = \frac{R}{2}} \qquad \text{or} \qquad R=2f$$

Note:

1. The rays from object actually meet at a point after reflection or refraction. That point is called image of the object.

2. The image is real if the rays actually converge to the point. Real image is always inverted.

3. The image is virtual, if the ray do not actually meet but appear to diverge from the point. Virtual image is always erect.

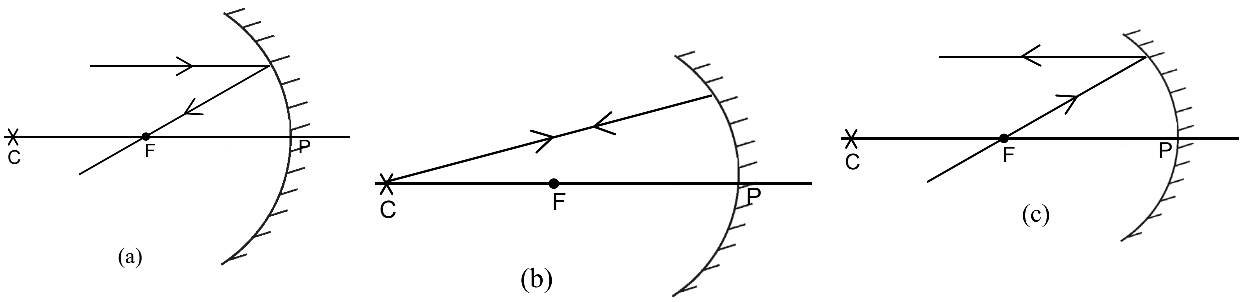
Note4: Ray diagram for image formation by a spherical mirror:

To obtain image of the point object, it is convenient to choose any two of the following rays.

a) The ray from the object which is parallel to principal axis passes through focus after reflection.

b) When a ray of light is passed through the centre of curvature the reflected ray retraces its path.

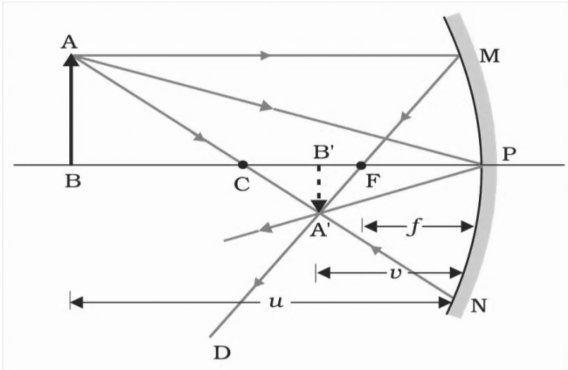
c) If the incident ray is passing through the focus the reflected ray is parallel to principal axis.



d) The ray incident at any angle at the pole, the reflected ray follows the laws of reflection.

Note: Focal length and radius of curvature for **plane mirror are ∞.**

Drive the mirror equation:



The geometry of reflections of incident rays is as shown in figure.

Let, MPN= spherical mirror

AB=object, $A'B'$=real image,

PF=f=focal length,

PC=R=radius of curvature

PB=u=object distance,

PB'=v=image distance

The two right angled triangles $A'B'F$ and MPF are similar.

$$\therefore \quad \frac{A'B'}{PM}=\frac{B'F}{PF}$$

But PM=AB

$$\therefore \quad \frac{A'B'}{AB}=\frac{FB'}{PF} \quad \text{------------- (1)}$$

Two right angled triangles $A'B'P$ and ABP are similar

$$\therefore \quad \frac{A'B'}{AB}=\frac{PB'}{PB} \quad \text{------------- (2)}$$

Comparing equations (1) and (2)

$$\frac{FB'}{PF}=\frac{PB'}{PB} \quad \text{but } \left(FB'=PB'-PF\right)$$

$$\frac{PB'-PF}{PF}=\frac{PB'}{PB} \quad \text{-------------- (3)}$$

Applying Cartesian sign convention, $PB'=-v$, $PB=-u$ and $PF=-f$

(-ve sign indicates that distances are taken opposite to the incident ray direction) $\therefore$ equation (3) becomes

$$\frac{-v-(-f)}{-f}=\frac{-v}{-u}$$

$$\frac{-v+f}{-f}=\frac{v}{u} \qquad \text{Or} \qquad \frac{v-f}{f}=\frac{v}{u}$$

$$\frac{v}{f}-1=\frac{v}{u} \qquad \frac{v}{f}=\frac{v}{u}+1 \quad \text{Dividing by 'v'}$$

$$\therefore \quad \boxed{\frac{1}{u}+\frac{1}{v}=\frac{1}{f}}$$

This is called mirror equation.

Linear magnification (m): The ratio of the height of the image to the height of the object is called linear magnification.

i.e. $m = \frac{h'}{h}$

Where, m=linear magnification, h'=height of image, h=height of object

Magnification produced by a mirror:

In figure, triangles $A'B'P$ and ABP are similar

$$\frac{A'B'}{AB} = \frac{PB'}{PB}$$

$$\frac{h'}{h} = -\frac{v}{u}$$ Where, v=image distance, u=object distance

$$\therefore \quad m = -\frac{v}{u}$$

This is the expression for linear magnification.

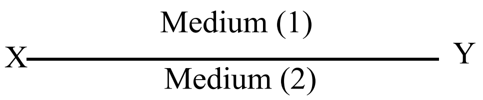
Note:

1. $m = \frac{f-v}{f} = \frac{f}{f-u}$
2. If m is positive then image formed is virtual and erect.
3. If m is negative then image formed is real, inverted.
4. If $|m| = +1$, then size of image= size of object.
5. Image formed is enlarged when $|m|>1$.
6. Image formed is diminished when $|m|<1$.

Refraction of light

Optical Medium: The medium which allows the light to pass through it is called optical medium.
Ex: Vacuum, Air, Glass, Pure water etc.

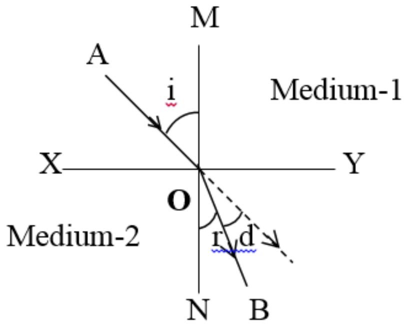
Interface:



The surface which separates two media is called interface.

Refraction of light: When a ray of light incident obliquely at the interface of two optical media, its direction of propagation changes at the interface. This phenomenon is called refraction.

Ray diagram to show refraction:



XY = Refracting surface(Interface), AO = Incident ray,
OB = Refracted ray, O = Point of Incidence,
MN = Normal at 'o', i = Angle of Incidence
r = Angle of Refraction, d = Angle of Deviation

Laws of refraction:

First law: The incident ray, refracted ray and normal to the interface at the point of incidence lie in the same plane.

Second law (Snell's Law): It states that, "The ratio of sine of angle of incidence to the sine of angle of refraction is constant for a pair of media".

i.e. $\frac{\sin i}{\sin r}$ = Constant Where, i = Angle of incidence, r=Angle of refraction

Relative refractive index: From Snell's law, $\frac{\sin i}{\sin r}$=Constant

This constant is called relative refractive index.

i.e. Relative refractive index = $\frac{\sin i}{\sin r}$

It is defined as "The ratio of sine of angle of incidence to the sine of angle of refraction".

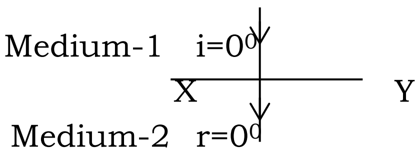
It can be represented as, ${}_1n_2$. It is read as n-two-one. i.e. $n_{21}=\frac{\sin i}{\sin r}$

$(n_{21} \text{ can also be writtten as } {}_1n_2)$

Note: The basic cause of refraction is that the speed of light changes as it goes from one medium to another

1. ${}_1n_2$ means R.I. of second medium with respect to first medium.
   e.g.: ${}_wn_g$=R.I. of glass w.r.t. water.
2. Refractive index measures the bending of light and it measures the velocity of light in the medium.

3. Normal incidence:



When a ray of light incident along the normal, then it is called normal incidence. In the normal incidence, the light enters the second medium without bending for normal incidence, $i=0^0$ and $r=0^0$.

1. **Limitation of Snell's law:**

   From Snell's law $n_{21}=\frac{\sin i}{\sin r}$

   For normal incidence, $i=0^o$ and $r=0^o$

   $\therefore n_{21}=\frac{\sin 0^0}{\sin 0^0}=\frac{0}{0}$ it is Indeterminent form.

   i.e. Snell's law does not hold good for normal incidence or for 0^0 angle of incidence.

Denser medium: The optical medium which has high R.I. w.r.t. another medium is called denser medium.

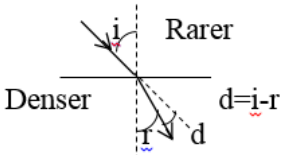
Rarer medium: The optical medium which has low R.I. w.r.t. another medium is called rarer medium.

Ex: For air and water media, air is rarer medium and water is denser medium.

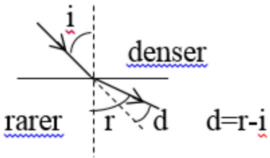
For water and glass media, water is rarer medium and glass is denser medium.

Note:

1. The velocity of light in rarer medium is greater than that in denser medium
2. When a ray of light travels from rarer to denser, then it bents towards the normal.



3. When a ray of light travels from denser to rarer, then it bents away from the normal



4. Relative RI $n_{21} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$
5. Absolute RI $n = \frac{c}{v} = \frac{\lambda}{\lambda_m}$
6. RI of a medium has no unit and no dimension. It is just a number.
7. Optical density of a medium determines the ability of the medium to refract light and it is directly proportional to RI of the medium.
8. Optical density and mass density are two different terms for ex, optical density of turpentine oil is more than that of water, where as mass density of turpentine oil is less than that of water.

Relation between n_{21} and n_{12}:

RI of medium 2 w.r.t medium 1 is related to RI of the medium 1 w.r.t medium 2 as

$$n_{21} = \frac{1}{n_{12}}$$

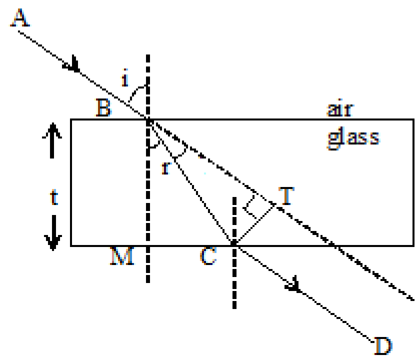
Expression for n_{32}:

RI of medium 3 w.r.t. medium 2 can be expressed as

$$n_{32} = \frac{n_{31}}{n_{21}} \quad \text{or} \quad n_{32} = n_{31} \times n_{12}$$

Note: $n_{gw} = n_{ga} \times n_{aw}$

Consequence of refraction: Lateral shift and normal shift are due to refraction.



Lateral shift:

In rectangular slab, the refraction takes place at two interfaces as shown in figure. In this case.

i.e. angle of emergence=angle of incidence

∴ Emergent ray is parallel to incident ray. The incident ray suffers lateral shift.

The perpendicular distance between emergent ray and incident ray direction is called lateral shift.

Note:

1. During lateral shift, net deviation=0
2. Expression for lateral shift is, $L_S = \frac{t}{\cos r}\sin(i-r)$

   Where, t = thickness of slab, i = angle of incidence, r = angle of refraction
3. If $r=0^0$ and $i=90^0$ then cosr=1, sin(i-r)=1,

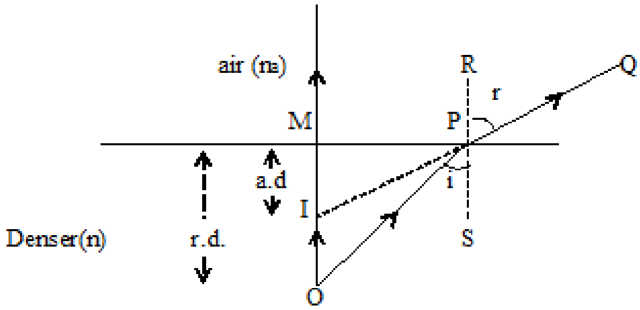
   ∴ L_s=t, i.e., Lateral shift is maximum and equal to thickness of slab.
4. If $i=0^0$, $r=0^0$ then cosr=1, sin(i-r)=0

   ∴ L_s=0 and lateral shift is minimum.

Normal shift: It is the apparent shift in the position of an object kept in one medium when viewed along normal through another medium.

Ex: The bottom of the tank filled with water appears to be raised.

It can be shown that,

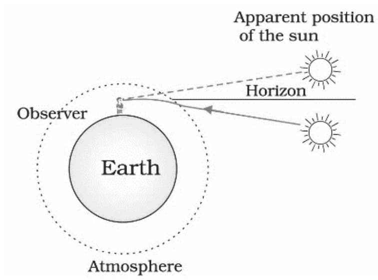


$$\text{Apparent depth} = \frac{\text{real depth}}{\text{R.I. of medium}}$$

$$a.\,d = \frac{r.d}{n},$$

n=R.I. of denser medium.

Note: Expression for normal shift is, $N_s = r.d\left(1 - \frac{1}{n}\right)$



Atmospheric refraction:

Sun is visible little before sunrise and little after sunset. Explain why?

The sun is visible little before the actual sunrise and little after the sunset. This is due to atmospheric refraction.

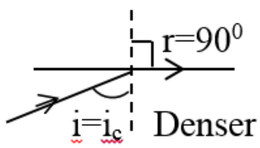
The RI of air with respect to vacuum is 1.0029. Due to this, the apparent shift in the direction of the sun is by about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 minutes.

The apparent flattening (Oval shape) of the sun at sunset and sunrise is also due to the same phenomenon.

Critical Angle: It is the angle of incidence in denser medium at which the angle of refraction becomes 90^0

Or

It is the angle of incidence in denser medium at which the refracted ray grazes the interface.



i.e. If $i=i_c$, then $r=90^0$. At critical angle the refracted ray grazes the interface.

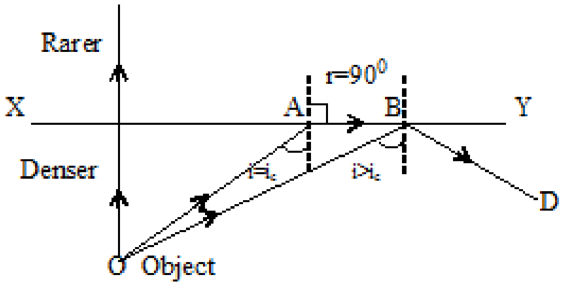
Relation between n and i_c:

$$n = \frac{1}{\sin i_c} \quad \text{or} \quad \sin i_c = \frac{1}{n}$$

Note: If rarer medium is other than air. $\therefore \sin i_c = \frac{n'}{n}$

Where, n'=R.I. of rarer

Total internal reflection (T.I.R):



XY=interface
OA=incident ray
AB=refracted ray
BD=reflected ray
i=angle of incidence
i_c=critical angle.

When a ray of light travels from denser to rarer medium and if an angle of incidence is greater than critical angle, then the light is totally reflected back into denser medium. This phenomenon is called TIR.

Conditions for TIR:

1. The light ray should travel from denser to rarer medium.
2. Angle of incidence should be greater than critical angle.

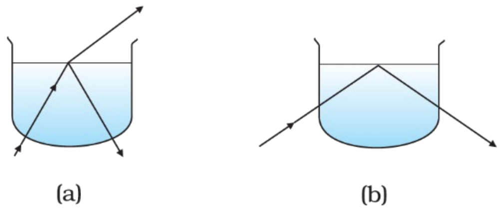
Explain Total internal reflection:

When a ray of light travels from denser to rarer medium, it bends away from the normal. On increasing the angle of incidence, angle of refraction also increases.

When angle of incidence is equal to critical angle the refracted ray just grazes the interface.

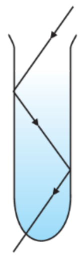
When the angle of incidence becomes greater than the critical angle, then the ray gets reflected back into the same medium, this phenomenon is called TIR.

Experimental demonstration of TIR:



- ❖ Take a laser pointer and shine its beam through the turbid water.
- ❖ When we shine the beam from below the beaker, we observe that the beam undergoes partial reflection and partial refraction at the free surface of water in the beaker. (fig a).
- ❖ When we direct the laser beam from one side of the beaker and adjust its direction so that refraction above the water surface is totally absent, we observe that the beam is totally reflected back to water from its upper surface. This is TIR (fig b)

Note: If we repeat this experiment by taking turbid water in a long test tube and shine the laser beam from the top at a suitable angle, the beam is totally internally reflected repeatedly from the walls of the test tube as shown in figure.



CRITICAL ANGLE OF SOME TRANSPARENT MEDIA

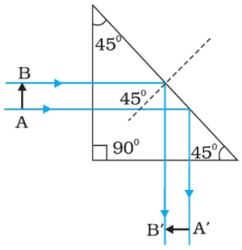
Substance medium	Refractive index	Critical angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.62	37.31°
Diamond	2.42	24.41°

Application of TIR

1) Diamond: Brilliance of diamond is mainly due to the TIR of light.

The RI of diamond is 2.42, hence if critical angle is very small (24.4^0). If it is cut suitably, so that the light falls at an angle greater than 24.4^0. Therefore it suffers multiple total internal reflections. Thus diamond sparkle brilliantly.

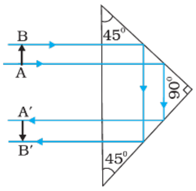
2) Porro-prism:(total reflection prism)



A right angled isosceles prism called porro-prism works on the principle of TIR of light. It can be used to bend the light through 90^0 and 180^0 and also to invert the image of an object without changing its size.

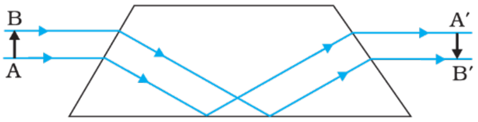
The incident beam is turned by an angle 90^0. This arrangement is used in periscope.

Note:

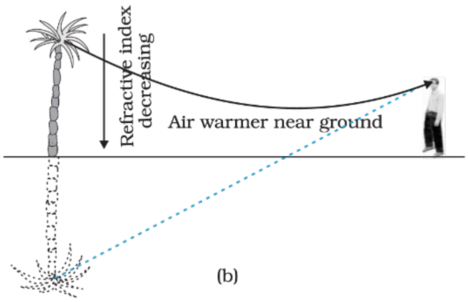


1) The incident beam is turned though 180^0, this arrangement is used in binoculars.

2) This arrangement is used to make inverted image.



3) Mirage:



(b)

Mirage is an optical illusion caused due to TIR and refraction.

On a hot day density of air layer increases with height. A day of light travelling from top of a tree downwards, more from denser medium to rarer medium. When the angle of incidence greater than critical angle, the ray undergoes TIR.

A distant observer sees the object as well as the image floating on the ground, he gets the impression of water pool near the object. This optical illusion is called mirage.

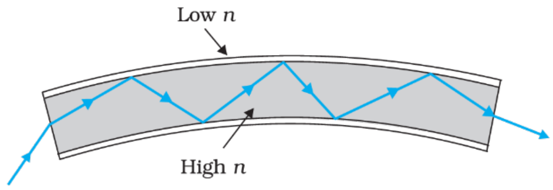
It is observed in deserts and in highways during hot days.

4) Optical fibres:

It is a transparent fibre which transmits light introduced at one end to the opposite end through repeated TIR.

It works on the principle of TIR.

It is made of high quality glass or quartz of high RI called core and coated with glass material of low RI called cladding.



A ray of light entering one end of the fibre at an angle greater than critical angle, undergoes multiple TIR and emerges through the other end without loss of energy.

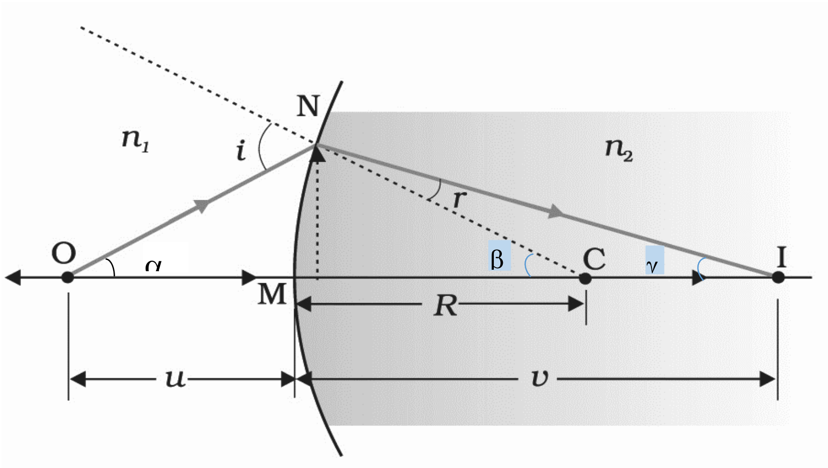
Note: The sleeve containing a bundle (10^4) of optical fibres is called as light pipe.

Uses of optical fibres: optical fibres are used

- To transmit light without any loss in its intensity.
- As light pipe to examine internal organs like stomach, esophagus, intestine. Etc.
- As decorative lamps.
- To transmit and receive electrical signals, which are converted into light by transducers.
- In the field of communication.

Refraction at spherical surface

Derive the relation between u, v, n and R For spherical surface:



The geometry of refraction of light at spherical surface is as shown.

Let, O=real object, I=real image of 'O', C=centre of curvature,

i=angle of incidence. r=angle of refraction, ON = incident ray

NI = Refracted ray, NC = normal, n_1=R.I. of medium (1),n_2=R.I. of medium (2), NM=perpendicular to principal axis.

From right angled triangle NMO, $\tan\alpha = \frac{MN}{MO}$

For small angle, $\alpha = \frac{MN}{MO}$ $(\tan\theta = \theta, \text{ if } \theta <<)$

From right angled triangle NMC $\tan\beta = \frac{MN}{MC}$ $\therefore$ $\beta = \frac{MN}{MC}$

From right angled triangle NMI, $\tan\gamma = \frac{MN}{MI}$ $\therefore$ $\gamma = \frac{MN}{MI}$

From triangle NOC,

$i = \alpha + \beta$ (exterior angle = sum of interior opposite angles)

$$\therefore \quad i = \frac{MN}{MO} + \frac{MN}{MC} \text{ ------------------ (1)}$$

From triangle NCI, $\beta = r + \gamma$ $(\beta \text{ is exterior angle})$ $\therefore$ $r = \beta - \gamma$ $(N\hat{I}C = N\hat{I}M)$

$$r = \frac{MN}{MC} - \frac{MN}{MI} \text{ ----------------- (2)}$$

From Snell's law $n_1 \sin i = n_2 \sin r$ For small angles sini = i and sinr = r

$$\therefore \quad n_1 \times i = n_2 \times r$$

From equation (1) and (2)

$$n_1\left[\frac{MN}{MO}+\frac{MN}{MC}\right]=n_2\left[\frac{MN}{MC}-\frac{MN}{MI}\right]$$

$$\therefore \qquad \frac{n_1}{MO}+\frac{n_2}{MI}=\frac{n_2}{MC}-\frac{n_1}{MC} \text{ -------------- (3)}$$

Applying Cartesian sign convention

MO = –u = object distance

MI = +v = image distance

MC = +R = radius of curvature

∴ Equation (3) becomes

$$\frac{n_1}{-u}+\frac{n_2}{v}=\frac{n_2}{R}-\frac{n_1}{R}$$

$$\boxed{\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}}$$

This is the relation between u, v, n and R.

Note:

1. If the object lies in the rarer medium, then $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$ is valid irrespective of the type of the spherical refracting surface.
2. If object lies in the denser medium, then $\frac{n_1}{v}-\frac{n_2}{u}=\frac{n_1-n_2}{R}$ is valid irrespective of the type of the spherical refracting surface.
3. The factor $\frac{n_2-n_1}{R}$ or $\frac{n_1-n_2}{R}$ measures the power of a refracting surface.

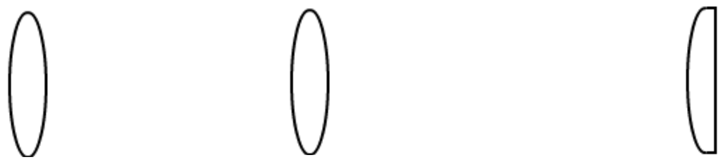
Lens:

Lens is an optical medium bound by two surfaces of which at least one is spherical.

There are two types,

1. **Convex lens:** A lens which is thinner at the edges and thicker at the middle is called convex lens.
2. **Concave lens:** A lens which is thinner at the middle and thicker at the edges is called concave lens.

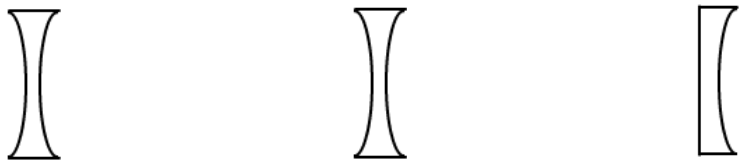
Types of convex lens



$R_2=\infty$

Biconvex lens (R_1 and R_2 may different) | Equiconvex lens (R_1 and R_2 are equal) | Plano-convex lens

Types of concave lens



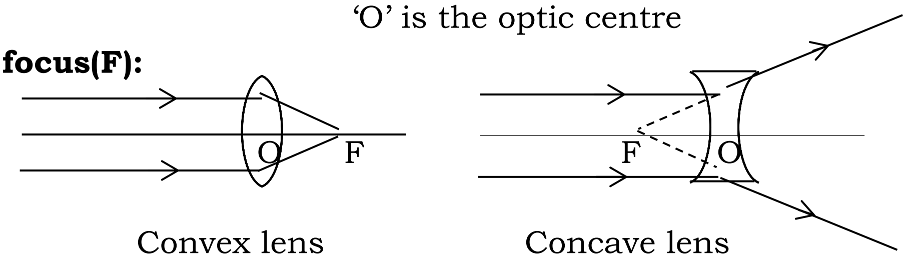
$R_2=\infty$

Biconcave ($R_1 \neq R_2$) | Equiconcave ($R_1=R_2$) | Plano concave

Optic centre of lens: It is a point in a lens on the principal axis through which a ray of light passes without deviation.

'O' is the optic centre

Principal focus(F):



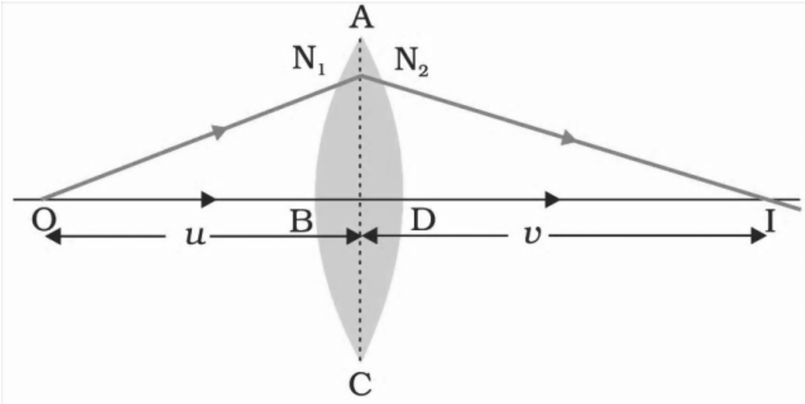
Convex lens | Concave lens

It is a point on the principal axis where the parallel the beam of light converge or appear to diverge after refraction.

In case of convex lens the light rays are converged at F. In case of concave lens the light rays appear to be diverged from F.

Focal length: It is the distance between optic centre and principal focus of a lens. SI unit is 'm'.

Derive the lens makers formula:



The geometry of image formation by a double convex lens is as shown in figure.

We have, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

Where, n_1=RI of object space (RI of surrounding), n_2=RI of image space (RI of lens)

u=object distance, v=image distance, R=radius of curvature.

The light ray undergoes two refractions at ABC and ADC surfaces.

Case1: Refraction at surface ABC

Let O=real object, I_1=real image of 'O', C_1=centre of curvature,

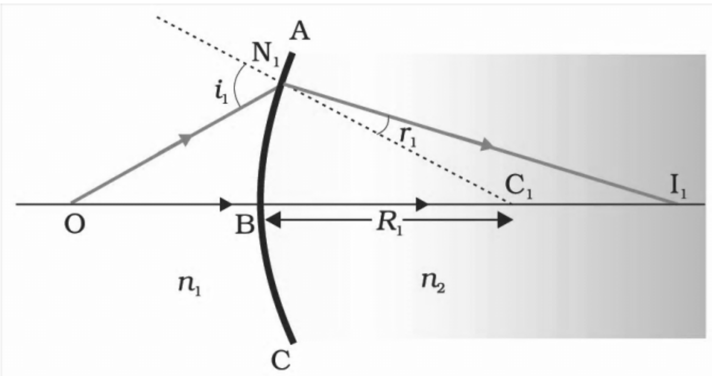
u = –BO, v = +BI_1, $n_2 \to n_2$, $n_1 \to n_1$,

R= BC_1=+R_1=radius of curvature of ABC

∴ equation (1) becomes

$$\frac{n_2}{BI_1} - \frac{n_1}{-BO} = \frac{n_2 - n_1}{R_1}$$

$$\frac{n_2}{BI_1} + \frac{n_1}{BO} = \frac{n_2 - n_1}{R_1} \quad \text{----------------(2)}$$



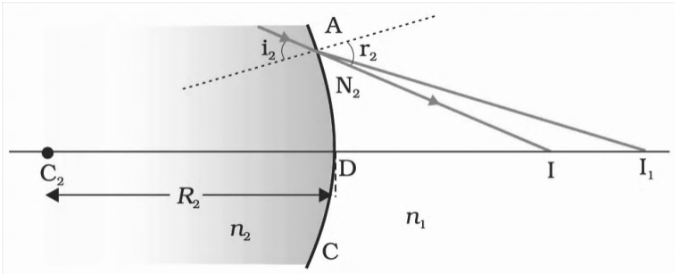
Case2: Refraction at surface ADC

In this case,

I_1 acts a virtual object for ADC surface.

I= Real image of I_1

u = +DI_1=+BI_1(for thin lens B and D are very close to each other)



v =+DI

R = DC_2 = – R_2=radius of curvature of ADC

∴ $n_2 \rightarrow n_1$ and $n_1 \rightarrow n_2$

∴ Equation (1) becomes

$$\frac{n_1}{DI} - \frac{n_2}{BI_1} = -\frac{n_2 - n_1}{R_2} \text{ ------------- (3)}$$

Adding equation (2) and (3)

$$\frac{n_2}{BI_1} + \frac{n_1}{BO} + \frac{n_1}{DI} - \frac{n_2}{BI_1} = \frac{n_2 - n_1}{R_1} - \frac{n_2 - n_1}{R_2}.$$

$$\frac{n_1}{BO} + \frac{n_1}{DI} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ --------- (4)}$$

If the object is at infinity

Then BO=∞, DI=f=focal length of lens

∴ equation (4) becomes

$$\frac{n_1}{f} = (n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{ ---------- (5)}$$

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = (n_{21} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \qquad \left(\because n_{21} = \frac{n_2}{n_1}\right)$$

This is called lens maker's formula.

Note-1: If the surrounding is air, $n_1=n_a=1$, then $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

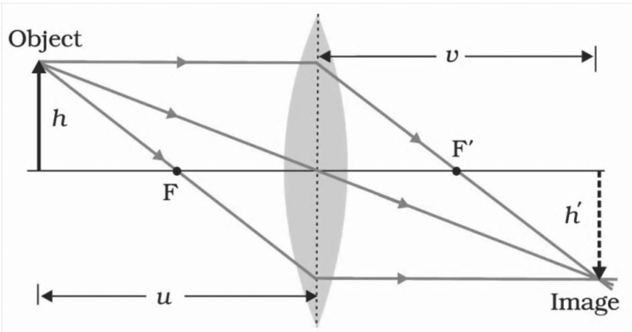
Where, n=RI of lens.

Note-2: If the thickness of lens is negligible compared to its radii of curvature then the lens is called thin lensx.

Note3: The position of an object on the principal axis of the lens for which the image is formed at ∞ is called first principal focus of the lens.

The position of an image on the principal axis of the lens whose object is lying at ∞ is called second principal focus of the lens.

Note-4: A ray emanating from the object parallel to the principal axis of the lens after refraction passes through the second principal focus. F^1 (in a convex lens) or appear to diverge (in a concave lens) from it.



A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.

A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.

Note-5: The thin lens formula is given by

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$$

Where, v=image distance, u=object distance, f=focal length.

Linear magnification of lens (m):

It is defined as the ratio of height of image to the height of object.

i.e. $m = \frac{h^{|}}{h}$ Where $h^{|}$=height of image, h=height of object

and also $m = \frac{v}{u}$ Where, v=image distance, u=object distance

$\therefore\ m = \frac{h^{|}}{h} = \frac{v}{u}$

Sign convension for linear magnification:

1. 'm' is positive for erect and virtual image formed by convex and concave lens.
2. m is negative for inverted and real image.

Power of lens: It is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.

Power of a lens is given by $P = \frac{1}{f}$ Where, f=focal length in metre.

S.I. unit of power is dioptre (D).

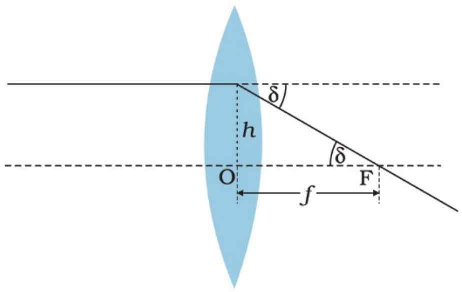
Define one dioptre:

We have, $P = \frac{1}{f}$, If, f=1m,

Then P=1D

i.e. The power of lens is said to be one dioptre if focal length is 1 metre.

Show that power of a lens is reciprocal of its focal length:



From the diagram, $\tan\delta = \frac{h}{f}$,

If, h=1m $\tan\delta = \frac{1}{f}$

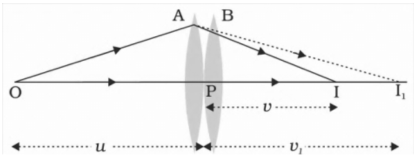
For small angles, $\tan\delta \approx \delta$ $\delta = \frac{1}{f}$,

since $\delta \approx p$ $p = \frac{1}{f}$

Note: 1) Power is also defined as the reciprocal of focal length.

2) Power is positive for convex lens and negative for concave lens.

Derive the expression for equivalent focal length of two thin lens in contact:



Consider two thin lens A and B in contact.

Let, f_1=focal length of A, f_2=focal length of B, O=real object

The thin lens formula is $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ ---------- (1)

Where, v=image distance, u=object distance, f=focal length.

The geometry of image formation is as shown.

Case1: Image formation by lens A.

Let, O=real object, I_1=real image, v=v_1, u=u and f=f_1

$\therefore$ equation (1) becomes $\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1}$ ----------- (2)

Case2: Image formation by lens B. Here, I_1 acts as virtual object and I is real image.

$\therefore$ v=v, u=v_1, and f=f_2 $\therefore$ equation (1) becomes,

$$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2} \text{ ----------- (3)}$$

Adding equation (2) and (3) $\frac{1}{v_1}-\frac{1}{u}+\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_1}+\frac{1}{f_2}$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \text{ -------- (4)}$$

But $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ f=equivalent focal length

$\therefore$ Equation (4) becomes $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

This is the expression for equivalent focal length of two thin lenses in contact.

If 'n' number of thin lens are in contact, then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+....+\frac{1}{f_n}$$

In terms of power, WKT $P=\frac{1}{f}$, $\therefore \frac{1}{f_1}=P, \frac{1}{f_2}=P_2, ...$

Thus, $P=P_1+P_2+..........+P_n$

Where, P is the net power of the lens combination.

Note:

1. Total magnification of combination of two lens is $m=m_1 \times m_2$

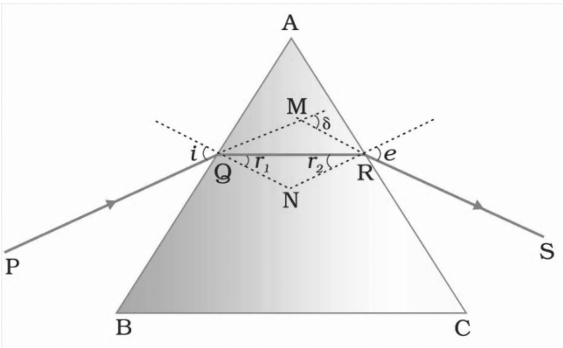
   Where, m_1=magnification of first lens, m_2=magnification of second lens

   for 'n' number of lenses, $m=m_1 \times m_2 \times m_3 \times .... \times m_n$.

   Thus the combination of lenses increases the magnification of the final image
2. System of combination of lenses is commonly used in designing lenses for cameras, microscopes, telescopes and other optical instruments.

Derive the expression for refractive index of the material of the prism:

Consider a prism ABC in air medium let n be the R.I. of prism and A be the angle of prism. The light ray undergoes refraction through a prism as shown in ray diagram.



For AB surface

Let PQ=incident ray

QR=refracted ray

i=angle of incidence

r_1=angle of refraction

d_1=angle of deviation

$\therefore d_1=i-r_1$

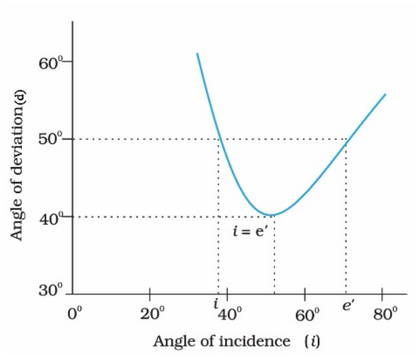
For AC surface

Let QR=incident ray, RS=emergent ray, r_2=angle of incidence

e=angle of emergence, d_2=angle of deviation

$\therefore d_2=e-r_2$

Total deviation



$d=d_1+d_2=(i-r_1)+(e-r_2)$

$d=i-r_1+e-r_2$

$d=i+e-r_1-r_2$

$d=i+e-(r_1+r_2)$ ---------- (1)

From quadrilateral AQNR

$A+\angle QNR=180^0$---------- (2)

From triangle QNR

$r_1+r_2+\angle QNR=180^0$---------- (3)

From equations (2) and (3)

$A+\angle QNR=r_1+r_2+\angle QNR$

$A=r_1+r_2$ ---------- (4)

Substituting (4) in (1)

$d=i+e-A$ -------------- (5)

As angle of incidence increases, angle of deviation varies as shown in graph.

At minimum deviation position of prism

$i=e$

$r_1=r_2=r$ and d=D=angle of minimum deviation.

$\therefore$ equation (5) becomes

$D=i+i-A$

$D=2i-A$

$A+D=2i$

$$i=\frac{A+D}{2}$$

Equation 4 becomes

$A=r+r$, $A=2r$

$$r=\frac{A}{2}$$

From Snell's law, $n=\frac{\sin i}{\sin r}$

$$\therefore n=\frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

This is called prism formula.

Angle of minimum deviation:

It is the minimum value of the angle of deviation for a ray passing through a prism.

Thin prism: The prism whose angle is about 8^0 is called thin prism or small angled prism.

Deviation produced by a thin prism:

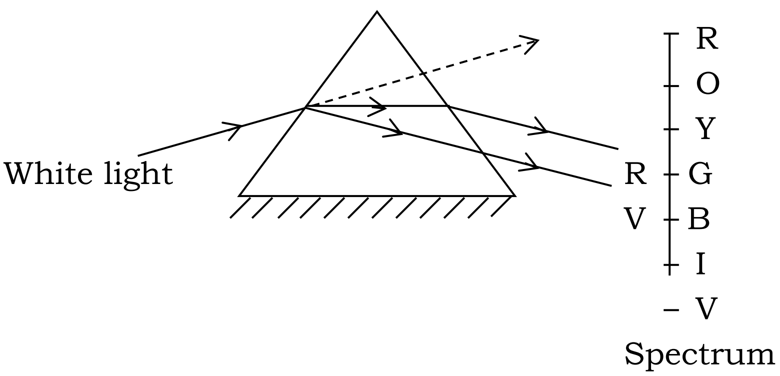
W.K.T. $$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

For a thin prism A and D are very small

$$\therefore \quad n = \frac{\frac{A+D}{2}}{\frac{A}{2}}$$

$$D = (n-1)A.$$

Dispersion by a prism:



When white light is passed through a prism it splits up into its constituents colours. This is called dispersion.

Spectrum: The band of colours obtained by a prism is called spectrum.

There are two types

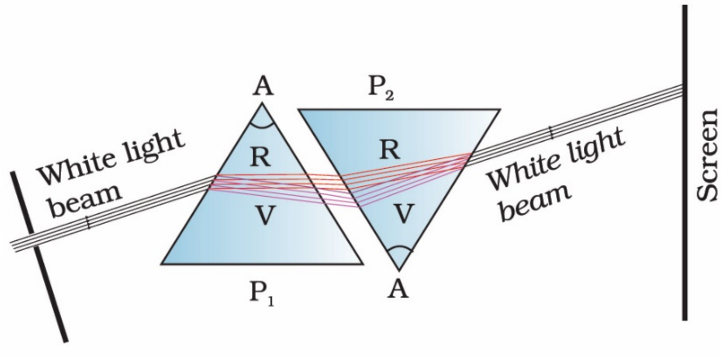
I. Pure spectrum: A spectrum in which colours are not overlapped is called pure spectrum.

Ex: Spectrum obtained by a prism.

II. Impure spectrum: A spectrum in which colours are overlapped is called impure spectrum

Ex: Rainbow.

Newton's classic experiment on dispersion: (A beam of light passing through a glass slab does not undergo dispersion. Explain?)



Arrange two prisms as shown in figure (Equivalent to glass slab)

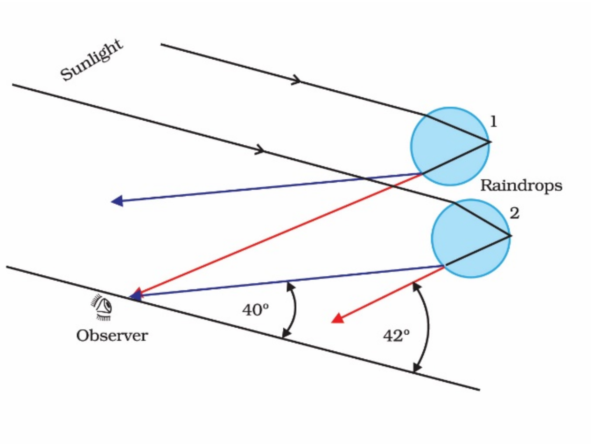
White light is split into its component colours by the first prism. Each color is due to some wavelength of light. RI of glass is different for different wavelengths. Red component of light bends the least, where, as violet component bends the most in glass. When an inverted prims is placed near the first prism, it reverses the action of the first prism. The output available from the second prism again white light.

Note:

1. Deviation of violet is greater than that of red. $d_V > d_R$.
2. $n_V > n_R$, Where, n_V=R.I. of prism for violet., n_R=R.I. of prism for red
3. $v_R > v_V$, where, v_R= velocity of red in prism, v_V=velocity of violet in prism
4. $\lambda_R > \lambda_V$, Where, λ_R=Wavelength of red, λ_V=Wavelength of violet.
5. The spectrum due to sunlight is called solar spectrum.
6. Red, blue and green are called primary colours. Other colours can be obtained by mixing primary colours.
7. Vacuum is a non dispersive medium, because all colours travel with the same speed.

The Rainbow:

It is the combined effect of refraction, total internal reflection and dispersion of light by water droplets of rain.

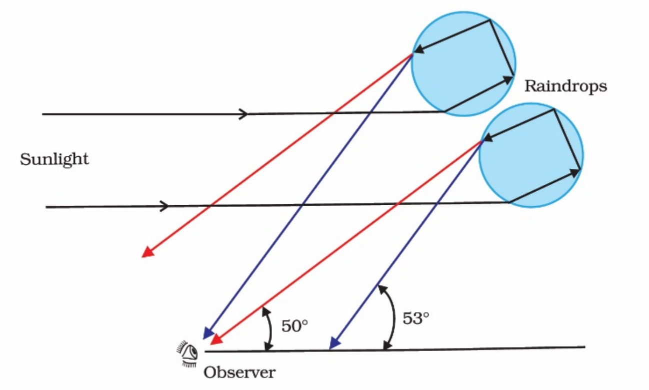


Primary rainbow: It is a result of three steps process, that is refraction, total internal reflection and refraction.

In primary rainbow, red colour present on the top and violet on the bottom.

Secondary rainbow: It is a result of four step process, that is refraction, two total internal reflections and refraction.

In secondary rainbow, the violet colour is present on top and red on the bottom.



Scattering of light

When a beam of light is passed through a material medium, the molecules absorb light, get excited and re-emit the light in all possible directions.

The phenomenon of absorption and re-emission of light in all possible direction by the molecules of the medium is called scattering of light.

The scattering of light depends upon the size of the scatterers in the atmosphere.

The scattered light is partially plane polarized.

Rayleigh's scattering law:

It states that, "the intensity of scattered light is inversely proportional to fourth power of the wavelength of incident light".

$I \alpha \frac{1}{\lambda^4}$ Where, I= intensity of scattered light, λ= wavelength of incident light.

Note:

Types of scattering:

Coherent scattering: The scattering in which the frequency of scattered light is same as that of incident light is called *coherent scattering*. It is also called *elastic scattering or Rayleigh's scattering*. **Ex:** atmospheric scattering

The coherent scattering takes place when the size of the scatterer is equal to or less than wavelength of incident light.

Incoherent scattering: The scattering in which the scattered light has lower and higher frequencies in addition to incident light frequency is called *incoherent scattering*. It is also called *inelastic scattering or Stokes scattering*. **Ex:** Raman effect, Compton Effect.

Incoherent scattering takes places when size of the scatterer is greater than wavelength of incident light.

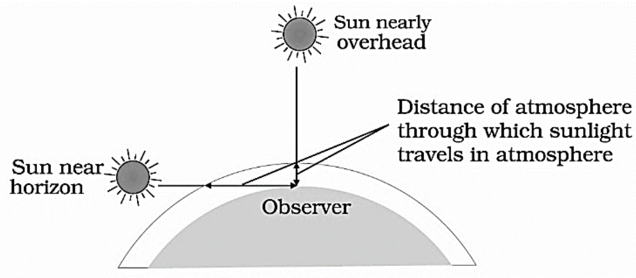
The blue colour of sky and sea:

The blue colour of sky and sea are due to coherent scattering by atmospheric molecules. From Rayleigh's scattering law. $I \alpha \frac{1}{\lambda^4}$

When sunlight passes through the atmosphere, the violet and blue colours are scattered more (due to less wavelength), but orange and red colours are scattered less (due to more wavelength). So molecules present in the atmosphere emit the light of violet and blue colours with maximum intensity. Our eyes are sensitive to blue colour. Thus the sky and sea appears blue colour.

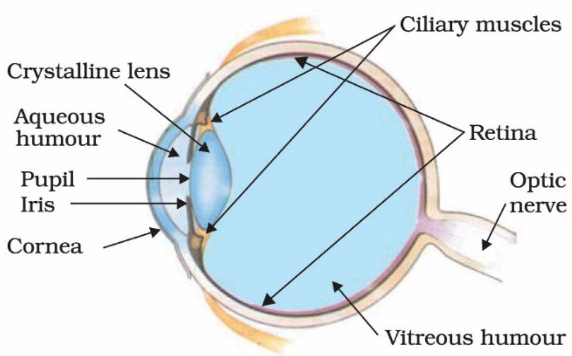
Reddish appearance of sun at sunrise and sunset:

Sunlight travels through a longer distance in the atmosphere at sunrise and sunset. Most of blue and other shorter wavelengths are removed by scattering. The Least scattered light (red to orange) reaching our eyes. Therefore sun looks reddish.



Optical instruments: Optical devices and instruments have been designed utilising reflecting and refracting properties of mirrors, lenses and prisms, periscope, kaleidoscope, binoculars, telescopes, microscopes are some examples of optical instruments.

The Eye: The eye is an optical device which acts as a natural camera. Light enters the eye through a curved front surface, cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under the control of muscles. The light is further focused by the eye lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and color respectively and transmit electric signals via the optic nerve to the brain, which finally processes this information.



The shape and focal length of eye lens can be modified by the ciliary muscles.

Note

1. The distance between retina and eye lens should be fixed i.e., V=2.5cm
2. The average RI of crystalline lens is 1.43
3. The average RI of aqueous humour and vitreous humour is 1.336

Accommadation: The modification of the focal length of the eye by the ciliary muscles to see the objects at all distances is called accommadation.

Least distance of distinct vision (D): (Near point)

The closest distance for which the lens can focus light on the retina is called the least distance of distinct vision.

D=25cm for normal vision (normal eye)

Note:

1. D=7 to 8cm in child ten years age. D≅200cm in a person at 60 years age
2. Eye has a capability to interpret incoming electromagnetic waves as image through a complex process.
3. **Far Point:**

   It is the largest distance for which the lens of the relaxed eye can focus light on the retina. For a person with normal vision far point is near ∞.
4. **Range of Vision:-**

   The distance between the near point and far point of an eye is known as range of vision. For a normal adult eye, the range of vision is 25cm to ∞.
5. **Power of accommodation:-**

   Power of accommodation of an eye is defined as the maximum variation in the power of the eye lens. For a normal eye the power of accommodation is about 4D.

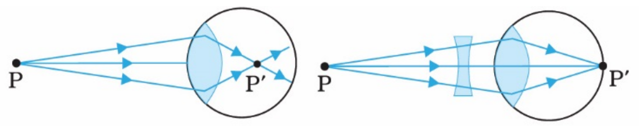
Common Defects of Vision:

(a) Short sightedness or Myopia. (b) long sightedness or Hypermetropia
(c) Presbyopia (d) Astigmatism

Correction of eye defects:

Nearsightedness or Myopia: If the eye lens converges light coming from distant object at a point in front of retina. This defect is called nearsightedness or myopia.

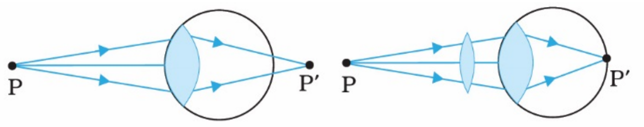
The myopia can be corrected by using concave lens between the eye and object.



Note: The person having shortsightedness can see near objects but far objects are blurred.

Farsightedness or Hypermetropia: If the eye lens converges light coming from near objects at a point behind the retina. This defect is called farsightedness or hypermetropia.

Hypermetropia can be corrected by using a convex lens between eye and object.



Note:

1. The person having farsightedness can see distant objects but near objects are blurred.

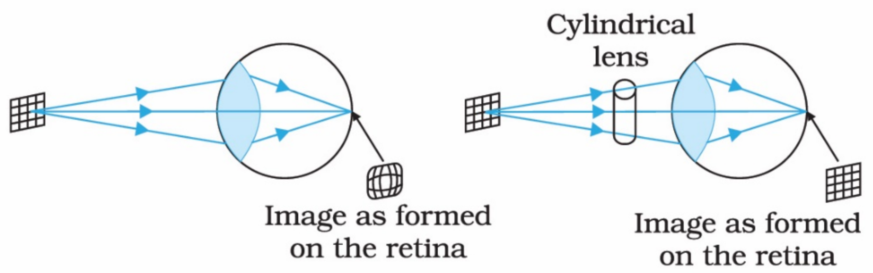
Presbyopia:

A human eye which cannot see near as well as far objects clearly is said to be suffering from a defect known as Presbyopia.

Presbyopia can be corrected by using bi-focal lens. i.e, upper surface of the lens is diverging (concave) and it corrects myopia. Lower surface of the lens is converging and it corrects Hypermetropia.

Astigmatism:

A human eye which cannot focus on both horizontal and vertical lines simultaneously is said to be suffering from a defect known as Astigmatism. It can be corrected by using a cylindrical lens of desired radius of curvature.



Microscope: The optical instrument used to see very small objects which cannot be seen by naked eye is called microscope.

Simple microscope: A simple microscope is a converging lens of small focal length.

Ex: Reading lens

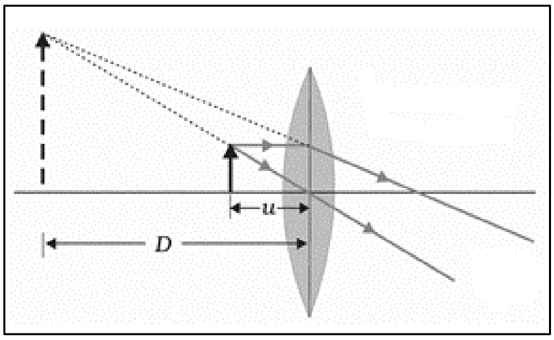
Expression for magnification of simple microscope when the image is at near point:-

When an object is placed between optical centre and principal focus, a virtual, erect and magnified image is obtained. The position of the object is adjusted so that, the image obtained at near point.

Linear magnification,

$$m = \frac{v}{u} \qquad \text{WKT} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\therefore m = v\left(\frac{1}{v} - \frac{1}{f}\right) = \left(1 - \frac{v}{f}\right)$$



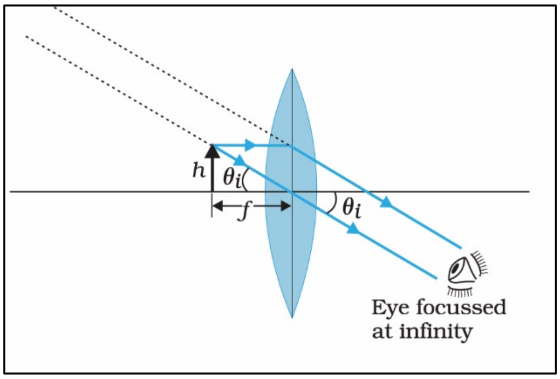
According to the sign convention. v = -D

$$\therefore m = 1 + \frac{D}{f}$$

Define magnifying power (Angular magnification) of a simple microscope and hence obtain an expression for it when the image is formed at ∞.

Magnifying power is defined as the ratio of the angle formed by the image(at ∞) to the angle formed by the object at the eye, when situated at least distance of distinct vision.

Magnifying Power= $M = \frac{\beta}{\alpha}$.



Where, α= angle formed by the object at the eye, when it is at D.

β= angle formed by the image at the eye.

Since α and β are small, tan α = α and tan β= β.

$$\therefore m = \frac{\tan\beta}{\tan\alpha}$$

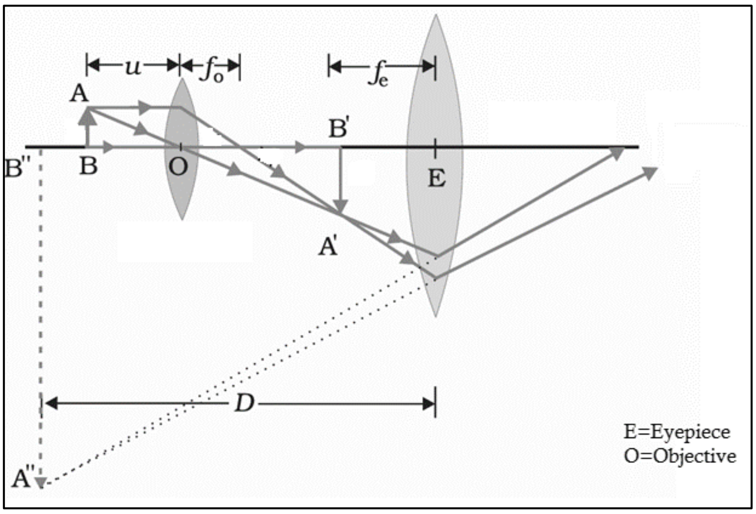
$$\tan\alpha = \frac{h_o}{D} \longrightarrow (1), \quad \tan\beta = \frac{h_i}{-v}$$

$$\tan\beta = \frac{h_o}{-v}\frac{v}{u} = \frac{-h_o}{u} \qquad \because \left[m = \frac{h_i}{h_o} = \frac{v}{u}\right]$$

Angle formed by the object, when it is at u = -f.

$$\tan\beta = \frac{h_o}{f} \longrightarrow (2) \quad \text{from (1) and (2)} \qquad m = \frac{\beta}{\alpha} = \frac{D}{f}$$

Compound microscope:



Compound microscope has two lenses. The lens nearest to the object is called objective. The lens nearest to the eye is called eye piece.

Objective is a convex lens of short focal length and small aperture. Eyepiece is also a convex lens but of large aperture and large focal length. When the object AB is placed at a distance greater than the focal length of the objective, a real inverted and magnified image $A'B'$ is formed. The position of the eye piece is adjusted such that $A'B'$ lies within its principal focus. Hence $A'B'$ serves as object to form a final image $A''B''$ which is virtual, erect enlarged image.

Magnification of objective , $m_0 = \frac{h_i}{h_0} = \frac{L}{f_0}$

Eye piece behave like a simple microscope $\therefore m_e = 1 + \frac{D}{f_e}$

Magnifying power of a compound microscope, $m = m_o m_e$

$\therefore m = m_o m_e = \left(\frac{L}{f_o}\right)\left(1 + \frac{D}{f_e}\right)$

When the final image is at ∞, then $m_e = \frac{D}{f_e}$

$\therefore$Magnifying power of a compound microscope is $m = m_o m_e = \left(\frac{L}{f_0}\right)\left(\frac{D}{f_e}\right)$

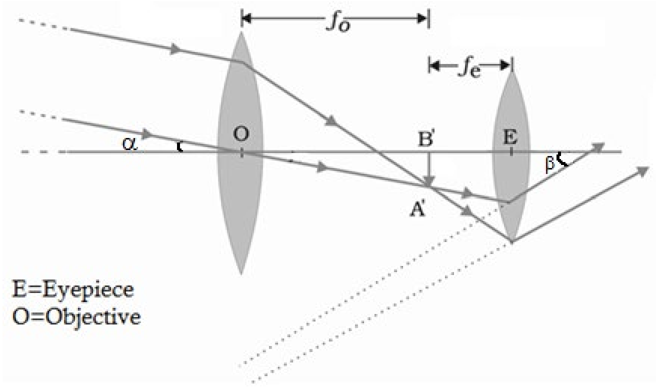
Where f_0 = focal length of the objective

f_e = focal length of the eye piece

L= length of the tube

D= least distance of distinct vision.

Telescope:-



The optical instrument used to see distant objects is called telescope. It has an objective and an eyepiece. Objective is a convex lens of large aperture and long focal length. The eye piece is also a convex lens but of small aperture and short focal length. The rays from a distant object converges to form an image $A'B'$. The eye piece is adjusted such that $A'B'$ lies exactly at its principal focus. Thus the final image is obtained at ∞.

Magnifying power of telescope:

It is defined as ratio of angle subtended by final image at eye to the angle subtended by the object at the eye.

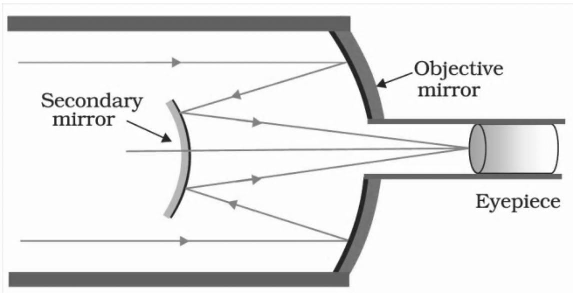
Magnifying power, $M = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$

length of the telescope, $L = f_0 + f_e$

Where f_0 = focal length of objective

f_e = focal length of eye piece

Reflecting Telescope (Cassegrain Telescope):-



Telescope with mirror objectives is called reflecting telescope.

Casse grain telescope consist of concave mirror of 5m in diameter with a narrow hole at its centre. Parallel rays from a distant star incident on the mirror, tends to converge at its principal focus. But before converging, they get reflected by the secondary convex mirror on the eyepiece. The final image is seen through the telescope.

Thus the magnifying power of reflecting telescope.

$$m = \frac{f_0}{f_e} = \frac{\frac{R}{2}}{f_e} = \frac{R}{2f_e}$$

f_e = focal length of eye piece

R= Radius of the curvature of the convex mirror.

Answer the following questions:

1. Define the terms (a) ray of light & (b) beam of light
2. Which are the common defects of human eye?
3. What is accommodation of eye?
4. Define linear magnification.
5. Write the expression for the magnification in terms of object and image distance
6. What is refraction of light?
7. Write the formula for refractive index for normal refraction.
8. Define critical angle.
9. Write the relation between refractive index and critical angle of a material
10. What is total internal reflection?
11. On what principle optical fibre does works?
12. What is a lens?
13. Write the expression for power of a lens
14. What is dispersion of light?
15. State Rayleigh's law of scattering.
16. Mention the expression for linear magnification of a simple microscope
17. Mention the expression for angular magnification of a simple microscope
18. State laws of reflection
19. Write the sign conventions used for measuring distances in case of spherical surfaces
20. State laws of refraction.
21. Draw diagram representing lateral shift (lateral displacement) of a ray passing through a parallel sided glass slab.
22. Draw diagram representing apparent depth for (a) normal and (b) oblique viewing
23. Mention a few illustrations that occur in nature due to refraction of light.
24. What happens to the direction of the incident ray when it travels from (a) optically denser medium to rarer medium & (b) optically rarer medium to denser medium?
25. Write the conditions to have total internal reflection
26. Mention a few illustrations of total internal reflection
27. Write the expression for the power of a combination of number of thin lenses
28. Define power of a lens. Write its S.I unit.
29. Why sky is blue in color?
30. Why sun is red at rise and set?

31. What is least distance of distinct vision? Write its value.
32. What is myopia? Why it occurs? How to correct it?
33. What is hypermetropia? Why it occurs? How to correct it?
34. Draw ray diagram of a simple microscope
35. Draw ray diagram showing the image formation in a compound microscope and label the parts.
36. Mention the expression for magnification of a compound microscope
37. Draw the ray diagram of a refracting telescope and label the parts.
38. Draw schematic diagram of a reflecting telescope
39. Derive the relation between focal length and radius of curvature of a spherical mirror
40. Derive mirror equation.
41. Derive the relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the spherical surface OR derive the relation between n, u, v, & R.
42. Derive lens maker's formula
43. Derive the expression for effective focal length of two thin lenses in contact
44. Arrive at the expression for refractive index of material of prism in terms of angle of the prism and angle of minimum deviation.

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Chapter 10:

WAVE OPTICS

Introduction to Theories of Light:

i) Descartes's and Newton's corpuscular theory:

Newton developed Descartes's corpuscular theory. According to Newton's theory, speed of light in a denser medium is greater than the speed of light in a rarer medium.

Later Foucault proved that speed of light is greater in a rarer medium than in a denser medium.

ii) Huygens's and Fresnel's Ether wave theory:

According to this theory, light is a periodic disturbance transmitted through a medium called ether in the form of waves and light is a mechanical longitudinal wave. Later on Fresnel concluded that light is a transverse wave, not longitudinal.

iii) Maxwell's electromagnetic wave theory:

According to this theory light is an electromagnetic wave which consists of mutually perpendicular time varying electric and magnetic fields which are also perpendicular to the direction of propagation of light wave. But the theory fails to explain photoelectric effect and Compton effect.

iv) Einstein's quantum theory:

According to this theory, light consists of packets of energy known as photons. This theory explains photoelectric effect and Compton effect but fails to explain interference, diffraction and polarization.

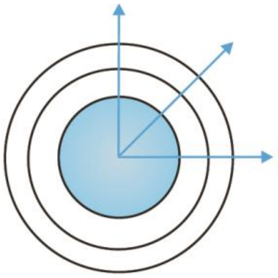
v) de-Broglie's theory of dual nature of light :

According to de-Broglie, a moving material particle is always associated with a wave known as matter waves or de-Broglie waves thus light has dual nature.

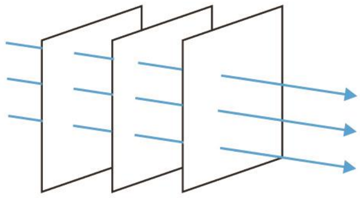
Wave front: The locus of points which oscillate in same phase is called a wave front. (OR) Surface of constant phase is called wave front.

Types of wave front:

1. **Spherical wave front**: The wave front formed due to a point source at finite distance is called spherical wave front.
2. **Plane wave front:** The wave front formed due to parallel beam of light or by the source which is kept at infinity is called plane wave front.
3. **Cylindrical wave front:** The wave front formed due to linear source is called cylindrical wave front.



Spherical wave front



Plane wave front

Note:

1) The energy of the wave travels in a direction perpendicular to the wave front.
2) The speed with which the wave front moves outwards from the source is called the speed of the wave.

Huygens principle:

Statement:

1) "Each point of the wave front is the source of a secondary disturbance, called secondary wavelets, which travel in all directions with the velocity of light in the medium".
2) The surface drawn tangentially to these secondary wavelets in the forward direction gives new wave front called secondary wave front.

Explanation:

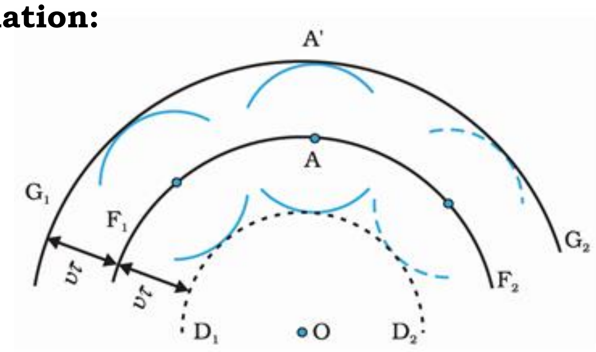


Fig. 1

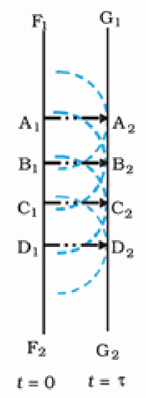


Fig.2

Let 'O' be a point source of light. It sends out disturbance with spherical symmetry in the homogeneous medium around it.

Let F_1F_2 be the position of spherical wavefront at t=0. After time t=τ, each secondary wavelet will travel a distance = vτ.

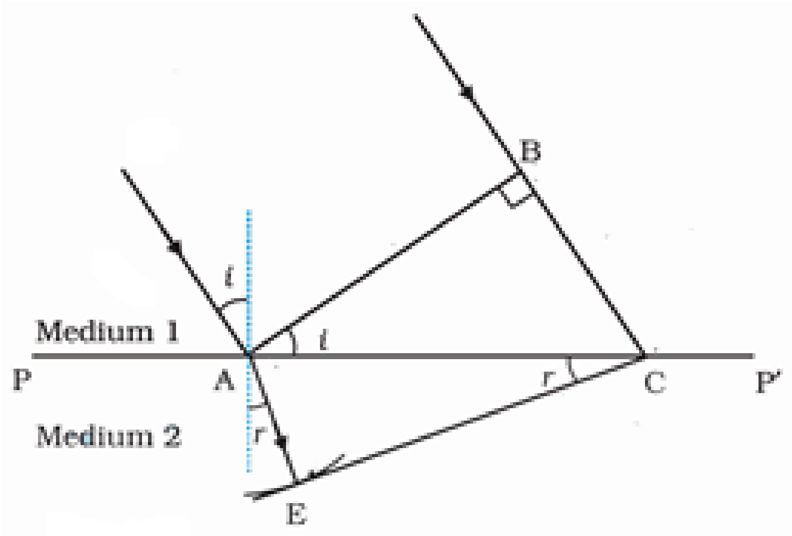
Where, 'v' is the speed of light in the given medium.

By taking every point as the centre, draw spheres of radii = vτ. Draw tangents on every sphere and join them. An envelope G_1 G_2 enclosing these tangents is the position of wave front F_1F_2 after time τ (fig1).

The position of a plane wave front after time 'τ' is shown in fig 2. A_1A_2, B_1B_2, C_1C_2 and D_1D_2 etc are the rays of light.

Note: There is no backward flow of energy when a wave travels in the forward direction. Therefore wavefront which shows backward propogation of light is not considered.

Derive Snell's law using Huygens principle:



Let PP^1 = Interface, n_1 = R.I of medium (1)

n_2 = R.I of medium (2) ($n_2>n_1$)

v_1 = speed of light in medium (1)

v_2 = speed of light in medium (2)

AB = plane wavefront incident on PP^1

i = angle of incidence

r = angle of refraction

CE = Refracted wavefront

The distances BC and AE are travelled by light in same time 't'

For medium (1) $v_1 = \frac{BC}{t}$

$\therefore$ BC = v_1 t

For medium (2) $v_2 = \frac{AE}{t}$

AE = v_2 t

$\therefore$ velocity = $\frac{distance}{time}$

$v = \frac{d}{t}$

From right angled triangles ABC, sin i = $\frac{BC}{AC}$

From right angled triangles AEC, sin r = $\frac{AE}{AC}$

Consider, $\frac{\sin i}{\sin r} = \frac{\frac{BC}{AC}}{\frac{AE}{AC}}$

$$\frac{\sin i}{\sin r} = \frac{BC}{AE}$$

$$= \frac{v_1 t}{v_2 t}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

But $\frac{v_1}{v_2} = \frac{n_2}{n_1}$

$$\therefore \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$$

$$\therefore \; n_{21} = \frac{\sin i}{\sin r}$$

This is Snell's law.

Note:

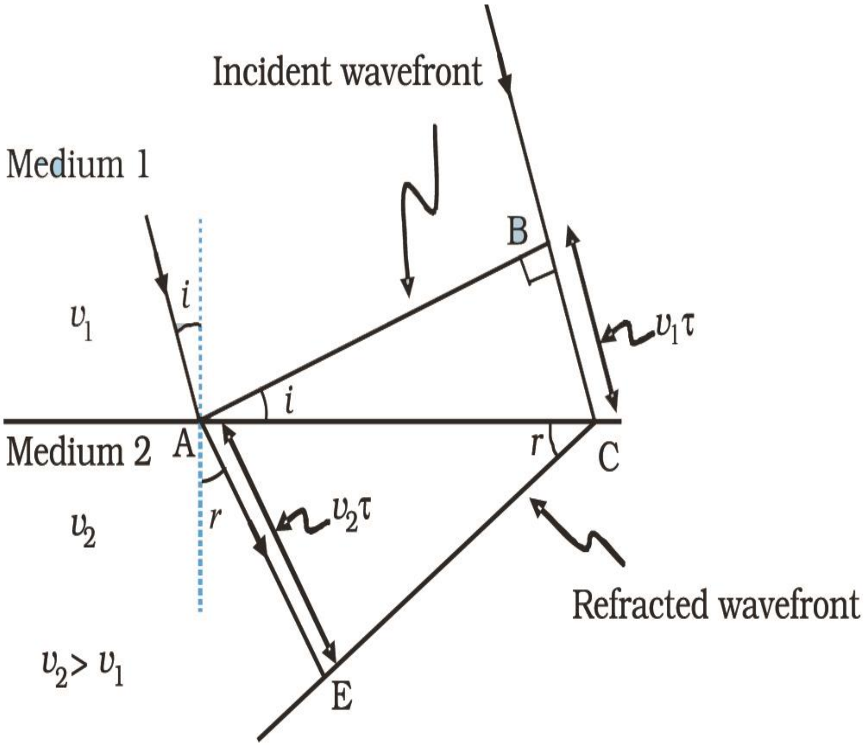
If λ_1 and λ_2 denote the wavelengths of light in medium (1) and medium (2) and if BC = λ_1 and AE = λ_2 then

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1 t}{v_2 t}$$

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

i.e., when a wave travels from rarer medium to denser medium, the speed of light decreases but frequency remains constant.

Refraction of a plane wave at a rarer medium:



A plane wave incident on a rarer medium for which, $v_2 > v_1$, bends away from the normal.

i.e., i < r

According to Snell's law,

$$\frac{\sin i}{\sin r} = n_{21} = \frac{n_2}{n_1}$$

If, $i=i_c$, then, $r=90^0$ & $\sin r=1$,

$$\therefore \quad \sin i_c = \frac{n_2}{n_1}$$

And for all angles of incidence, $i>i_c$ the wave will undergo total internal reflection.

Derive law of reflection using Huygens principle:

Let AB = Incident plane wavefront

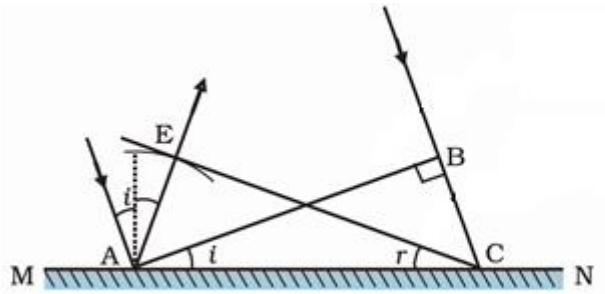
CE = reflected plane wavefront

i = angle of incidence

r = angle of reflection

t = time taken by light to travel from B to C and A to E.

v = speed of light



But $v = \frac{BC}{t}$ and $v = \frac{AE}{t}$

i.e., BC = vt i.e., AE = vt

$\therefore$ BC = AE

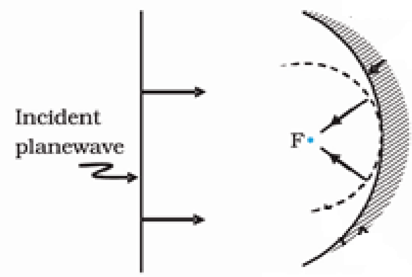
The two triangles EAC and BAC are congruent.

Therefore, i = r

This is the law of reflection.

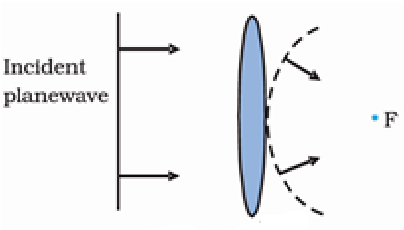
Explain reflection of plane wavefront by concave mirror:

When a plane wavefront is incident on a concave mirror then the reflected wavefront will be spherical wavefront of radius $\frac{R}{2}$ where, R=radius of concave mirror.



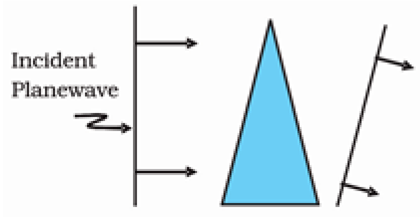
Explain refraction of plane wavefront through a convex lens:

When a plane wavefront is incident on a convex lens then the refracted wavefront will be spherical wavefront of radius ‘f’ where, f=focal length of lens.



Explain refraction of plane wavefront through a prism:

When a plane wavefront is incident on a prism then the refracted wavefront will be plane wavefront.



The Doppler effect:

Whenever there is a relative motion between the source of light and the observer, the frequency of light received by the observer is different from the actual frequency of the light emitted by the source. This effect is known as Doppler's effect.

Apparent frequency: The frequency of light received, when the source of light and the observer are in relative motion is known as apparent frequency.

Doppler's shift: The apparent change in the frequency or wavelength of the light due to the relative motion between the source and the observer is called Doppler shift.

The fractional change in frequency, $\frac{\Delta\nu}{\nu} = -\frac{V_{radial}}{C}$

Where, V_{radial}= component of the source velocity along the line joining the observer to the source relative to the observer.

V_{radial} is considered positive when the source moves away from the observer.

Blue shift: When the source of light moves towards the observer, there is an apparent decrease in wavelength, thus the spectrum of the radiation from the source shift towards the blue end. This is called blue shift.

Red shift: When the source of light moves away from the observer, there is an apparent increase in wavelength. Thus the spectrum of the radiation from the source shift towards the red end. This is called red shift.

Note:

- Red shift confirms that the universe is expanding.
- Doppler effect is the basis of the measurements of the radial velocities of distant galaxies.

INTERFERENCE OF LIGHT

The modification in the distribution of light energy due to superposition of two or more light waves is called Interference.

Ex. 1) A thin layer of oil spreaded over a water appears coloured due to interference.

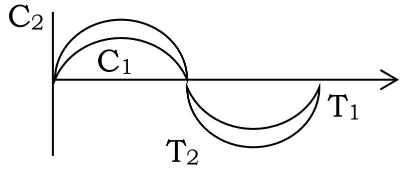
2) A thin soap film appears coloured due to interference.

Note: Interference was first observed by Newton and it was first explained by Thamos Young.

Types of Interference: There are two types

I. Constructive interference: When crest of one wave falls on crest of another and trough of one wave falls on trough of another wave. Then the interference is called Constructive Interference. In constructive interference intensity of light is maximum. Therefore bright fringe is formed.

In constructive interference, two waves are in phase,

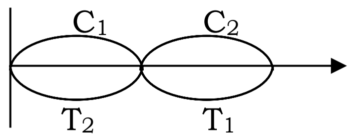


C_1 and C_2 = crest
T_1 and T_2 = trough

II. Destructive Interference: When crest of one wave falls on trough of another and trough of one wave falls on crest of another. Then the interference is called destructive interference.

In destructive interference, the intensity is minimum or zero. Therefore dark fringe is formed.

In destructive interference, the two waves are out of phase.



Note:

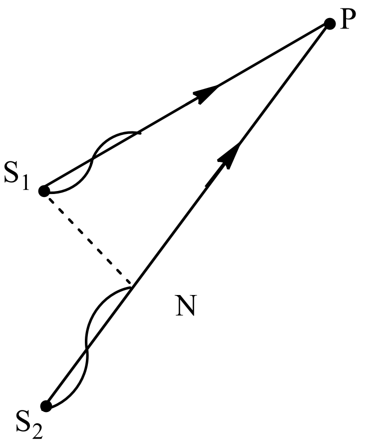
1. During interference, energy can neither be created nor be destroyed. But it is redistributed. i.e. light energy is conserved during interference.
2. In order to get interference the light waves should have same amplitude, same wavelength, same frequency and of constant phase difference.

Coherent sources: Two sources are said to be coherent sources if they emit light waves of same amplitude, same wavelength, and same frequency and of constant phase difference.

Two independent identical sources cannot act as a coherent sources, because they donot maintain constant phase difference.

Theory of interference:

Consider two coherent sources S_1 and S_2. The two waves coming from S_1 and S_2 are superposed at 'P'.



The displacement of wave at 'P' due to S_1 is

$$Y_1 = a\sin\omega t$$

Where, a=amplitude, $\omega=2\pi f$=angular frequency, ωt=phase

The displacement of wave at 'P' due to S_2 is

$$Y_2 = a\sin(\omega t+\phi)$$

Where, ϕ=phase difference between two waves.

From superposition principle,

The resultant displacement is

$Y = Y_1 + Y_2$

$Y = a\sin\omega t + a\sin(\omega t+\phi)$

$Y = a[\sin\omega t + \sin(\omega t+\phi)]$

$$Y = a\left[2\sin\left(\frac{\omega t+\omega t+\phi}{2}\right)\times\cos\left(\frac{\omega t-(\omega t+\phi)}{2}\right)\right]$$

$\because \quad \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

but, $\cos(-\theta)=\cos\theta$

$$Y = 2a\sin\left(\frac{2\omega t+\phi}{2}\right)\cos\left(-\frac{\phi}{2}\right)$$

$$Y = 2a\sin\left(\omega t+\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)$$

$$Y = 2a\cos\left(\frac{\phi}{2}\right)\sin\left(\omega t+\frac{\phi}{2}\right)$$

$$Y = R\sin\left(\omega t+\frac{\phi}{2}\right)$$

Where, $R = 2a\cos\left(\frac{\phi}{2}\right)$ = Resultant amplitude

This is the theory of interference

Obtain the condition for constructive interference and destructive interference:

We have resultant amplitude

$$R = 2a\cos\left(\frac{\phi}{2}\right)$$

Where, a=amplitude, ϕ=phase difference

But resultant intensity $\propto$ (resultant amplitude)^2 , $I \propto R^2$

Condition for constructive interference: For constructive interference I is maximum, therefore R is also maximum.

When $\cos\left(\frac{\phi}{2}\right) = \pm 1$

i.e. $\frac{\phi}{2} = 0, \ \pi, \ 2\pi, \ 3\pi, ...$

or $\phi = 0, 2\pi, 4\pi, 6\pi.....$

in general, $\phi = 2n\pi$

Where, n=0, 1, 2, 3, ...

This is condition for constructive interference in terms of phase difference

$I \propto R^2$ $R^2 = 4a^2\cos^2\frac{\phi}{2}$ R is maximum, When, $\cos\frac{\phi}{2} = \pm 1$

but path difference=$\frac{\lambda}{2\pi} \times$ Phase difference

path difference=$\frac{\lambda}{2\pi} \times \phi$

path difference=$\frac{\lambda}{2\pi} \times 2n\pi$

Path difference = $n\lambda$

Where, λ=wavelength.

R is minimum, When, $\cos\frac{\phi}{2} = 0$

This is the condition for constructive interference in terms of path difference.

Condition for destructive interference: For destructive interference, I is minimum, therefore R is also minimum.

When, $\cos\left(\frac{\phi}{2}\right)=0$

or $\frac{\phi}{2}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},......$

$\phi=\pi, 3\pi, 5\pi, ...$

In general $\phi=(2n+1)\pi$

Where, n=0, 1, 2, 3.....

This is the condition for destructive interference in terms of phase difference.

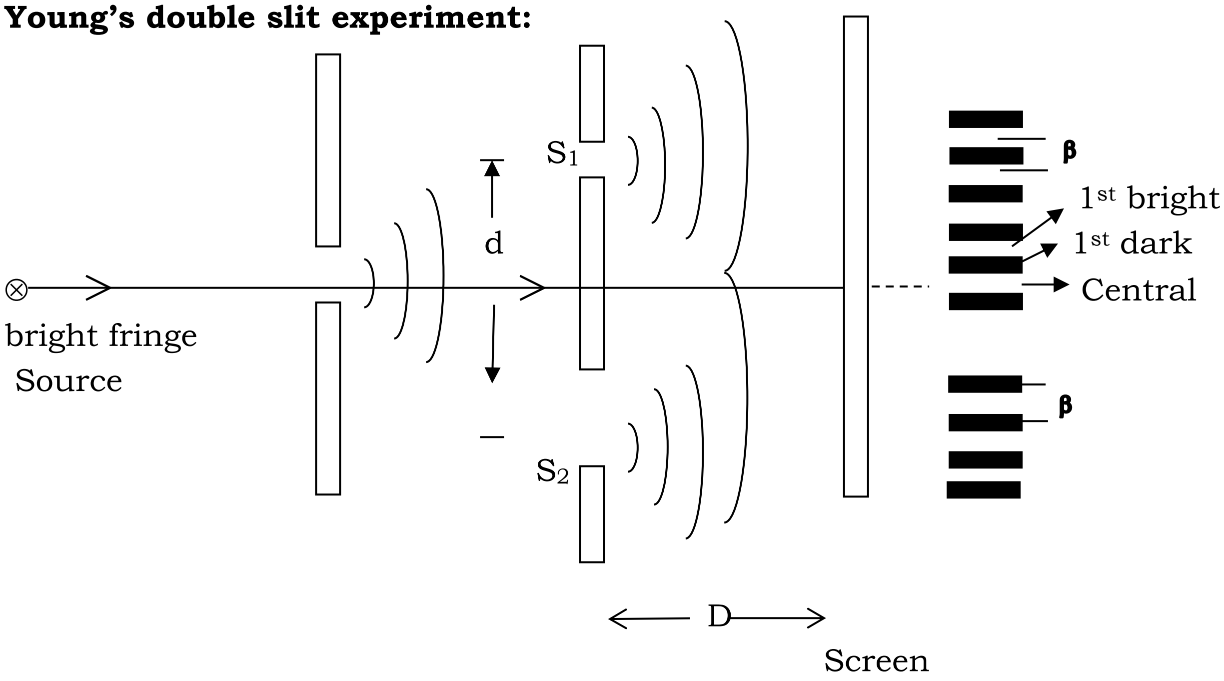
But, path difference = $\frac{\lambda}{2\pi}\times\phi$

Path difference= $\frac{\lambda}{2\pi}\times(2n+1)\pi$

Path difference= $(2n+1)\frac{\lambda}{2}$

This is the condition for destructive interference in terms of path difference.

Young's double slit experiment:



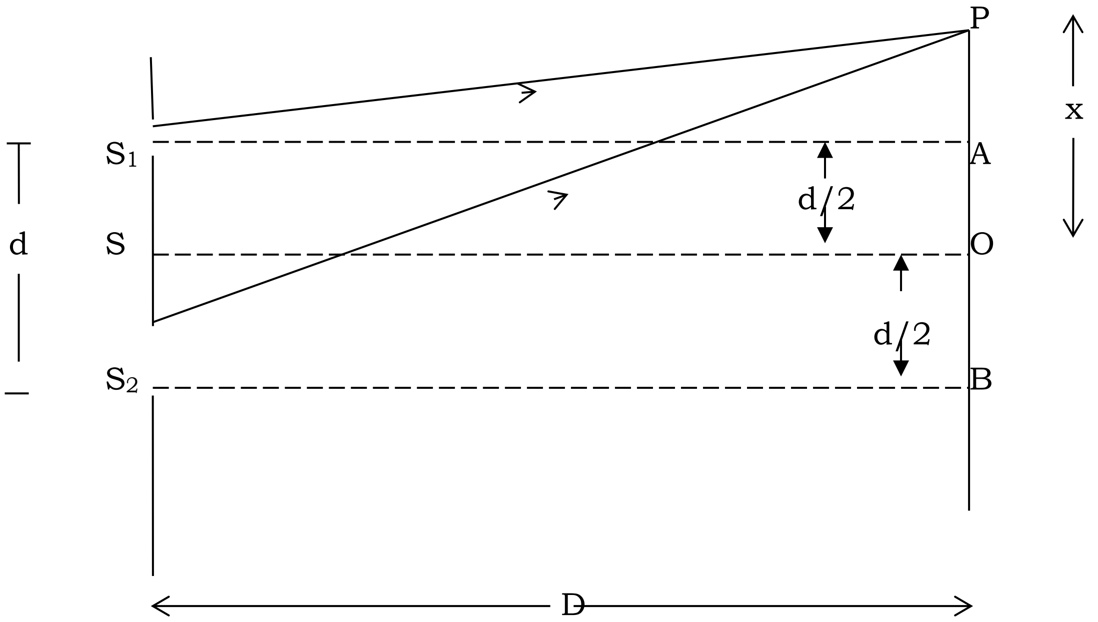
The monochromatic light coming from the source is made to fall on a narrow rectangular slit 'S'. The light coming from 'S' is made to fall on two narrow closely spaced parallel slits S_1 and S_2. S_1 and S_2 acts as coherent sources. The light waves (wave front) coming from S_1 and S_2 are superposed each other and produce interference pattern on the screen. The pattern consists of alternate bright and dark fringes. The bright fringe is due to constructive interference and dark fringe is due to destructive interference. The central fringe is always bright.

The distance between centres of two consecutive bright or dark fringes is called fringe width (β).

Fringe width: $\beta = \frac{\lambda D}{d}$,

Where, λ= wavelength of light, d= distance between double slits
D= distance between screen and double slits.

Obtain an expression for fringe width:



Let S_1 and S_2 be the two coherent sources separated by a distance 'd'. Let S be the midpoint between S_1 and S_2. Let 'D' be the distance between double slits and screen. Consider any point 'p' on the screen at a distance 'x' from 'o'. Two light waves coming from S_1 and S_2 are superposing to produce interference pattern. The central bright fringe is formed at 'o'.

From figure

$S_2P - S_1P$ = path difference

From right angle triangle S_1AP

$S_1P^2 = S_1A^2 + AP^2$ because, AP=OP-OA, AP=(x-d/2)

$\therefore S_1P^2 = D^2 + (x - d/2)^2$

$S_1P^2 = D^2 + x^2 + \frac{d^2}{4} - dx \rightarrow$ (1)

From right angle triangle S_2BP

$S_2P^2 = S_2B^2 + BP^2$ because, BP=OP+OB, BP= $x + \frac{d}{2}$

$S_2P^2 = D^2 + \left(x + \frac{d}{2}\right)^2$ $\therefore S_2P^2 = D^2 + x^2 + \frac{d^2}{4} + dx \rightarrow (2)$

Equation (2) – (1).

$$S_2P^2 - S_1P^2 = D^2 + x^2 + \frac{d^2}{4} + dx - D^2 - x^2 - \frac{d^2}{4} + dx$$

$$S_2P^2 - S_1P^2 = 2dx$$

$$(S_2P + S_1P)(S_2P - S_1P) = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P} \qquad \therefore \text{Path difference } = \frac{2xd}{S_2P + S_1P}$$

If d is very small then x is very small

$S_1P \cong S_2P \cong D$

$\therefore$ **Path difference** $= \frac{\mathbf{2xd}}{\mathbf{D + D}} = \frac{2xd}{2D}$ $\therefore$ **Path difference** $= \frac{xd}{D} \rightarrow$ (3)

For constructive interference, path difference = $n\lambda$ $\longrightarrow$ (4)

Where, n=0,1,2,3,....................(Fringe number),

From equation (3) and (4)

$$\frac{xd}{D} = n\lambda$$

$$x = \frac{n\lambda D}{d}$$

$\therefore$Distance of n^{th} bright fringe from ‘O’

$$x_n = \frac{n\lambda D}{d}$$

Similarly for $(n-1)^{th}$ bright fringe

$$x_{(n-1)} = \frac{(n-1)\lambda D}{d}$$

But fringe width= $x_n - x_{n-1}$

$$\beta = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d}$$

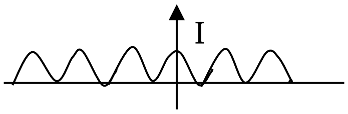
$$\beta = \frac{\lambda D}{d}(n - n + 1)$$

$$\beta = \frac{\lambda D}{d}$$

This is the expression for fringe width.

Note:

1. The intensity distribution curve of interference pattern is as shown.



2. Width of bright fringe = width of dark fringe
3. If two waves have different amplitudes 'a' and 'b', then $R=\sqrt{a^2+b^2+2ab\cos\phi}$
4. We know that $R=\sqrt{a^2+b^2+2ab\cos\phi}$, if 'R' is maximum

   When $\cos\phi = +1$, $R_{max}=\sqrt{a^2+b^2+2ab\times 1}=\sqrt{(a+b)^2}$, $R_{max}=a+b$
5. If 'R' is minimum, when $\cos\phi = -1$

   $\therefore$ $R_{min}=a-b$
6. $\frac{R_{max}}{R_{min}}=\frac{a+b}{a-b}$
7. $I_1 \alpha a^2$, $I_2 \alpha b^2$

   $\sqrt{I_1}\,\alpha a$ $\sqrt{I_2}\,\alpha b$, And $I \alpha R^2$

   But $R^2 = a^2+b^2+2ab\cos\phi$

   $\therefore$ $I = I_1+I_2+2\sqrt{I_1 I_2}\cos\phi$
8. I is maximum when $\cos\phi=+1$

   $I_{max}=I_1+I_2+2\sqrt{I_1 I_2}$

   I is minimum when $\cos\phi = -1$ $I_{min}=I_1+I_2-2\sqrt{I_1 I_2}$
9. $I \alpha R^2$

   $I_{max} \alpha R^2_{max}$

   $\therefore$ $I_{max} \alpha (a+b)^2$

   Similarly $I_{min} \alpha (a-b)^2$

   $\therefore \frac{I_{max}}{I_{min}}=\frac{(a+b)^2}{(a-b)^2}$

DIFFRACTION OF LIGHT

The phenomenon of bending of light waves around the edges of obstacle is called diffraction.

The diffraction was first observed by F.M. Grimaldi

Examples for diffraction:

1. The different colures observed in a spider web are due to diffraction.
2. Coloured rings around the moon are due to diffraction.
3. Luminous border rounding the profile of a mountain is due to diffraction.

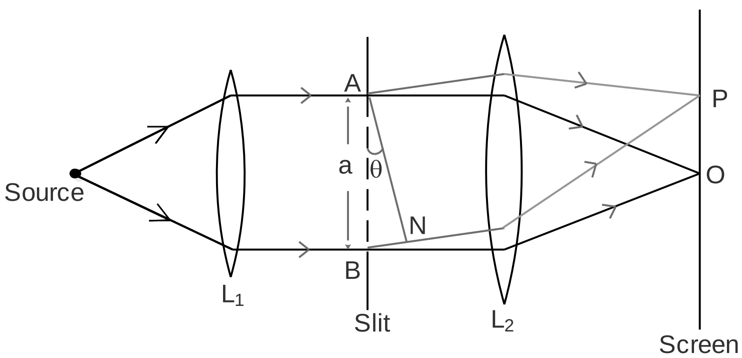
Note:

1. The diffraction is common to all types of waves.
2. The diffraction is observed when the wave length of wave is comparable to the size of obstacle.
3. Diffraction of sound is easily observed in nature because wave length of sound is comparable with size of the obstacle present around us.
4. Diffraction of light is not so easy in nature because the wave length of light is very small compare to the size of the obstacle present around us.
5. **Types of diffraction:** There are two types.
    1. **Fresnel's diffraction:** If the spherical or cylindrical wavefront incident on the obstacle, the diffraction is called Fresnel's diffraction.

       In Fresnel's diffraction, the obstacle is at a finite distance from source and screen.
    2. **Fraunhoffer diffraction:** If the plane wavefront incident on obstacle, the diffraction is called Fraunhoffer diffraction.

       In Fraunhoffer diffraction, the obstacle is at infinite distance from the source and screen.

Diffraction at a single slit (Fraunhoffer diffraction):



The monochromatic light coming from source is made to incident on a convex lens L_1 to obtain parallel beam of light. The parallel beam of light is made to incident on a narrow slit 'AB' of width 'a'. The diffracted waves are passed through another convex lens 'L_2'.

Since point O on the screen is equidistant from points A and B, the secondary wavelets from A and B reach the point O in the same phase and hence constructive interference takes place.

$\therefore$ O is position of central maximum

The secondary wavelets will also be diffracted through an angle θ and reach the point P on the screen. The point P will be of maximum or minimum intensity depending on the path difference between the secondary wavelets.

From figure, path difference = $BN = a\sin\theta$

Where, θ=angle of diffraction

i) Condition for diffraction minima: Let, path difference=λ

i.e., $a\sin\theta = \lambda$

Divide the slit into two equal halves. So path difference between secondary wavelets originating from each half is $\frac{\lambda}{2}$. Therefore, these wavelets will meet point P out of phase, hence destructive interference takes place at P. Thus position P will be of minimum intensity.

i.e., For Ist minima, path difference $= a\sin\theta_1 = \lambda$

since, θ_1 is very small

$$a\theta_1 = \lambda$$

$$\theta_1 = \frac{\lambda}{a}$$

In general, for n^{th} minima

Path difference = $a \sin\theta_n = n\lambda$

Since, θ_n is very small

$$a\theta_n = n\lambda$$

$$\theta_n = \frac{n\lambda}{a}$$

Phase difference = $2n\pi$

Where, n = 1, 2, 3, ……….

λ = wavelength of light.

ii) condition for secondary diffraction maxima:

If path difference = a sinθ is an odd multiple of $\frac{\lambda}{2}$, then the constructive interference takes place at P. Hence point P is the position of secondary maxima.

For n^{th} maxima.

Path difference = $a \sin\theta_n = \left(n + \frac{1}{2}\right)\lambda$.

θ_n is small

$$\therefore \theta_n = \left(n + \frac{1}{2}\right)\frac{\lambda}{a} \quad \text{where, } n = 1, 2, 3, \ldots\ldots\ldots.$$

Note-1: The distance of 1^{st} minimum on either side from the centre of the central maxima is

$$x = \frac{\lambda D}{a}$$

Where, λ = wavelength of light

a = width of slit

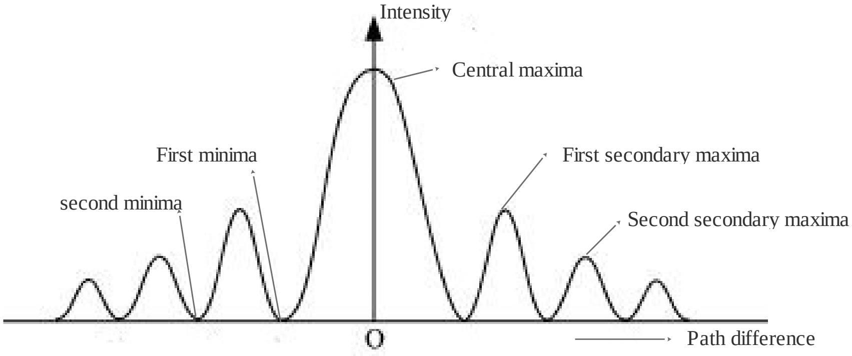
D = distance between slit and screen.

Note-2: Width of central maxima: It is the distance between the two first minima on either side of the central maxima.

i.e., $2x = \frac{2\lambda D}{a}$

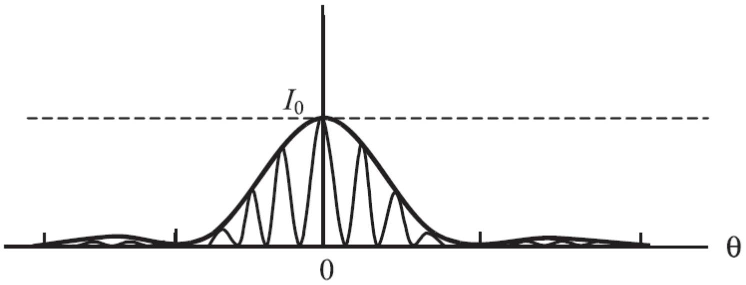
Note-3: Angular width of central maximum = $2\theta = \frac{2x}{f} = \frac{2x}{D} = \frac{2\lambda}{a}$

Intensity distribution curve:



The diffraction pattern consist of a very bright band called central maxima or principal maxima. On either side of the principal maxima, alternate minima and secondary maxima are present.

Comparison of interference and diffraction of light: Interference and diffraction both are the result of superposition of waves. Therefore they are not easily distinguishable. If there are number of sources then the resultant effect of the superposition of waves emanating from them is comparable to diffraction. But if there are two coherent sources then the term interference is used for the result.



The figure shows double slit interference pattern. Solid thick line enclosing this pattern is comparable to single slit diffraction.

Difference between interference and Diffraction patterns:

Interference	Diffraction
➢ It is due to superposition of two wavefronts originating from two coherent sources.	It is due to superposition of secondary wavelets originating from the different points of the same wavefront.
➢ All the maxima i.e., bright fringes are of the same intensity.	The bright fringes are of varying intensity. (intensity decreases away from the central maxima on each side).
➢ The dark fringes are usually almost perfectly dark.	The dark fringes are not perfectly dark.
➢ The width of fringes is equal	Width of central fringes is double than the width of other maxima.
➢ Bands are large in number and are equally spaced.	Bands are few in number and are unequally spaced.

Observing single slit diffraction pattern:

It can be observed by using two razor blades and an electric lamp with straight filament.

The blades are held to maintain a narrow slit just near the eyes and if we see the filament through the slit, bright and dark fringes are observed with slight adjustments of the slits.

Slits made by aluminum foil, two fingers can also be used to observe diffraction.

Multi colors shown by CD is because of diffraction of light.

Important point:

In interference and diffraction, light energy is redistributed, if it reduces in one region, producing a dark fringe, it increases in another region producing a bright fringe. There is no gain or loss of energy, which is consistent with the principle of conservation of energy.

Limit of resolution of optical Instrument

It is the minimum separation between two points at which they can be seen as distinct.

Resolving power of optical instrument:

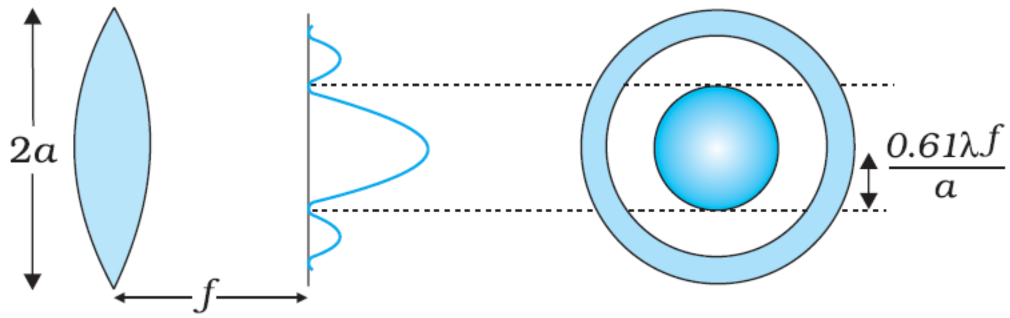
It is the reciprocal of minimum separation between two points at which they can be seen as distinct.

Image of a point object is not sharp due to diffraction. Explain? (Airy disc)

Due to diffraction of light at a circular aperture, a converging lens can not form a point image of an object, rather it produces a brighter central disc (known as airy disc) surrounded by alternate dark and bright concentric rings.

The radius of the central bright region is given by

$$r_0 = \frac{1.22\lambda f}{2a} = \frac{0.61\lambda f}{a}$$



The lateral width of the image = $f\Delta\theta = \frac{1.22\lambda f}{2a} = \frac{0.61\lambda f}{a}$

$$\therefore \Delta\theta = \frac{0.61\lambda}{a}$$

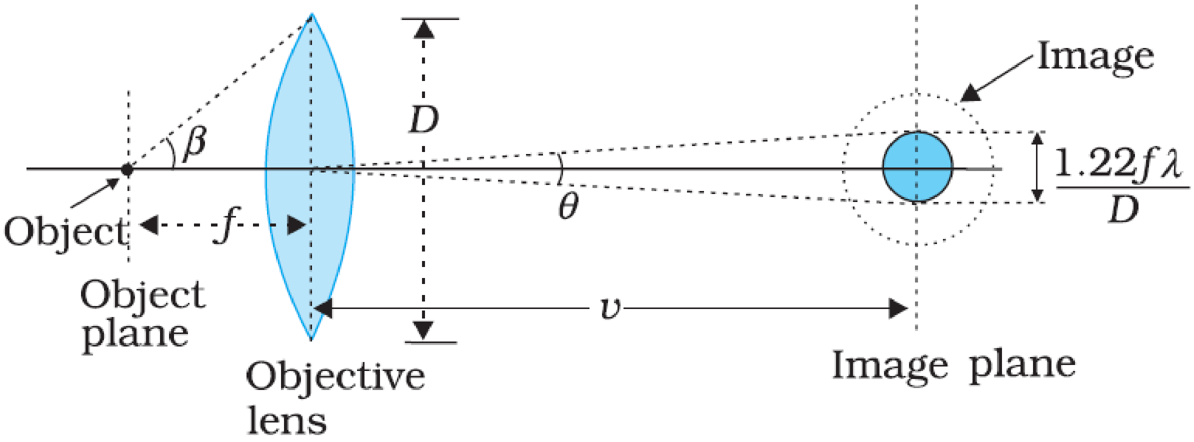
Where f = focal length of the lens

2a = D = diameter of the lens or circular aperture.

$\Delta\theta$ = limit of resolution of optical instruments.

Obtain an expression for limit of resolution of a microscope:

Let an object be placed just away from the focus of circular non corrected objective lens of a microscope (u = f). It will produce a diffracted real image at V.



From figure, $\tan\beta = \frac{D}{2}\left(\frac{1}{u}\right) \approx \frac{D}{2f}$

Then $\frac{D}{u} \approx \frac{D}{f} \approx 2\tan\beta \longrightarrow (1)$

The size of image due to diffraction

$$V\theta = V\left(\frac{1.22\lambda}{D}\right)$$

Two objects whose images are closer than this distance will not be resolved, they will be seen as one. The corresponding minimum separation d_{min}, in the object plane is given by

$$d_{min} = \frac{V\left(\frac{1.22\lambda}{D}\right)}{m}$$

$$d_{min} = \frac{1.22\lambda}{D}\frac{V}{m}$$

But $m = \frac{V}{u} = \frac{V}{f} \quad \therefore \frac{V}{m} = f$

Hence $d_{min} = \frac{1.22\lambda f}{D} \longrightarrow (2)$

From (1) and (2)

$$d_{min} = \frac{1.22\lambda}{2\tan\beta}$$

Since β is small, $\tan\beta \approx \sin\beta$

$$\therefore d_{min} = \frac{1.22\lambda}{2\sin\beta}$$

If the medium between the object and the objective lens is other than air with refractive index 'n' then,

Limit of resolution of a microscope is given by

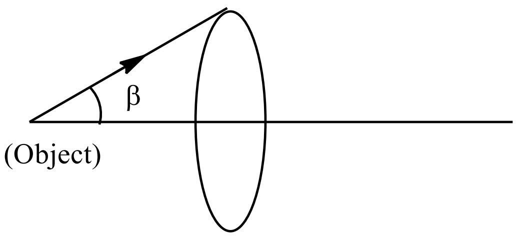
$$d_{min} = \frac{1.22\lambda}{2n\sin\beta}$$

Where λ = wavelength of light

nsinβ = numerical aperture

Mention the expression for limit of resolution of microscope:

$$d_{min} = \frac{1.22\lambda}{2n\sin\beta}$$



Where, d_{min} = limit of resolution

λ = wavelength of light

β = Semirevrtical angle

n = R.I of medium between object and objective

S.I. unit of limit of resolution of microscope is meter.

Resolving power of microscope:

$$R.P = \frac{1}{d_{min}}$$

$$\therefore\ R.P = \frac{2n\sin\beta}{1.22\lambda}$$

Method of increasing resolving power of microscope :

Resolving power of microscope can be increased,

i. by increasing refractive index of medium (n)
ii. by increasing semivertical angle (β)
iii. by decreasing wavelength of light (λ)

Limit of resolution of Telescope: $\Delta\theta = \frac{0.61\lambda}{a}$

Where, Δθ = limit of resolution, λ = wavelength, a = radius of objective

S.I. unit of Limit of resolution of Telescope is radian.

Note: $\Delta\theta = \frac{1.22\lambda}{D}$, Where, D= diameter of objective.

Resolving power of Telescope:

$$R.P = \frac{1}{\Delta\theta}$$

$$R.P = \frac{a}{0.61\lambda}$$

Note: $R.P = \frac{D}{1.22\lambda}$, Where, D= diameter of objective.

Methods of increasing resolving power of telescope:

Resolving power of telescope can be increased,

1. By increasing diameter of telescope.
2. By decreasing wavelength of light.

Oil – immersion objective: The objective of microscope in which oil is filled between the object and the objective lens.

Note: Resolving power of a microscope mainly depends on wavelength of light and the RI of the medium because sinβ cannot be more than one.

Validity of ray optics (Fresnel distance)

Fresnel distance is difined as the distance of the screen from the slit or aperture when the spreading of light due to diffraction from the centre of the screen is equal to the size of the slit.

It is given by $z_F = \frac{a^2}{\lambda}$

Where, a = width of the slit
λ = wavelength of light
D = distance between screen and the slit

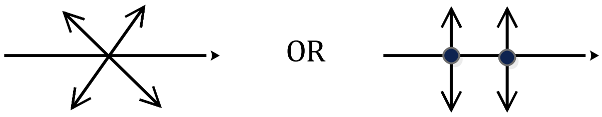
- If $D < Z_F$, the spreading due to diffraction is smaller compared to the size of the beam.
- If $D = Z_F$, the spreading due to diffraction becomes comparable to the size of the beam.
- If $D > Z_F$, the spreading due to diffraction dominates over that due to ray optics.

i.e., ray optics is valid in the limit of wavelength tending to zero.

POLARISATION

Unpolarised light:

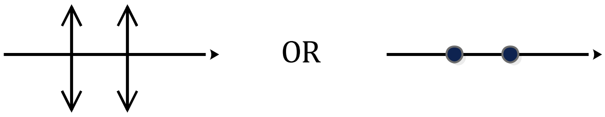
It is the light which has vibrations in all planes perpendicular to direction of propagation.

It can be represented 

as

Polarisation: When a beam of unnpolarised light is passed through a polaroid, the electric vectors are restricted to a single plane perpendicular to direction of propagation. This is called polarisation.

Polarised light: It is the light in which the electric vectors are restricted to a single plane perpendicular to direction of propagation.

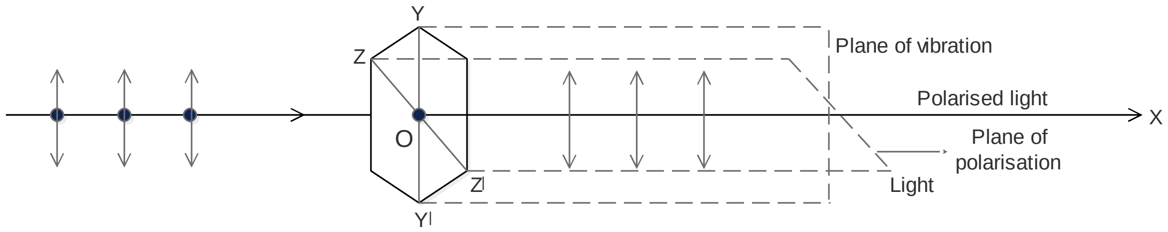
It can be 

represented as

Note:

1. Light wave is a transverse wave, the electric field vibrations (electric vector) are right angles to the direction of propagation of light.
2. An instrument used to produce polarised light from unpolarised light is called a polarizer.

Plane of vibration and plane of polarization:



Plane of vibration: The plane in which vibration of polarized light are confined (XY plane).

Plane of polarization: A plane perpendicular to the plane of vibration (XZ plane).

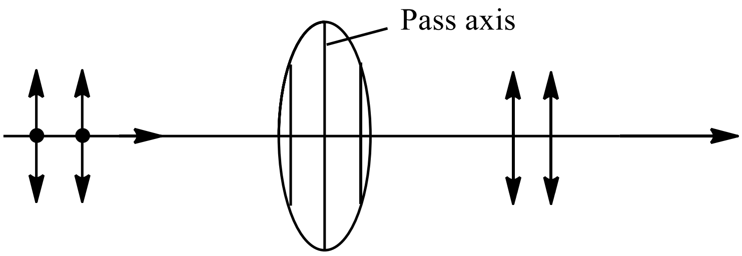
Polaroid:

Polaroids are the synthetic crystal sheets used produce and analyse polarised light.

A polaroid consists of long chain molecules aligned in a particular direction.

When unpolarised light is passed through a polaroid, the electric vector along the direction of aligned molecules get absorbed and electric vector perpendicular to the direction of aligned molecules are emerged out. Therefore the vibration of emerged light are restricted to a single plane. So emerged light is plane polarised light (or linearly polarised light).

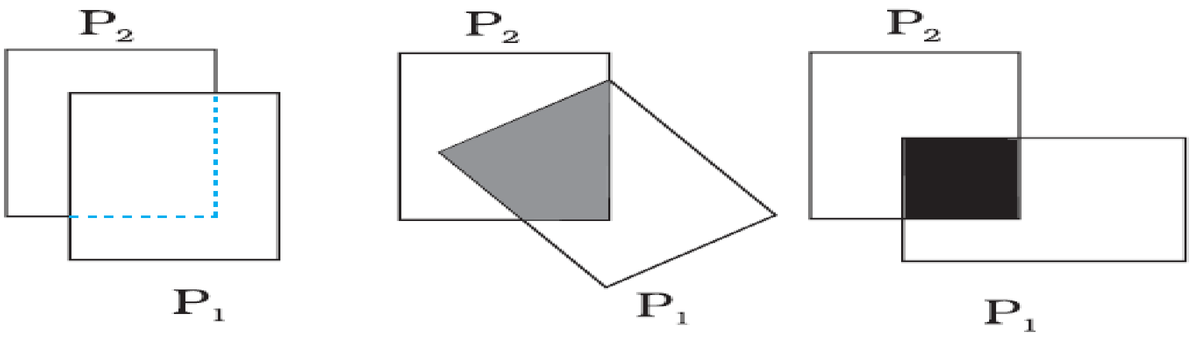
Pass axis: The direction perpendicular to the direction of aligned molecules is called pass axis.



Uses of polaroid:

1. Polaroid can be used to control the intensity of light.
2. It is used in sun glass.
3. It is used in windowpanes of aeroplanes and trains.
4. Used in photographic cameras and 3D movie cameras.

Describe an experiment to demonstrate the transverse nature of light:

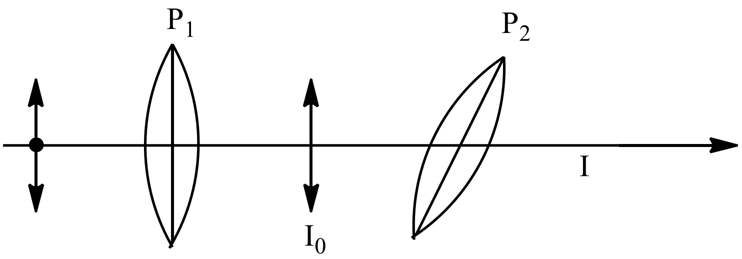


If the light from the source passes through a single polaroid P_1, its intensity reduced by half. Rotating P_1 has no effect on the transmitted beam and transmitted intensity remains constant.

If the light passes through two polaroids P_2 and P_1. Its intensity varies as the angle between them varies from O° to 90°.

The intensity of the transmitted beam of light is maximum when P_2 and P_1 are parallel. And zero when P_2 and P_1 are perpendicular.

Malus law:



If we rotate P_1 or P_2 then intensity of emerged light varies.

$$I=I_0\cos^2\theta$$

This is called Malus law.

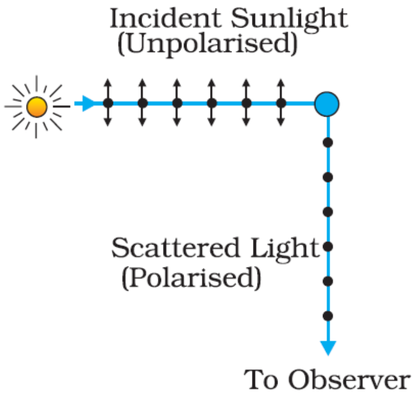
Where, I=Intensity of polarised light after passing through P_2.

(Intensity of emergent polarised light)

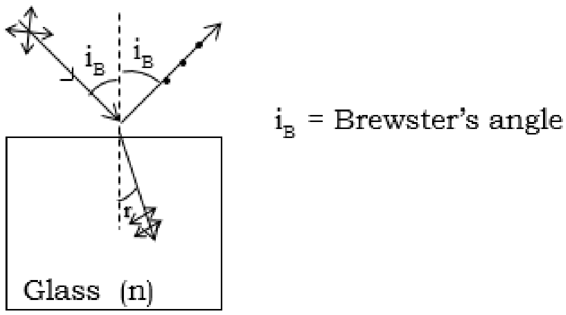
I_0= Intensity of polarised light after passing through P_1. (intensity of incident polarised light)

θ=angle between two pass axes of P_1 and P_2.

Polarisation by scattering: When an unpolarised light is incident on a molecule then scattering take place. It is observed that the light which is scattered in the direction perpendicular to incident direction is plane polarized. Sunlight gets polarized by the molecules of atmosphere.



Polarisation by reflection:



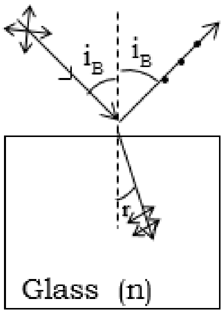
When a beam of unpolarised light is incident on a glass surface, the reflected light is partially polarized, the degree of polarization varies with angle of incidence. At a particular angle of incidence, the reflected light is completely polarized and it this condition reflected wave becomes perpendicular to refracted wave.

Brewster's angle (i_B):
The angle of incidence at which the reflected light is completely polarised is called Brewster's angle.

State Brewster's law:
It states that refractive index of refracting medium is equal to tangent of Brewster angle.

i.e. $n = \tan i_B$

Arrive Brewster's law:



i_B = Brewster's angle

If angle of incidence is equal to Brewster's angle, i.e. $i = i_B$,

then $r + i_B = 90^0$

$\therefore \quad r = 90^0 - i_B$

According to Snell's law $n = \dfrac{\sin i}{\sin r}$, n=R.I. of refracting medium.

$$\therefore \quad n = \frac{\sin i_B}{\sin(90 - i_B)}$$

$$n = \frac{\sin i_B}{\cos i_B}$$

$$n = \tan i_B$$

This is called Brewster's law.

One Mark Questions:

1. Define wave front.
2. What is the shape of wavefront obtained from a point source at a (i) small distance (ii) large distance?
3. Under what conditions a cylindrical wave front is obtained?
4. What type of wave front is obtained when a plane wave is reflected by a concave mirror?
5. Who proposed the wave theory of light?
6. Name the physicist who experimentally studied the interference of light for the first time.
7. What is interference of light?
8. What is the maximum intensity of light in Young's double slit experiment if the intensity of light emerging from each slit is I_o?

OR What is the intensity of light due to constructive interference in Young's double slit experiment if the intensity of light emerging from each slit is I_o?

9. Define fringe width.
10. Instead of using two slits as in Young's experiment, if two separate but identical sodium lamps are used, what is the result on interference pattern?
11. What is the effect on interference fringes when yellow light is replaced by blue light in Young's double slit experiment?
12. How does the fringe width in interference pattern vary with the wavelength of incident light?
13. What is the effect on the interference fringes in a Young's double-slit experiment when the monochromatic source is replaced by a source of white light?
14. How does the fringe width in interference vary with the intensity of incident light?
15. Which colour of light undergoes diffraction to maximum extent?
16. Name a factor which affects the resolving power of a microscope.
17. How will the diffraction pattern due single slit change when violet light replaces green light?
18. Do all waves exhibit diffraction or only light?
19. We do not encounter diffraction effects of light in everyday observations. Why?
20. Why are diffraction effects due to sound waves more noticeable than those due to light waves?
21. Is the width of all secondary maxima in diffraction at slit same? If not how does it vary?
22. What is resolving power of microscope?
23. What about the consistency of the principle of conservation of energy in interference and in diffraction? **OR** Does the law of conservation of energy holds good in interference and in diffraction?
24. How can the resolving power of a telescope be increased?
25. Which phenomenon confirms the transverse nature of light?
26. What is meant by plane polarised light?
27. What is pass axis?

28. By what percentage the intensity of light decreases when an ordinary unpolarised (like from sodium lamp) light is passed through a polaroid sheet?
29. Let the intensity of unpolarised light incident on P_1 be I. What is the intensity of light crossing polaroid P_2 , when the pass-axis of P_2 makes an angle 90^o with the pass-axis of P_1?
30. What should be the angle between the pass axes of two polaroids so that the intensity of transmitted light form the second polaroid will be maximum?
31. State Brewster's Law.
32. Write the relation between refractive index of a reflector and polarising angle.
33. Define Brewster's angle (OR Polarising angle).

Two Marks Questions

1. State Huygen's principle.
2. Name the wavefront obtained when a plane wave passed through (i) a thin convex lens (ii) thin prism.
3. What is the shape of the wavefront in each of the following cases:
   (a) Light emerging out of a convex lens when a point source is placed at its focus.
   (b)The portion of the wavefront of light from a distant star intercepted by the Earth.
4. What are coherent sources? Give an example.
5. Can two sodium vapour lamps be considered as coherent sources? Why?
6. Write the expression for fringe width in Young's double slit experiment.
7. What are the factors which affect the fringe width in Young's double slit experiment?
8. Let the fringe width in Young's double slit experiment be 0. What is the fringe width if the distance between the slits and the screen is doubled and slit separation is halved?
9. What is diffraction of light? Give an example.
10. Mention the conditions for diffraction minima and maxima.
11. Give the graphical representation to show the variation of intensity of light in single slit diffraction.
12. Mention the expression for limit of resolution of microscope.
13. Write the expression for limit of resolution of telescope.
14. Give the two methods of increasing the resolving power of microscope.
15. Write the mathematical expression for Malus law. Explain the terms.
16. Represent polarised light and unpolarised light.

Three Marks Questions

1. Using Huygen's wave theory of light, show that the angle of incidence is equal to angle of reflection in case of reflection of a plane wave by a plane surface.
2. Illustrate with the help of suitable diagram, action of the following when a plane wavefront incidents.
   (i) a prism (ii) a convex lens and (iii) a concave mirror. (each three marks)

3. Briefly describe Young's experiment with the help of a schematic diagram.
4. Distinguish between interference of light and diffraction of light.
5. Briefly explain Polarisation by reflection with the help of a diagram.
6. Show that the refractive index of a reflector is equal to tangent of the polarising angle. OR Arrive at Brewster's law.
7. What are Polaroids? Mention any two uses of polaroids.

Five Marks Questions

1. Using Huygen's wave theory of light, derive Snell's law of refraction.
2. Obtain the expressions for resultant displacement and amplitude when two waves having same amplitude and a phase difference ϕ superpose. Hence give the conditions for constructive and destructive interference. **OR** Give the theory of interference. Hence arrive at the conditions for constructive and destructive interferences.
3. Derive an expression for the width of interference fringes in a double slit experiment.
4. Explain the phenomenon of diffraction of light due to a single slit and mention of the conditions for diffraction minima and maxima.

Five Marks Numerical Problems

1. Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.
2. In a Young's double slit experiment, the angular width of a fringe formed on distant screen is 0.1^0. The wavelength of light used is 6000 Å. What is the spacing between the slits?
3. In a double slit experiment angular width of a fringe is found to be 0.2^0 on a screen placed 80 cm away. The wave length of light used is 600 nm. Find the fringe width.
   What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.
4. A beam of light consisting of two wavelengths 650 nm and 520 nm, is used to obtain interference fringes in Young's double slit experiment with D = 60 cm and d = 1 mm.
   a) Find the distance of third bright fringe on the screen from central maximum for wavelength 650nm.
   b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?
5. In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is (i) $\lambda/3$ (ii) $\lambda/2$?
6. In a Young’s double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.
   Also find the distance of fifth dark fringe from the central bright fringe.

7. In Young's double slit experiment with monochromatic light and slit separation of 1mm, the fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5cm towards the slits, the change in fringe width is 30 μm. Calculate the wavelength of the light used.
8. What is the shape of the wavefront in each of the following cases: (a) Light diverging from a point source. (b) Light emerging out of a convex lens when a point source is placed at its focus. (c) The portion of the wavefront of light from a distant star intercepted by the Earth.
9. (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0×108 m s^{-1}) (b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?
10. What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)
11. Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?
12. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.25 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen.
    Find (i) the width of the slit and (ii) angular position of the first secondary maximum.
13. Assume that light of wavelength 5000 Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 5.08m ?
14. Unpolarised light is incident on a plane glass surface. What should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other? (For glass refractive index = 1.5).

    OR

    What is the Brewster angle for air to glass transition? (For glass refractive index = 1.5).
15. A beam of unpolarised is incident on an arrangement of two polaroids successively. If the angle between the pass axes of the two polaroids is 60°, then what percentage of light intensity emerges out of the second polaroid sheet?

~*~*~*~*~*~

Chapter 11:

DUAL NATURE OF RADIATION AND MATTER

Work function:

"The minimum energy required by a free electron to just liberate from the metal surface is called Work function."

It is denoted by ϕ_0.

SI unit of work function is joule (J).

Practical unit is eV (electron volt).

Define eV: It is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt.

$$1eV = 1.602\times10^{-19}\ J$$

Note:

1. Work function depends on nature of metal surface and properties of metal.
2. Platinum has highest work function ($\phi_0 = 5.65\ eV$)

   Caesium has lowest work function ($\phi_0 = 2.14 eV$)

Electron emission:

The liberation of free electrons from a metal surface is called electron emission.

Types of electron emission:

According to the electron theory of metals, a large number of loosely bound electrons (free electrons) exist in a metal. Thus electrons can be liberated from a metal surface by any one of the following methods.

1. **Thermionic emission**
2. **Photoelectric emission**
3. **Field emission**
4. **Secondary emission**

1. The process of emission of free electrons from the metal surface by suitably heating a metal is called ***Thermionic emission.***
2. The process of emission of free electrons from the metal surface when a light of suitable frequency incident on it is called ***Photoelectric emission.***

   The electrons emitted from this method are called photoelectrons.
3. The process of emission of free electrons from the metal surface by applying a strong electric field to a metal is called ***Field emission*** or ***cold cathode emission.*** (Electric field is of the order of 10^8 V/m.)
4. The process of emission of electrons from the surface of a metal, using a beam of accelerated charged particles (like electrons) is called ***Secondary emission.***

PHOTO ELECTRIC EFFECT:

"*The phenomenon of emission of free electrons from metal surface when a light of suitable frequency incident on it is called Photoelectric effect.*"

Photoelectric effect was discovered by Heinrich Hertz.

The current constituted by photoelectrons is called *photoelectric current* or photocurrent.

In photoelectric effect light energy is converted into electrical energy. i.e. Photoelectric effect follows law of conservation of energy.

Hertz's observations on photoelectric effect:

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857-1894). In his experimental investigation on the production of electromagnetic waves by means of a spark discharge, Hertz observed that high voltage sparks across the detector loop were enhanced (increased) when the emitter plate was illuminated by ultraviolet light from an arc lamp.

When light falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface. After gaining sufficient energy from the incident light, the electrons escape from the surface of the metal into the surrounding space.

Lenard's observations on Photoelectric effect

Lenard observed that

(i) When ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit.

(ii) As soon as the ultraviolet radiations were stopped, the current flow also stopped.

These observations indicate that when ultraviolet radiations fall on the emitter plate (C), electrons are ejected from it which are attracted towards the positive collector plate (A).

Thus, light falling on the surface of the emitter causes current in the external circuit.

Hallwachs observations on photoelectric effect

Hallwachs, in 1888, undertook the study further and connected a negatively charged zinc plate to an electroscope. He observed that,

(i) The zinc plate lost its charge when it was illuminated by ultraviolet light.

(ii) The uncharged zinc plate became positively charged when it was irradiated by ultraviolet light.

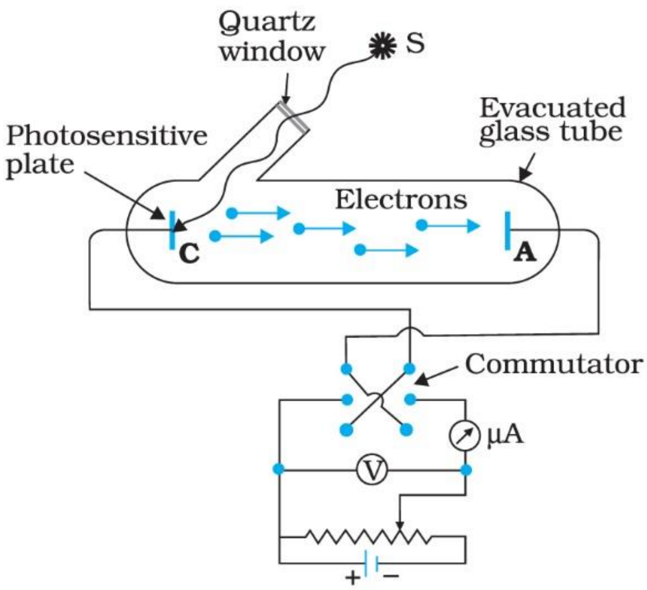
(iii) Positive charge on a positively charged zinc plate was found to be further enhanced when it was illuminated by ultraviolet light.

From these observations he concluded that negatively charged particles (electrons) were emitted from the zinc plate under the action of ultraviolet light.

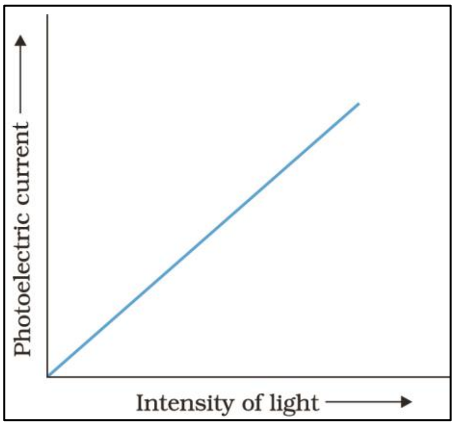
Note: It was found that certain metals like zinc, cadmium, magnesium, etc., responded only to ultraviolet light, having short wavelength, to cause electron emission from the surface. However, some alkali metals such as lithium, sodium, potassium, caesium and rubidium were sensitive even to visible light. All these *photosensitive substances* emit electrons when they are illuminated by light. After the discovery of electrons, these electrons were termed as *photoelectrons.* The phenomenon is called *photoelectric effect.*

Experimental study of photoelectric effect

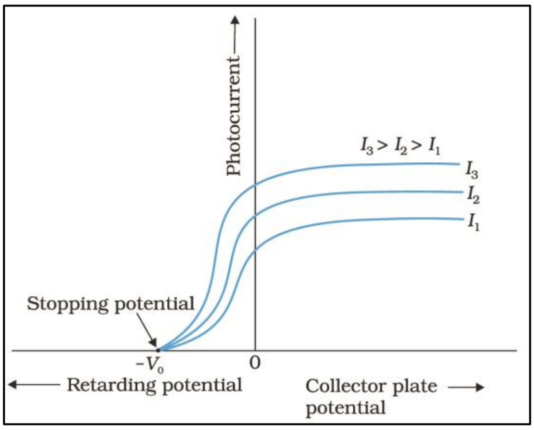
Figure shows an experimental arrangement to study photoelectric effect. C is the cathode (emitter plate) coated with a photosensitive material. A is the anode or collector plate placed in front of C. Both these electrodes are enclosed in a evacuated glass tube G provided with Quartz window. The battery maintains potential difference between plates A and C that can be varied. The polarity of the A and C can be reversed by a Commutator. When the anode plate A is positive with respect to the cathode plate C then electrons are attracted to it. Through the quartz window radiation of suitable frequency is made to fall on the cathode. As soon as the radiation falls on the cathode photoelectrons are liberated. These photoelectrons move towards the anode and produce photoelectric current. This current is recorded by a microammeter.



Experimental observations made on photoelectric effect:

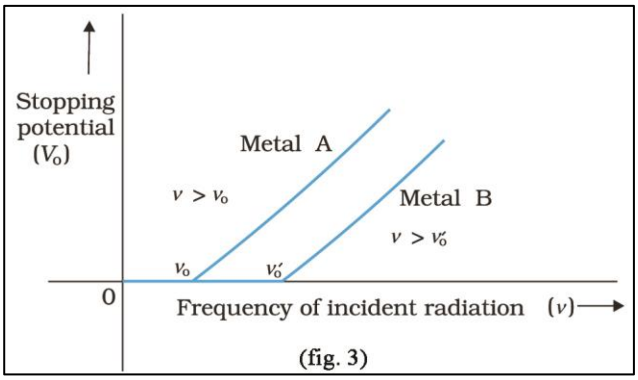


(fig. 1)



(fig. 2)

1. For a given photosensitive material and frequency of incident radiation (above the threshold frequency), the photoelectric current is directly proportional to the intensity of incident light (fig. 1).
2. For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity. (fig.2)
3. For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the *threshold frequency*, below which no emission of photoelectrons takes place, no matter how intense the incident light is.
4. Above the threshold frequency, the stopping potential or the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity. (fig.3)



(fig. 3)

5. The photoelectric emission is an instantaneous process without any apparent time lag (10^{-9} s or less), even when the incident radiation is made exceedingly dim.

Note:

1. The positive potential given to A with respect to C is called accelerating potential. The negative potential given to A with respect to C is called retarding potential.

2. As accelerating potential increases, photoelectric current also increases and reaches maximum value. If we increase accelerating potential for that, the photoelectric current does not increases. This maximum value of photoelectric current is called saturation current.

Laws of photo electric effect

1. Photo electron emission is an instantaneous process.
2. For every metal, there is a certain minimum frequency of the incident radiation called threshold frequency below which photoelectric emission does not take place.
3. Photoelectric current is directly proportional to the intensity of the incident radiation provided the frequency is greater than threshold frequency and is independent of frequency.
4. At constant frequency of incident radiation and for a given photosensitive metal, saturation current is directly proportional to intensity of incident radiation.
5. The maximum kinetic energy and the stopping potential of the photoelectric increases linearly with the frequency of the incident radiation and is independent of intensity.

Threshold frequency:

The minimum frequency of incident radiation at which or above which photoelectric emission takes place is called *Threshold frequency*.

It is denoted by ν_0.

Threshold wavelength:

The maximum wavelength of incident radiation at which or below which photoelectric emission takes place is called *Threshold wavelength*.

It is denoted by λ_0.

Relation between Threshold frequency (ν_0) and Threshold wavelength (λ_0):

$$C = \nu_0 \lambda_0 \qquad \text{Or} \qquad \nu_0 = \frac{C}{\lambda_0}$$

Note: ν_0 and λ_0 depends on nature of metal surface and properties of metal.

Expression for work function:

$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$, Where C=speed of light in free space.

Stopping potential (Cut off potential):

The minimum negative potential given to the anode for which the photo electric current becomes zero is called the stopping potential.

If V_s is the stopping potential for photoelectrons emitted with maximum velocity V_{max} or maximum kinetic energy E_{max} then $E_m = \frac{1}{2} m v_{max}^2 = eV_s$ Where, m is the mass of the electron, e is its charge.

Note:

1. If anode is made negative with respect to cathode, then photoelectric current decreases. If negative potential of A increases then photoelectric current decreases.
2. The stopping potential is directly proportional to frequency of incident radiation. It depends on nature of emitter material. It does not depend on intensity of incident radiation.
3. The photoelectric current is directly proportional to the number of photoelectrons emitted per second.

Failure of wave theory of light to explain Photoelectric effect:

The wave nature of light was well established by the end of the nineteenth century. The phenomena of interference, diffraction and polarisation were explained in a natural and satisfactory way by the wave picture of light. According to this picture, light is an electromagnetic wave consisting of electric and magnetic fields with continuous distribution of energy over the region of space over which the wave is extended. Let us now see if this wave picture of light can explain the observations on photoelectric emission given in the previous section.

We should note that in the wave picture, the absorption of energy by electron takes place continuously over the entire wavefront of the radiation. Since a large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small. Explicit calculations estimate that it can take hours or more for a single electron to pick up sufficient energy to overcome the work function and come out of the metal. This conclusion is again in striking contrast to observation (iv) that the photoelectric emission is instantaneous. In short, the wave picture is unable to explain the most basic features of photoelectric emission.

Einstein's explanation of photoelectric effect:

In 1905, Einstein explained the photoelectric effect using quantum theory. According to which the radiation consists of discrete energy packets or bundles called quanta or photons. Energy of each such photon is given by **E=h** $\boldsymbol{\nu}$**,** where h is planck's constant $\boldsymbol{\nu}$ is the frequency of the radiation. Einstein considered that the emission of photoelectron is the result of an elastic collision between a photon of incident radiation and free electron inside the metal. In this process the entire photon energy is transferred to the electron. A part of the energy gained by the electron is used in doing work against the surface forces of the metal (surface barrier). This part of the energy represents the work function of the metal.

The remaining energy is available to the electron as its maximum kinetic energy, Thus according to Einstein's explanation.

$$\text{Photon Energy} = \begin{matrix}\text{Work function of}\\ \text{the photo cathode}\end{matrix} + \begin{matrix}\text{maximum kinetic energy}\\ \text{of the photo electrons}\end{matrix}$$

i.e. $$\mathbf{h\nu = \phi_0 + \frac{1}{2} mV_{max}^2}$$

Where, h is Planck's constant, ν is the frequency of the incident radiation, m is the mass and V_{max} is the maximum velocity of the electron.

This relation is called ***Einstein's photoelectric equation.***

Note: We have, $\mathbf{h\nu = \phi_0 + \frac{1}{2} mV_{max}^2}$, but $\phi_0 = h\nu_0$, $\therefore$ $h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2$

$$\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0 , \qquad \frac{1}{2}mv_{max}^2 = h\left(\nu - \nu_0\right)$$

Explanation of experimental observations based on Einstein's photoelectric equation:

1. If $\nu<\nu_0$, then kinetic energy will be negative which is not possible because kinetic energy cannot be negative. This shows that photoelectric emission is not possible if frequency of incident light is less than the threshold frequency of the metal.
2. One photon can emit only one electron from the metal surface, so the number of photo-electrons emitted per second is directly proportional to the intensity of incident light which depends upon number of photons present in the incident light.
3. If $\nu>\nu_0$, maximum kinetic energy increases with increase of frequency of incident radiation.
4. Since the interaction between photon and electron is treated as electric collision between two macro particles, i.e. photo electric emission is instantaneous.

Determination of Plank's constant and work function of a metal:

According to Einstein's photoelectric equation.

We have, $\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$, but, $\frac{1}{2}mv_{max}^2 = eV_0$

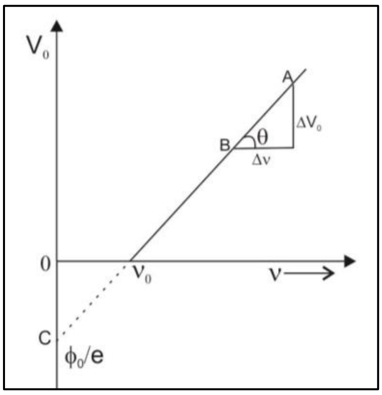
Where, e is the charge of an electron and V_0 is stopping potential.

$$\therefore \quad eV_0 = h\nu - h\nu_0$$

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{h\nu_0}{e}$$

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e} \text{ --------- (1)} \qquad [\because h\nu_0 = \phi_0]$$

Equation (1) can be compared with the equation of a straight line y=mx+c, where m is the slope of the line and c is the intercept on y-axis. Thus, graph between V_0 and ν is a straight line having slope= $\left(\frac{h}{e}\right)$ and intercept $OC = \left(\frac{\phi_0}{2}\right)$



From figure,

$$\frac{h}{e} = \tan\theta = \frac{\Delta V_0}{\Delta \nu}$$

h=e× tanθ ---------(2)

h=e × slope of V_0 versus ν graph

Using equation (2) value of h can be determined.

Intercept, $OC = \left(\frac{\phi}{2}\right)$ i.e, $[\phi_0 = OC \times e]$.

This is the expression for workfunction.

Particle nature of light:

Light exhibits the phenomenon like reflection, refraction, interference, diffraction and polarization. These phenomena can be explained successfully by treating light as a wave.

Light also exhibits the phenomena like photoelectric effect and Raman effect. These phenomena can be explained by treating the light as particles in the form of Quanta or photons.

Thus, **radiations have dual nature i.e., wave and particle nature.**

What are photons?

The discrete energy packets of the radiation are called photons.

Characteristics of Photons:

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

(ii) Each photon has energy $E = h\nu = hc/\lambda$ and momentum $p = h\nu/c = h/\lambda$, and speed c, the speed of light.

(iii) All photons of light of a particular frequency ν, or wavelength λ, have the same energy (E) and momentum (p), whatever the intensity of radiation may be. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

(iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.

(v) In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

Wave nature of matter: De – Broglie hypothesis

A French physicist ***Louis de-Broglie*** suggested that the particles like electron, protons, neutrons etc have also dual nature i.e., the material particle can behave both wave as well as particle.

According to de-Broglie, a moving particles can be associated with a wave and is known as **de-Broglie waves or matter waves .**

Expression for de-Broglie Wavelength

According to Quantum theory, the energy of a photon is given by

$$\mathbf{E = h\nu} \quad \text{........(1)}$$

where h is a Planck's constant , ν is the frequency of a photon.

According to Einstein's mass energy equivalence, the energy of a photon is

$$\mathbf{E = mc^2} \quad \text{......(2)}$$

Where m is the mass of a photon, c is a velocity of light

From (1) and (2) we have $mc^2 = h\nu$ or $mc^2 = \frac{hc}{\lambda}$

$$mc = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mc} \qquad \text{since, } mc = p = \text{momentum of photon}$$

$$\therefore \boxed{\lambda = \frac{h}{p}}$$

This is the expression for de-Broglie wavelength.

If instead of photon, a material particle of mass m moving with velocity v is considered then

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{is de-Broglie wavelength.}$$

The kinetic energy of the particle is $E = \frac{1}{2}mv^2$ hence $mv = \sqrt{2mE}$

$$\therefore \quad \boldsymbol{\lambda = \frac{h}{\sqrt{2mE}}}$$

For a charged particle accelerated by a potential difference V the kinetic energy gained is $E = \frac{1}{2}mv^2 = Vq$ hence $\boldsymbol{\lambda = \frac{h}{\sqrt{2mVq}}}$

Note:

1. de-Broglie wavelength 'λ' is smaller for a heavier particle.
2. $h = \frac{1.227}{\sqrt{V}}$ nm

Heisenberg's uncertainty principle:

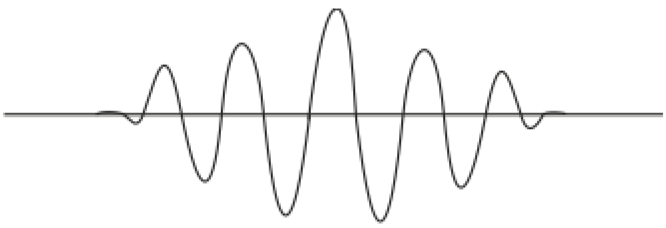
According to this principle. It is not possible to measure both the position and momentum of a particles at the same time exactly.

The product of the uncertainty in the measurement of the position and uncertainty in the measurement of the momentum as equal to ħ.

i.e. $\Delta x\ \Delta p \approx \hbar$. $\left(\because\ \hbar = \frac{h}{2\pi}\right)$

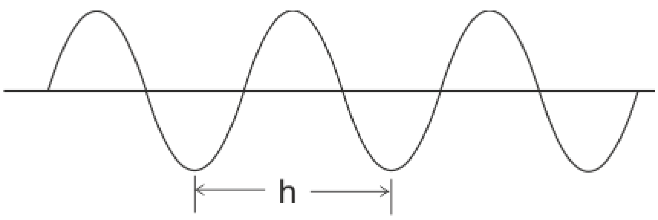
Probability interpretation of matter waves:

Max Born, proposed the probability interpretation according to him the electron is not localized in any finite region of space. That is, its position uncertainty is infinite ($\Delta x \to \infty$), which is consistent with uncertainty principle.

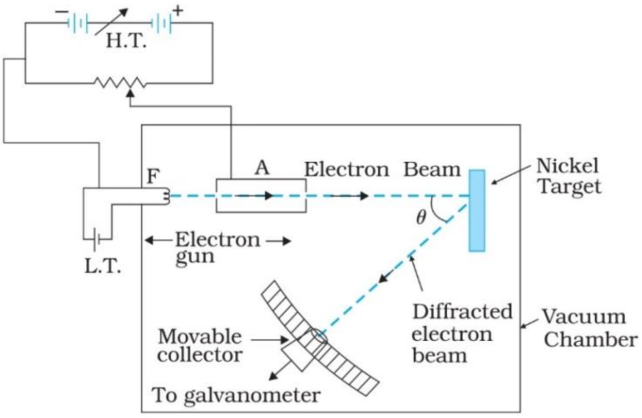


In general, the matter wave associated with the electron is not extended all over space. It is a wave packet extending over some finite region of space in that case Δx is not infinite but has some finite value depending on the extension of the wave packet. Also wave packet of finite wavelength spread around central wavelength as shown below.

Note: The matter wave corresponding to ta definite momentum of an electron extends all over space as shown Fig.



Davisson and Germer experiment:



The wave nature of electrons was first experimentally verified by C.J. Davisson and L.H. Germer in 1927 and independently by G.P. Thomson, in 1928, who observed diffraction effects with beams of electrons scattered by crystals. Davisson and Thomson shared the Nobel Prize in 1937 for their experimental discovery of diffraction of electrons by crystals.

The experimental arrangement used by Davisson and Germer is schematically shown in Fig. It consists of an electron gun which comprises of a tungsten filament F, coated with barium oxide and heated by a low voltage power supply (L.T. or battery). Electrons emitted by the filament are accelerated to a desired velocity by applying suitable potential/voltage from a high voltage power supply (H.T. or battery). They are made to pass through a cylinder with fine holes along its axis, producing a fine collimated beam. The beam is made to fall on the surface of a nickel crystal. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam, scattered in a given direction, is measured by the electron detector (collector). The detector can be moved on a circular scale and is connected to a sensitive galvanometer, which records the current. The deflection of the galvanometer is proportional to the intensity of the electron beam entering the collector. The apparatus is enclosed in an evacuated chamber. By moving the detector on the circular scale at different positions, the intensity of the scattered electron beam is measured for different values of angle of scattering θ which is the angle between the incident and the scattered electron beams. The variation of the intensity(I) of the scattered electrons with the angle of scattering θ is obtained for different accelerating voltages from 44V to 68V.

The de Broglie wavelength λ associated with electrons, , for V=54V is given by

$$\lambda = h/p = \frac{1.227}{\sqrt{V}}\text{nm} \qquad \lambda = \frac{1.227}{\sqrt{54}}\text{nm} = 0.167\text{nm}$$

Thus, there is an excellent agreement between the theoretical value and the experimentally obtained value of de Broglie wavelength. Davisson-Germer experiment thus strikingly confirms the wave nature of electrons and the de Broglie relation.

One mark questions:

1. What is electron emission?
2. What is thermionic emission?
3. What is field emission?
4. What is photoelectric emission?
5. What are photoelectrons?
6. What is photoelectric current?
7. Define threshold frequency.
8. Define threshold wavelength.
9. Define electron volt.
10. What is stopping potential?
11. Who proposed the matter waves?
12. What are mater waves? (2016-S)
13. What is de-Broglie wavelength?
14. What is the conclusion of Davisson and Germer experiment? (2015-A)
15. Write the expression for de-Broglie wavelength of a particle. (2017-A)

Two mark questions:

1. What are photosensitive substances? Give one example
2. What is photoelectric workfunction? Mention expression for it.
3. Explain variation of photoelectric current with intensity of incident radiation.
4. Explain how stopping potential varies with intensity of incident radiation.
5. Mention Einstein's photoelectric equation and explain the symbols.
6. Mention the expression for de-Broglie wavelength of a particle and explain the symbols. 2016-S.
7. Mention the expression for de-Broglie wavelength interms of kinetic energy of the particle and explain the symbols.
8. Mention the expression for de-Broglie wavelength in terms of acceleration potential and explain the symbols.
9. Define: a) Photoelectric work function, b) Electron volt (eV). (2016-S)
10. Write any two types of electron emission. (2014-A)

Three marks questions:

1. What is electron emission? Mention different types of electron emission?
2. Mention the Hertz observations on photoelectric effect.
3. Mention the Lenard's observations on photoelectric effect. (2015-S)
4. Mention the Halwachs observations on photoelectric effect. (2015-S)
5. Describe an experiment to study the photoelectric effect.

Five mark questions:

1. Mention the experimental observational of photoelectric effect or (state the laws of photoelectric effect). (2017-A)
2. Mention Einstein's photoelectric equation and explain the symbols. Explain the experimental results using this relation. (2015-A)
3. Mention any five characteristics of photon. (2014-A)

Problems:

1. Calculate the frequency associated with a photon of energy 3.3×10^{-20}J. Plank's constant h=6.6×10^{-34} Js. What is the wavelength of photon in air?
2. Ultraviolet light of wavelength 350 nm and intensity 1 W/m^2 is directed at a potassium surface. (a) Find the maximum kinetic energy of the photoelectrons. (b) If 0.5 percent of the incident photons produce photoelectrons, how many photoelectrons per second are emitted if the potassium surface has an area of 1cm^2? Work function=2.2 eV. h=6.63×10^{-34}Js.
3. Estimate the number of photons emitted by a 1000 W lamp emitting light of average wavelength 6000 Å. H=6.6×10^{-34} Js, c= 3×10^8m/s.
4. A metal whose work function is 3.0eV is illuminated b light of wavelength 3×10^{-7} m. calculate: (a) Threshold frequency (b)Maximum energy of the photoelectrons (c) retarding potential, assume h and c.
5. Ultraviolet radiation of wavelength 800Å and 700 Å when allowed to fall on a photosensitive metal surface are found to liberate electrons with maximum kinetic energy of 1.8 eV and 4 eV respectively. Calculate the value of Planck's constant.
6. Solar radiation falls on the earth at a rate of 2.0 cal/ cm^2 – min. how many photons / cm^2 –min is this, assuming an average wavelength of 5500Å.
7. The energy required to remove an electron from sodium is 2.3 eV. Does sodium show a photoelectric effect for orange light, with λ =680 nm?
8. Photoelectric work function of a metal is 2.5 eV. Calculate the threshold frequency and limiting wavelength for the metal.
9. A filament emits light of wavelength 600nm. Calculate the frequency of the light and also the energy of the emitted photon h= 6625×10^{-34}Js.
10. A metal whose work function is 2.5 eV is illuminated by light of wavelength 2500 Å. Calculate the threshold frequency and the maximum kinetic energy of the photoelectrons.
11. Radiations of wavelength 500 nm and 331 nm when incident on a photosensitive metal surface are found to liberate electrons with maximum kinetic energy of 0.061 eV and 1.87 eV respectively. calculate the value of Planks constant.

12. Calculate the kinetic energy of photoelectron emitted from a metal whose work function is 2.2 eV when irradiated with light of wavelength 450 nm and a retarding potential of 0.2 V is applied.
13. The speed of a photoelectron is $10^4 ms^{-1}$. What should be the frequency of the incident radiation on a potassium metal whose work function is 2.3eV?
14. Light of wavelength 350 nm is incident on two metals A and B whose work functions are 4.2 eV and 1.9 eV respectively. Which metal will emit photoelectrons?
15. Monochromatic light of wavelength 450 nm is incident on a clean sodium surface of work function 2.3 eV. Determine a) the energy of a photon of this light b) the maximum K.E. of emitted electron c) the threshold frequency for sodium, and d) the magnitude of the momentum of photon in the incident light.
16. A 100 watt sodium vapour lamp radiates uniformly in all directions. a) At what distance from the lamp will the average density of photons be $10^7 m^{-3}$ b) What is the average density of photons 2.0 m from the lamp? Assume the light to be monochromatic, with $\lambda = 589$ nm.
17. Calculate the number of photons emitted per second by a 600nm falling on a photosensitive material of work function 2eV. Assuming the efficiency of one percent for electron emission, calculate the current produced.
18. Calculate the wavelength of a photon of energy 10^{-19}J.
19. The work function of caesium metal is 2.14eV, when light of frequency 6×10^{14}Hz is incident on the metal surface, photoemission of electrons occurs find:

    a) Energy of incident photons

    b) Maximum kinetic energy of photoelectrons

    c) Give Plank's constant $=h=6.63\times10^{-34}$JS, $1eV=1.6\times10^{-19}$J. (2014-S)

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Chapter 12:

ATOMS

The matter is made up of atoms. To explain the structure of atom, various atomic models were proposed. The important atomic models are

- Thomson's atom model (palm pudding model)
- Rutherford atom model (Planetary model)
- Bohr's atom model
- Bohr – Summerfield atom model
- Vector atom model
- Wave mechanical atom model

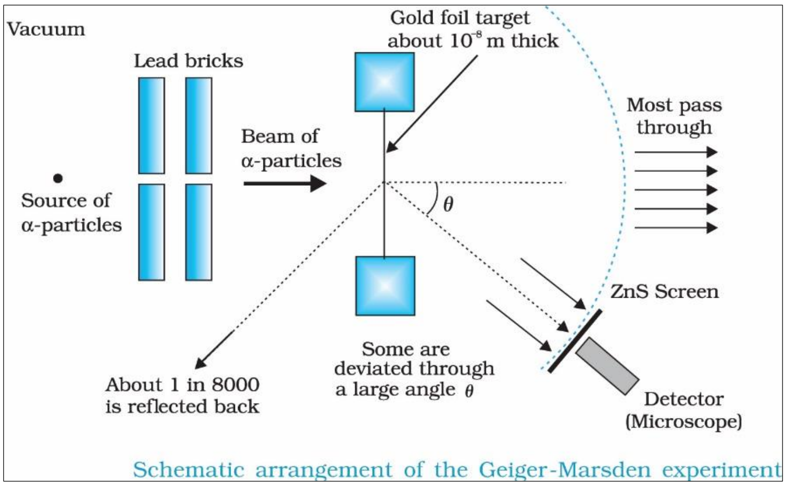
Thomson's atom model (Plum pudding model):

This is the first model proposed by J.J. Thomson According to Thomson's atom model, the positive charge of an atom is uniformly distributed throughout the volume of an atom. The negatively charged particulars called electrons are embedded in it like seeds in watermelon, therefore this model is also called plum pudding model.

Drawback: This model fails to explain alpha particle scattering experiment

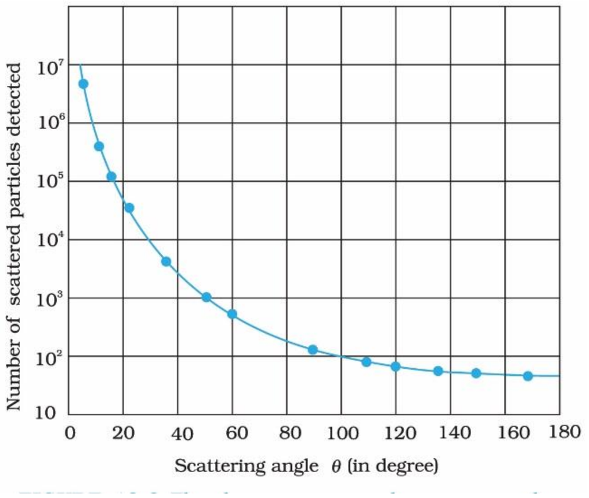
Alpha particle scattering experiment (Geiger-Marsden experiment):

Geiger and Marsden did the experiment under the suggestion of Rutherford. The α–particles emitted by Bismuth–214 (Radioactive sample) are collimated into a narrow beam by passing through a passage of lead bricks. A narrow beam of α-particles are made to incident on a thin gold foil of thickness 2.1×10^{-7}m. The scattered α-particles are detected by rotatable detector.



Schematic arrangement of the Geiger-Marsden experiment

Observation obtained by α-particle scattering Experiment:



- Maximum number of incident α - particles passed through gold foil without deflection (without scattering, because there is no collision ∴ θ = 0°)
- Only about 0.14% of incident α – particles scattered by an angle more than 1°.
- About 1 in 8000 incident α– particles deflected (Scattered) by an angle more than 90°.
- Only a small fraction of number of incident α – particles rebound back (i.e., θ = 180°) i.e. a few number of incident α – particles undergoing headon collision with nucleus.

Conclusions from α – particle scattering experiment:

1. Most of α-particle passed through foil un-deflected. Most of space in atom is empty.
2. A few positively charged α-particles were deflected.
3. Positive charge is concentrated in a very small volume that repelled and deflected the positively charged α-particles.
4. Calculations by Rutherford showed that the volume occupied by the nucleus is negligibly small as compared to the total volume of the atom. The radius of the atom is about 10^{-10}m while that of nucleus is 10^{-15}m.

Rutherford's atomic model:

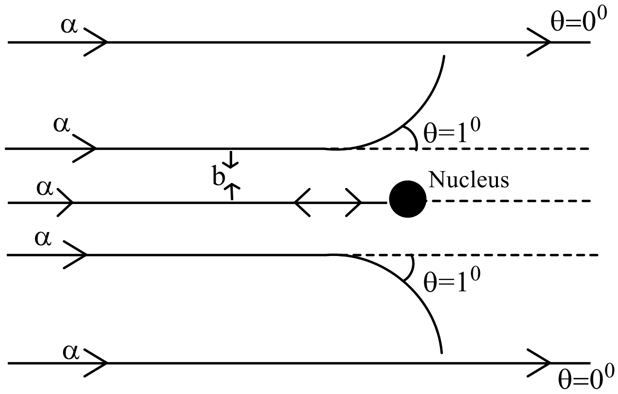
1. Almost all the mass of an atom and entire positive charge of an atom is concentrated at the centre. The central part is called nucleus. The negatively Charge electrons are revolving around the nucleus just like planets around the sun.
2. The size of a nucleus to be about 10^{-15}m to 10^{-14}m.
3. The size of an atom was known to be 10^{-10}m, about 10,000 to 1,00,000 times larger than the size of the nucleus, thus most of an atom is empty space.
4. Electrons and the nucleus are held together by electrostatic forces of attraction between nucleus and electrons.

Note:

Electrostatic force of repulsion between the alpha-particle and the positively charged nucleus is given by $F = \frac{1}{4\pi\varepsilon_0} \times \frac{(2e)(ze)}{r^2}$

Where, ‘r’ is the distance between the α-particle and the nucleus.

α - particle trajectory:



The trajectory traced by and α-particle depends on the impact parameter of collision.

The perpendicular distance between the initial velocity vector of the α-particles and centre of the nucleus is called impact parameters.

Note:

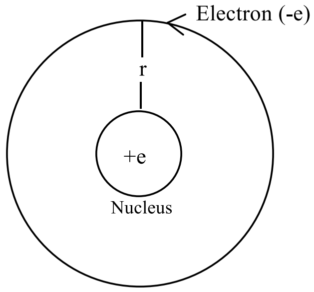
- It b is nearly equal to zero, then scattering angle θ = 180° i.e., incident α-particle rebound back.
- It b is very large, then θ□ 0^0 i.e., the incident α-particles pass through atom without scattering it is found that b α $\frac{1}{\theta}$ where b=impact parameter and θ =scattering angle.

Distance of closest approach:

It is the centre-to-centre distance between the α-particle and the gold nucleus when the α-particle is at its stopping point,

Distance of closet approach is, $d = \frac{2ze^2}{4\pi\varepsilon_0 k}$.

Derive an expression for total energy of an electron in hydrogen atom using Rutherford atomic model:



Consider hydrogen atom it consists of one proton and one electron revolving around the nucleus. According to Rutherford's atom model, for a revolution of electron

Centripetal force = Electrostatic force

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r^2}$$

$$mv^2 = \frac{e^2}{4\pi\varepsilon_0 r} \longrightarrow (1) \qquad \left[F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}\right]$$

Where, m = mass of electron, v = velocity of electron, r = radius of orbit, e = elementary charge

Kinetic energy of electron is

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2} \times \frac{e^2}{4\pi\varepsilon_0 r} \text{ [from equation (1)]}$$

Potential energy of electron

$$U = -\frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r} = \frac{-e^2}{4\pi\varepsilon_0 r}$$

$$\left[U = -\frac{1}{4\pi\varepsilon_0} \times \frac{q_1q_2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{(e)(-e)}{r}\right]$$

Total energy is given by

$$E = K + U$$

$$E = \frac{1}{2} \times \frac{e^2}{4\pi\varepsilon_0 r} - \frac{e^e}{4\pi\varepsilon_0 r}$$

$$E = \frac{e^2}{4\pi\varepsilon_0 r}\left[\frac{1}{2} - 1\right]$$

$$E = \frac{e^2}{4\pi\varepsilon_0 r}\left[-\frac{1}{2}\right]$$

$$E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

This is the expression for total energy of an electron of Hydrogen atom negative sign indicates that electron is bound to the nucleus.

Mention the limitations of Rutherford atom model:

1. It could not explain the different spectral series obtained by Hydrogen atom.
2. It could not explain why atoms emit radiation of only discreet wavelength (line emission spectra).
3. It could not explain stability of an atom.

Bohr's Postulates (Bohr's Atom Model):

1) The electron in an atom could revolve around nucleus in a certain stable orbits without the emission of radiant energy.

i.e., the stable orbits have definite fixed total energy and they are called as stationary orbits an non radiating orbits.

2) The angular momentum of an electron in a stationary orbit is the integral multiple of $\frac{h}{2\pi}$

$$\text{Angular momentum} = n \times \frac{h}{2\pi}$$

$$mV_n r_n = \frac{nh}{2\pi}$$ h = Planks constant, n=principal quantum number

n = 1, 2, 3, ...

i.e., Angular momentum of an electron is quantized therefore second postulate is called Bohr's quantum condition.

3) An electron might make a transition from higher energy orbit to lower energy orbit. During this transition the photon is emitted the energy of emitted photon is

$$h\nu = E_{n_2} - E_{n_1}$$

Where, ν=frequency of radiation, E_{n_1} = Energy of lower orbit,

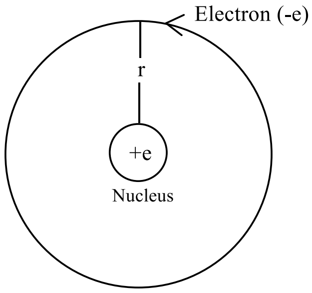
E_{n_1} =Energy of higher orbit

Limitations of Bohr's theory:

1. This theory is applicable only to hydrogen atom and hydrogen like atoms (hydrogenic atoms).
2. It could not explain relative intensity of spectral series.
3. It could not explain the elliptical orbits of the electron.

Derive an expression for Bohr radius or Derive expression for radius of n^{th} orbit of hydrogen atom:

Consider hydrogen atom. It consists of one proton in the nucleus and one electron is revolving around the nucleus for revolution of electron.



Centripetal force = electrostatic force

$$\frac{mv_n^2}{r_n^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2}$$

$$mv_n^2 = \frac{e^2}{4\pi\varepsilon_0 r_n} \quad \text{———— (1)}$$

Where, m = mass of an electron, v_n = velocity of an electron in n^{th} orbit r_n = radius of n^{th} orbit.

According to Bohr's atomic 2^{nd} postulate

$$mv_n r_n = \frac{nh}{2\pi} \quad n = 1, 2, 3, 4 \text{ .... (orbit number)}$$

$$v_n = \frac{nh}{2\pi mr_n} \quad \text{———— (2) n= principle quantum number}$$

Substituting equation (2) in (1)

$$m\left(\frac{nh}{2\pi mr_n}\right)^2 = \frac{e^2}{4\pi\varepsilon_0 r_n}$$

$$\frac{mn^2h^2}{4\pi^2m^2r_n^2} = \frac{e^2}{4\pi\varepsilon_0 r_n}$$

$$\frac{n^2h^2}{\pi mr_n} = \frac{e^2}{\varepsilon_0}$$

$$\frac{n^2h^2\varepsilon_0}{\pi me^2} = r_n$$

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

This is the expression for radius of n^{th} orbit

If n = 1 then $r_n = r_1$

$\therefore$ above equation become $r_1 = \frac{1^2h^2\varepsilon_0}{\pi me^2}$ ($r_1 = a_0$ = Bohr radius)

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2}$$

This is expression for the Bohr radius.

To find the value of Bohr's radius

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2}$$

$$a_0 =$$

$$a_0 =$$

$$a_0 = 5.2992\times10^{-11}\,m$$

Where, $h = 6.625 \times 10^{-34}$ JS
$\varepsilon_0 = 8.854 \times 10^{-12}$F/m
$\pi = 3.14$
$m = 9.1 \times 10^{-31}$ Kg
$e = 1.602 \times 10^{-19}$ C

Note : 1) For H-atom, radius of n^{th} orbit is

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

$$r_n = n^2\left(\frac{h^2\varepsilon_0}{\pi me^2}\right)$$

$$r_n = n^2\left(r_1\right)$$

For second orbit n = 2

$$r_2 = 2^2 \times r_1 = 4r_1$$

If, n = 3 (3^{rd} orbit)

$$r_3 = 3^2 \times r_1 = 9r_1$$

$\therefore$ r_1: r_2:r_3:........ = 1:2^2:3^2:.........

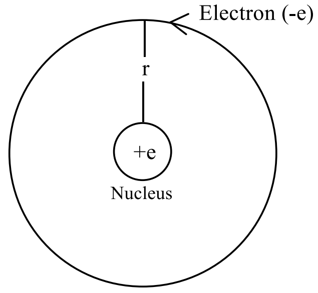
2) For hydrogen like atoms radius of n^{th} orbit is

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$$

Where, Z = atomic number of hydrogenic atoms

Derive an expression for total energy of an electron revolving in n^{th} orbit of an atom using Bohr's theory:

Consider hydrogen atom. It consists of proton in the nucleus and one electron revolving around the nucleus. For revolution of an electron



Centripetal force = Electrostatic force

$$\frac{mv_n^2}{r} = \frac{1}{4\pi\varepsilon_0}\times\frac{e^2}{r_n^2}$$

$$mv_n^2 = \frac{e^2}{4\pi\varepsilon_0 r_n}$$

Where m = mass of an electron, r_n = radius of n^{th} orbit, V = velocity of an electron in n^{th} orbit

The Kinetic energy of an electron in n^{th} orbit is K_n

$$K_n = \frac{1}{2} \times \frac{e^2}{4\pi\varepsilon_0 r_n}$$

The potential energy of an electron in n^{th} orbit is

$$U_n = \frac{1}{4\pi\varepsilon_0} \times \frac{(+e)(-e)}{r_n}$$

$$U_n = -\frac{e^2}{4\pi\varepsilon_0 r_n}$$

The total energy is in electron in n^{th} orbit is

$$E_n = K_n + U_n$$

$$= \frac{1}{2} \times \frac{e^2}{4\pi\varepsilon_0 r_n} - \frac{e^2}{4\pi\varepsilon_0 r_n}$$

$$= \frac{e^2}{4\pi\varepsilon_0 r_n}\left[\frac{1}{2} - 1\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0 r_n}\left(\frac{-1}{2}\right)$$

$$E_n = \frac{-e^2}{8\pi\varepsilon_0 r_n} \quad \text{——— (1)}$$

But $r_n = \dfrac{n^2 h^2 \varepsilon_0}{\pi m e^2}$

Equation (1) becomes

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 \times \dfrac{n^2 h^2 \varepsilon_0}{\pi m e^2}}$$

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

This is the expression for total energy of an electron in an n^{th} orbit of an atom.

Negative sign indicates that the electron is bound to the atom.

Note:

1. For hydrogenic atom (Hydrogen like atom) H_e^+, Li^{++}

$$E_n = -\frac{me^4 Z^2}{8\varepsilon_0^2 h^2 n^2}$$

Z = atomic number

2. For hydrogen atom

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2 n^2}$$

For first orbit n = 1, E_1 = energy of electron in first orbit of hydrogenic atom

$$E_1 = -\frac{me^4}{8\varepsilon_0^2 h^2 (1)^2}$$

$$E_1 = \frac{9.1\times10^{-31}\times\left(1.602\times10^{-19}\right)^4}{8\times\left(8.854\times10^{-12}\right)^2\times\left(6.625\times10^{-34}\right)^2}$$

$$E_1 = -\frac{59.936\times10^{-107}}{27534.16\times10^{-92}}$$

$$E_1 = -0.002176\times10^{-15}\,J$$

$$E_1 = -21.76\times10^{-19}\,J$$

$$E_1 = \frac{-21.76\times10^{-19}}{1.602\times10^{-19}}\,ev$$

E_1= −13.583 eV

E_1 = 13.6 eV

3. Energy of an electron in the n^{th} orbit of hydrogen atom is

$$E_n = -\frac{13.6eV}{n^2} \quad \text{Where n = 1, 2, 3 ....}$$

4. Energy of an electron in n^{th} orbit of hydrogen like atom

$$E_n = -\frac{13.6eV}{n^2}\times Z^2$$

5. To find energy of electron in different orbit of hydrogen atom

WKT $\therefore$ $E_n = -\frac{13.6eV}{n^2}$

For first orbit n = 1, $\therefore\ E_1 = \frac{-13.6eV}{1^2} = -13.6eV$

For second orbit n = 2, $\therefore\ E_2 = \frac{-13.6eV}{2^2} = -3.4eV$

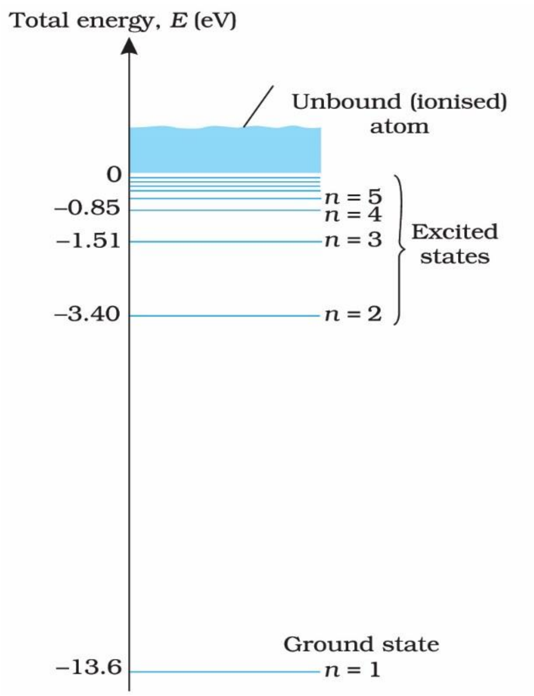
For third orbit n = 3, $\therefore\ E_3 = \frac{-13.6eV}{3^2} = -1.51eV$

For fourth orbit n = 4, $\therefore\ E_4 = \frac{-13.6eV}{4^2} = -0.8eV$

For fifth orbit n = 5, $\therefore\ E_5 = \frac{-13.6eV}{5^2} = -0.54eV$

For n = ∞, $\therefore E_\infty = \frac{-13.6eV}{\infty^2}$ $\left[\frac{1}{\infty} = 0\right]$ $\therefore$ $E_\infty = 0$

Draw the energy level diagram for Hydrogen atom:



From the above diagram it is confirmed that the total energy of an electron goes on increases as orbit number increases.

Note:

1. The lowest energy state for an electron in an atom is called ground state.
2. The higher energy state for an electron in an atom is called exited state.
3. Higher energy states are closer lower energy state are farther.
4. **One electron volt (eV):** The kinetic energy gained by an electron when one volt potential difference is applied is called one eV.
5. **Ionization:** The process of removal of electron from the atom is called ionization.

OR

The process of shifting of electron from original state to infinity level is called ionization energy the minimum amount

Ionization energy: The minimum amount of energy required to remove an electron from the ground state of the atom is called the ionization energy.

Wave number:

The reciprocal of wave length is called wave number.

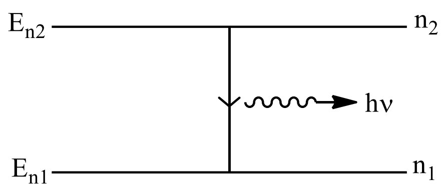
$$\text{Wavenumber} = \frac{1}{\text{wave length}}$$

$$\bar{\nu} = \frac{1}{\lambda}$$ S.I. unit of wave number is m^{-1}

Note: Wave number can also be defined as number of waves present in 1m length.

Derive an expression for frequency and wave number of radiation using Bohr's model

Consider a hydrogen atom, energy of electron in n^{th} orbit is



$$E_n = -\frac{me^4}{8\in_0^2 h^2 n^2}$$

Where m = mass of electron

e = elementary charge

$\in_0$ = permittivity of free space

h = plank's constant

n = energy level

Energy of electron in n_1 energy state is

$$E_{n_1} = -\frac{me^4}{8\varepsilon_0^2 h^2 n_1^2}$$

Energy of electron in n_2 energy state is

$$E_{n_2} = -\frac{me^4}{8\varepsilon_0^2 h^2 n_2^2}$$

According to 3rd postulate of Bohr's model when an electron jumps from n_2 orbit to n_1 orbit. The photon is emitted the energy of photon is

$$h\nu = E_{n_2} - E_{n_1}$$

Where ν= Frequency of radiation (photon)

$$h\nu = -\frac{me^4}{8\varepsilon_0^2 h^2 n_2^2} - \left[-\frac{me^4}{8\varepsilon_0^2 h^2 n_1^2}\right]$$

$$h\nu = -\frac{me^4}{8\varepsilon_0^2 h^2 n_2^2} + \frac{me^4}{8\varepsilon_0^2 h^2 n_1^2}$$

$$h\nu = \frac{me^4}{8\varepsilon_0^2 h^2}\left[-\frac{1}{n_2^2} + \frac{1}{n_1^2}\right]$$

$$h\nu = \frac{me^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\nu = \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

This is expression for frequency of radiation

but $c = \nu\lambda$

$$\nu = \frac{c}{\lambda}$$

Where, c = speed of light, ν = wave length of radiation

Above equation can be written as

$$\frac{c}{\lambda}=\frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda}=\frac{me^4}{8\varepsilon_0^2h^3c}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$ Where, $R=\frac{me^4}{8\varepsilon_0^2h^3c}$ =Rydberg constant.

This is expression for waves number.

Write the expression for Rydberg constant:

$$R=\frac{me^4}{8\varepsilon_0^2h^3c}$$

R = Rydberg constant, m = mass of electron,

e = charge on electron, ε_0 = Permittivity of free space

h = Planks constant c = speed of light

S.I. Unit of Rydberg constant is m^{-1}.

To find the value of Rydberg constant:

We have, $R=\frac{me^4}{8\varepsilon_0^2h^3c}$

$$=\frac{9.1\times10^{-31}(1.602\times10-19)^4}{8\times(8.854\times10^{-12})^2\times(6.625\times10^{-34})^3\times3\times10^8}$$

=

=

$R=1.03\times10^7\ m^{-1}$

This value is very close to the value ($1.097\times10^7 m^{-1}$) obtained from the empirical Balmer's formula.

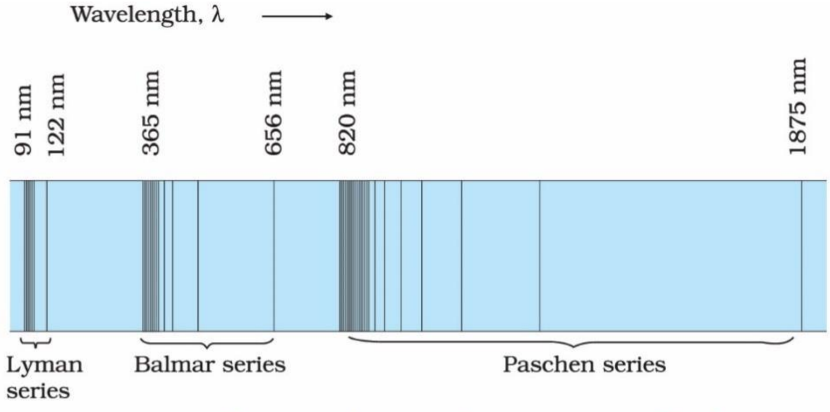
Note: for hydrogen like atoms, $\frac{1}{\lambda}=RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$,Where, Z = atomic number

Atomic spectra or emission line spectra:

When a gas or vapour is exited, then atoms of vapour emit the radiation which contain certain specific wavelengths. This spectra is called atomic spectra or emission line spectra.

The spectra which contains number of brightlines on dark background is called atomic spectra or emission line spectra.

Each elements has its own emission line spectra, i.e. the emission line spectra is different for different elements (gas), therefore, emission line spectra is a finger print for identification of element.



Emission lines in the spectrum of hydrogen.

Note: When white light is passed through vapour at low temperature then the transmitted light consists of dark lines. This spectra is called absorption line spectra.

Spectral series:

The set of spectral lines is called spectral series

Spectral series of Hydrogen atom

There are five spectral series in Hydrogen atom

- Lyman series
- Balmer series
- Paschen series
- Bracket series
- Pfund series

Note:

1. Balmar series was the first spectral series discovered by JJ Balmer.
2. The other spectral series of hydrogen atom were named after their discoverer Lyman, Paschan Bracket and Pfund series.

Lyman Series: When an electron jumps from higher energy orbits to first energy orbit then lyman series is obtained.

Formula for wave number in lyman series is

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

Where, n=2, 3, 5, …..∞, R=Rydbergs constant, λ=Wavelength

Lyman series belongs to ultraviolet region.

Balmer series: When an electron jumps from higher energy orbits to second energy orbit then Balmer series is obtained.

Formula for wave number is Balmer series is

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

Where, n=3, 4, 5, …..∞, R=Rydbergs constant, λ=Wavelength

Balmer series belong to visible region.

Paschen series: When an electron jumps from higher energy orbits to third energy orbit then Paschen series is obtained.

Formula for wavenumber is Paschen series is

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n^2}\right]$$

Where, n=4, 5, 6, …..∞, R=Rydbergs constant, λ=Wavelength

Paschen series belong to infrared region.

Bracket series: When an electron jumps from higher energy orbits to fourth energy orbit then Bracket series is obtained.

Formula for wavenumber in Bracket series is

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n^2}\right]$$

Where, n=5, 6, 7, …..∞, R=Rydbergs constant, λ=Wavelength

Bracket series belongs to middle infrared region.

Pfund series: When an electron jumps from higher energy orbits to fifth energy orbit then p-fund series is obtained.

Formula for Pfund series.

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n^2}\right]$$

Where, n=5, 6, 7, …..∞, R=Rydbergs constant, λ=Wavelength

P-fund series belongs to far infrared region.

Note:

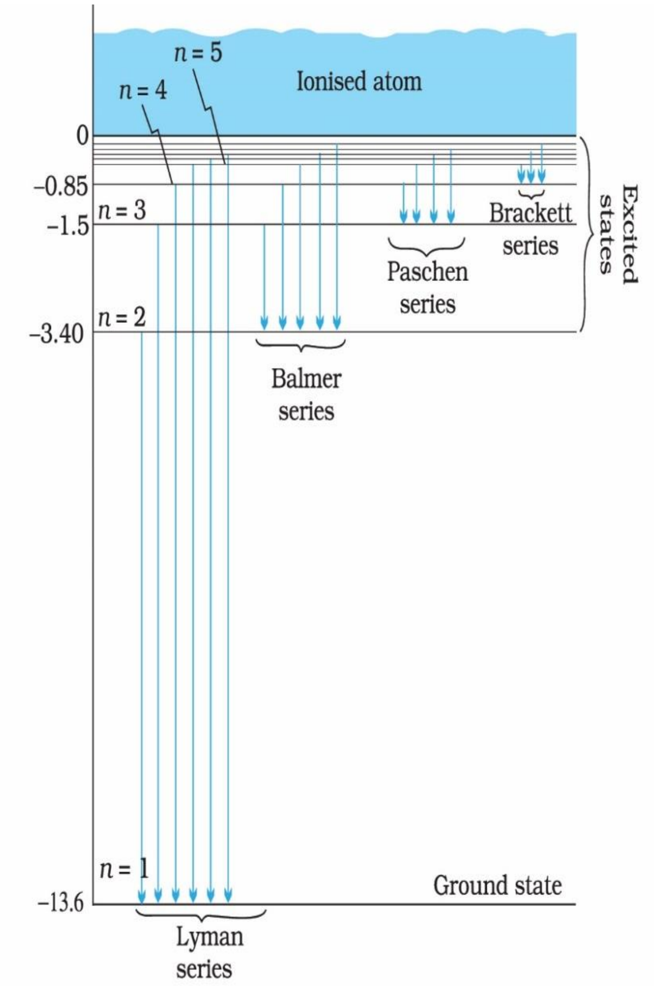
1. **Energy level diagram**
2. **The last line of spectral** series is called as series **limit.**
3. **Series limit is a shortest** wavelength spectral line.
4. **For series limit n=∞.**
5. **For Balmer series** $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

   We have

   $$C = \nu\lambda, \quad \lambda = \frac{C}{\nu}$$

   $$\frac{\nu}{C} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

   $$\nu = RC\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$



6. **Balmer series in the emission spectrum of hydrogen:** The first line of Balmer series is called H_α line, second line=H_β, third line=H_γ.

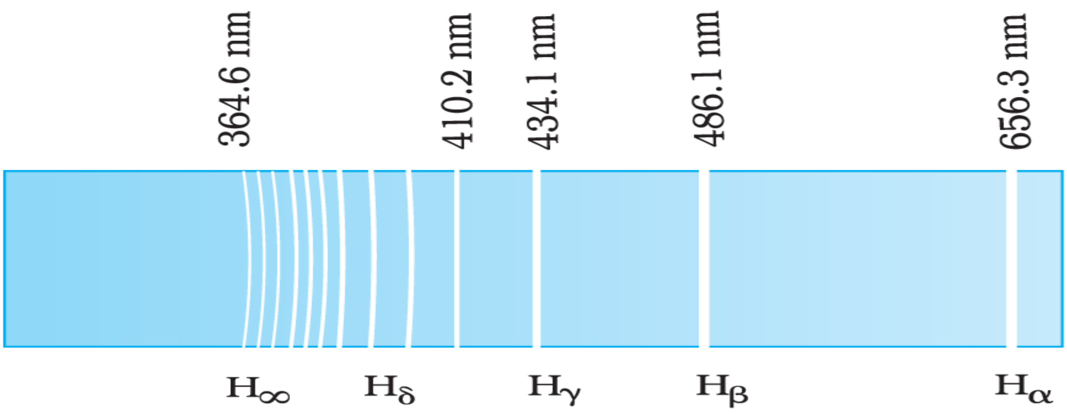
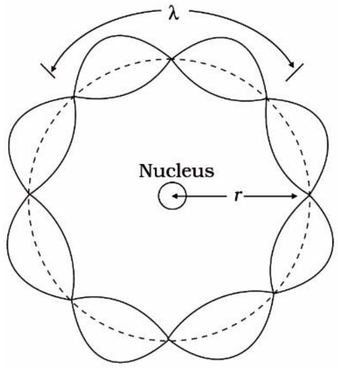


FIGURE : Balmer series in the emission spectrum of hydrogen.

de Broglies explanation of Bohr's Second Postulate (Bohr's Quantum Condition):



According to de Broglies hypothesis, the electron revolving in the orbit has wave nature. Therefore electron in its circular orbit must be seen as a particle wave. The particle wave produce the standing waves in n^{th} orbit.

The standing waves are formed if,

Total distance travelled by a particle wave=Integral multiple of wavelength.

$$2\pi r_n = n\lambda \text{ ------------- (1)}$$

(Total distance travelled by an electron in n^{th} orbit= Circumstance of n^{th} orbit = $2\pi r_n$)

According to de Broglies hypothesis

$$\lambda = \frac{h}{mv_n}$$

Where, λ=wavelength of particle wave, v_n=velocity of an electron

$\therefore$ Equation (1) becomes

$$2\pi r_n = n \times \frac{h}{mv_n}$$

$$mv_n r_n = n\frac{h}{2\pi}$$

$$\text{Angular momentum} = n \times \frac{h}{2\pi}$$

This is called Bohr's second postulate.

ONE MARK QUESTIONS WITH ANSWER:

1. Who discovered electrons?

Ans: Electrons were discovered by J.J Thomason in the year 1897.

2. What is the electric charge on an atom?

Ans: At atom of an element is electrically neutral.

3. Who proposed the first model of an atom?

Ans: J.J. Thomson proposed the first model of an atom in the year 1898.

4. Name the sources which emit electromagnetic radiations forming a continuous emission spectrum.

Ans: Condensed matter like solids and liquids and non-condensed matter lie dense gases at all temperatures emit electromagnetic radiations of several wavelengths as a continuous spectrum.

5. How does the spectrum emitted by rarefied gases different from those dense gases?

Ans: In the rarefied gases ,the separation between atoms or molecules are farther apart. Hence the atoms give discrete wavelengths without any interaction with the neighbouring atoms.

6. Give any one difference between Thomson's model and Rutherford's model of an atom.

Ans: In the Thomson's atom model, electrons are in stable equilibrium while in the Rutherford's atom model electrons always experience a net force $\frac{mv^2}{r}$ due to electrostatic force of attraction between electron and nucleus.

7. In which model atoms become unstable?

Ans: In Rutherford atom model. (An accelerating electron radiates energy and spiral around the nucleus. Ultimately electrons should fall inside the nucleus.)

8. What is a stationary orbit?

Ans: A stationary orbit is one in which the revolving electron does not radiate energy.

9. Give the relation between radius and principle Quantum number of an atom. Ans: $r_n \propto n^2$

10. Are the electron orbits equally spaced? Ans: No. Electron orbits are unequally spaced.

11. What is the relation between the energy of an electron and the principle quantum, number? Ans: $E_n \propto \frac{1}{n^2}$

12. What is excited state of an atom?

Ans: When atom is given sufficient energy, the transition takes place to an orbit of higher energy. The atom is then said to be in an excited state.

13. What is wave number of spectral line?

Ans: Wave number represents number of waves present in one metre length of the medium.

14. What is the value of Rydberg's constant? Ans: $R = 1.097 \times 10^7\ m^{-1}$.

15. **Name the spectral series of hydrogen which lies in the ultraviolet region** of electromagnetic spectrum. (2015-A)

16. **Name the spectral series of hydrogen atom in the visible region of electromagnetic spectrum. (2015-S)**

TWO MARKS QUESTIONS:

1. Name the two quantised conditions proposed by Bohr in the atom model.
2. Write the mathematical conditions for quantisation of orbits and energy states.
3. Write the expression for the radius of n^{th} orbit. Give the meaning of symbols used.
4. Give the expression for velocity of an electron in the n^{th} orbit. Give the meaning of symbols used.
5. Write the formula for the wave number of a spectral line.
6. What is the expression for the Rydberg's constant ? Give the meaning of the symbols used.
7. Write the formula for wave number of the spectral lines of Lyman series.
8. Write the formula for wave number of the spectral lines of Balmer series.
9. Mention any two demerits of Bohr's Theory.
10. How does Rydberg's constant vary with atomic number?
11. What is the value of ionization potential of ${}^{4}_{2}He$ atom?

THREE MARK QUESTIONS:

1. Explain briefly 1)Bohr's Quantisation rule and 2) Bohr's frequency condition.
2. Write de-Broglie wavelength associated with 3^{rd} and ^{4th} orbit in Bohr's atom model.
3. Give de-Broglie's explanation of quantisation of angular momentum as proposed by Bohr.
4. What are hydrogenic atoms?
5. Relate KE, PE and total energy of electron of an hydrogenic atom.

6. How is frequency of radiation different from that of frequency of electron in its orbit?
7. Why do we use gold in Rutherford's α-particle scattering experiment?
8. Using Balmer empirical formula, obtain the wavelengths of $H_\alpha, H_\beta, H_\gamma, \ldots H_\infty$.
9. By assuming Bohr's postulates derive an expression for radius of n^{th} orbit of electron, revolving round the nucleus of hydrogen atom. (2015-A)
10. State Bohr's postulates. (2015-S)

FIVE MARKS QUESTIONS:

1. State the postulates of a Bohr's theory of hydrogen atom.
2. Derive an expression for the radius of n^{th} Bohr's orbit of H_2 atom.
3. Obtain an expression for the energy of an electron in the nth orbit of hydrogen atom in terms of the radius of the orbit and absolute constants.
4. Give an account of the spectral series of hydrogen atom.
5. Explain energy level diagram of hydrogen atom.
6. Derive an expression for the frequency of spectral series by assuming the expression for energy.
7. Outline the experiment study of α- scattering by a gold foil.
8. Give the experimental conclusions arrived by Rutherford in the α- scattering experiment.
9. Assuming the expression for radius of the orbit, derive an expression for total energy of an electron in hydrogen atom. (2016-S, 2014-S)
10. Write three postulates of Bohr. Mention two limitation of Bohr model. (2014-A)

PROBLEM:

1. Calculate the shortest and longest wavelength of Balmer series of hydrogen atom. Given $R=1.097\times10^{7}m^{-1}$. (2016-A)
2. The first member of the Balmer series of hydrogen atom has wavelength of 6563Å. Calculate the wavelength an frequency of the second member of the same series. Given $C=3\times10^{8}$ ms^{-1}. (2017-A)

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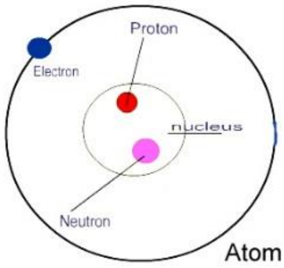
Chapter 15:

NUCLEI



The matter consists of number of atoms. The central part of the atom is called 'Nucleus'.

The Nucleus is a massive & positively charged hard core present at the centre of the atom.

Generally nucleus is spherical in shape. Radius of the nucleus is of the order of 10^{-15}m (Fermi).

More than 99.9% of the mass of an atom is concentrated in the nucleus. Hence nucleus is very massive and has high density.

Note:

1. Nucleus was discovered by 'Rutherford'.
2. The radius of the nucleus is 10^4 times smaller than radius of atom.
3. **Nuclear physics**: The branch of physics which deals with the study of Nucleus & its properties.

Composition of Nucleus:

The nucleus consists of protons and neutrons. Protons and neutrons are together called nucleons.

The number of protons present in the nucleus is called Atomic Number' (Z).

The total number of protons and neutrons present in the nucleus is called 'mass number' (A).

$\therefore$ Number of neutrons = A-Z

Note:

i) Neutron was discovered by James Chadwick.

ii) Protons carry positive charge & Neutrons carry no charge (Neutral)

iii) Free neutron is very unstable compared to free proton.

Representation of Nucleus:

The nucleus of an atom is represented as ${}^{A}_{Z}X$

Where, Z = Atomic number, A = Mass number, X = Chemical symbol of element.

For example.

The nucleus of Uranium is represented as ${}^{238}_{92}U$

Where, U = Uranium nucleus, A = 238 (Protons + Neutrons),

Z = 92 = proton number, A-Z = 146 = Neutrons number

i.e. Uranium nucleus contains 92 protons & 146 neutrons.

Types of Nuclei:

i) Isotopes: the nuclei having same atomic number but different mass number are called Isotopes.

Ex: i) ${}_1^1H, {}_1^2H, {}_1^3H$

ii) ${}_6^{12}C, {}_6^{13}C, {}_6^{14}C$

iii) ${}_{92}^{235}U, {}_{92}^{236}U, {}_{92}^{237}U$

${}_1^1H \rightarrow$ proton
${}_1^2H \rightarrow$ Deuteriun
${}_1^3H \rightarrow$ Tritium

ii) Isobars: The nuclei having same mass number but different atomic number are called Isobars.

Ex: ${}_6^{14}C$ & ${}_7^{14}N$

iii) Isotones: The nuclei having same number of neutron are called Isotones.

Ex: ${}_1^3H$ & ${}_2^4He$

A= 3	A = 4
Z = 1	Z = 2
A – Z = 2	A – Z = 2

Note 1:

1. **Isomers**: The nuclei having same mass number and same atomic number but different energy states are called isomers.

   Ex: $\left({}_{92}^{235}U\right)^* \xrightarrow[\text{(energy)}]{\gamma-\text{ray}} {}_{92}^{235}U$

   Excited state Ground state

2. Mass of an atom is very small kilogram (Kg) is not convenient unit to express such small masses. therefore atomic mass unit (u) is used to express mass of atom or nucleus.

Define atomic mass unit(u):It is defined as $1/12$ th of the mass of carbon-12 $\left({}_6^{12}C\right)$ atom.

$$1 \text{ atomic mass unit} = 1/12 \times \text{mass of } C^{12} \text{ Atom}$$

Note 2:

i) Mass of carbon – 12 atom = 19.92647×10^{-27} kg

But $1u = 1/12 \times$ mass of C^{12} atom

$1u = 1/12 \times 19.92647 \times 10^{-27}$ kg

$1u = 1.6605391 \times 10^{-27}$ kg

ii) Mass of electron = $m_e = 0.00055u$

Mass of proton = $m_p = 1.00727u$

Mass of Neutron = $m_n = 1.00866u$

Nuclear size: The shape of the nucleus is spherical. The volume of nucleus is directly proportional to mass number.

i.e., Volume of nucleus ∝ mass number (volume of sphere $\frac{4}{3}\pi R^3$)

$$V \propto A$$
$$\frac{4}{3}\pi R^3 \propto A$$
$$R \propto A^{1/3}$$
$$R = R_0 A^{1/3}$$

Where, $R_0 = 1.2 \times 10^{-15}$m = Nuclear constant ($10^{-15}$m= fermi=femtometer)

R = Radius of nucleus, A = Mass number

Nuclear charge: Nucleus consists of protons and neutrons. Protons carry positive charge and neutrons carry no charge.

Charge an each proton = + e

∴ Charge on nucleus = + Ze

Where, Z = No. of protons (atomic number), $e = 1.602 \times 10^{-19}$ C

Nuclear mass: Let m_p = mass of proton, Zm_p = total mass of protons,

m_n = mass of neutron, $(A - Z)m_n$ = total mass of neutrons

Nuclear mass = $zm_p + (A - Z)\, m_n$

Nuclear density: Nuclear density is the ratio of nuclear mass to the nuclear volume.

Nuclear density is found to be 2.31×10^{17} kg/m^3

This shows that protons and neutrons are tightly packed. Nuclear density is independent of mass number.

Einstein's mass – Energy Relation: It is given by

$$E = mc^2$$

Where, E = energy, m = mass equivalent to energy,

c = speed of light in vacuum

Note:

1. According to $E = mc^2$, energy is converted into mass and mass can be converted into energy. Mass an another form of energy.
   Ex: During nuclear fission and fusion mass is converted into energy.
2. Mass spectrometer is a device used to measure mass of the nucleus.
3. eV = electron volt is the unit for energy

$$1\text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

4. Calculate the energy equivalent of 1 amu.

$$E = mc^2$$

But $m = 1u = 1.6605 \times 10^{-27}$ kg = 1 amu

$C = 2.9979 \times 10^8\ ms^{-1}$ = speed of light

$E = 1.6605 \times 10^{-27} \times (2.9979 \times 10^8)^2$

$E = 1.6605 \times 10^{-27} \times 8.9874 \times 10^{16}$ J

$E = 14.9235 \times 10^{-11}$ J

$\left\{ 1eV = 1.602 \times 10^{-19} J \quad 10^6 = \text{Mega (M)} \right\}$

$$E = \frac{14.9235 \times 10^{-11}}{1.602 \times 10^{-19}} eV$$

$E = 9.315 \times 10^8$ eV

$E = 931.5 \times 10^6$ eV

$$\boxed{E = 931.5\ MeV}$$

i.e, 1u = 931.5 MeV energy

Mass defect (Δm): It is found that mass of the nucleus is less than the sum of the masses of its constituents.

The difference in mass of a nucleus and its constituent is called the mass defect.

The expression for mass defect is given by

$$\Delta m = [zm_p + (A - Z)\, m_N] - M$$

Where, Δm=mass defect, M=mass of nucleus, m_p=mass of proton, m_N=mass of neutron

Z=Atomic number, A=Mass number

Note:

1. Mass defect is converted into energy during the formation of nucleus. This energy is responsible for binding the nucleons inside the nucleus.

∴ The nucleus is stable. Some amount of energy is required to break the nucleus.

Binding energy (E_b): It is defined as the minimum energy required to split the nucleus into its constituent nucleons.

Expression for binding energy is given by,

$$\boxed{E_b = \Delta m \times c^2}$$

Where, Δm=Mass defect in Kg, C=Speed of light in vacuum in ms^{-1}

E_b=Binding energy in Joule

Note:

1. Binding energy can also be expressed as

$$\boxed{E_b = \Delta m \times 931.5 MeV}$$

Where, Δm=mass defect in amu (u), E_b =Binding energy in MeV

Binding energy per nucleon [E_{bn}]:

It is defined as the ratio of binding energy to the mass number.

$$\text{Binding energy per nucleon} = \frac{\text{binding energy}}{\text{mass number}}$$

$$E_{bn} = \frac{E_b}{A}$$

Binding energy per nucleon is the measure of stability of the nucleus.

Binding energy curve: A graph of platting binding energy per nucleon along y-axis and mass number along x – axis is called Binding energy curve.

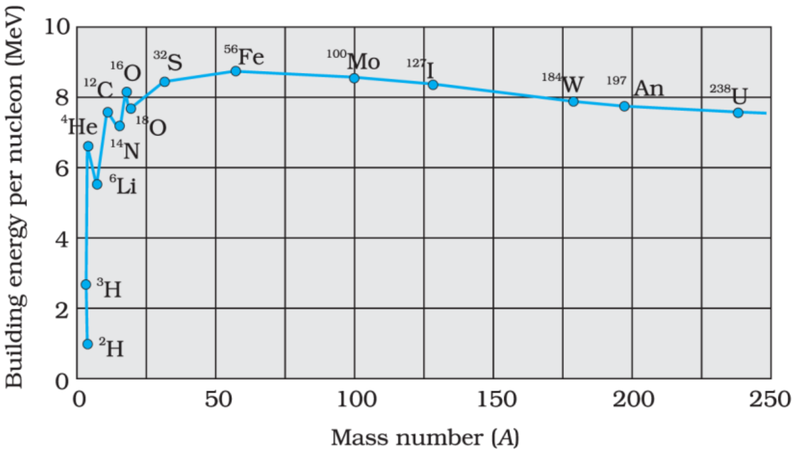


FIGURE : The binding energy per nucleon as a function of mass number.

Significance of Binding – energy curve:

1. Binding energy per nucleon is constant for a nuclei having mass number in between 30 and 170.
2. Binding energy per Nucleon is minimum for both lighter nuclei (A < 30) and heavy nuclei (A > 170).
3. The Binding energy curve explains why the energy is released during nuclear fission and fusion.
4. Binding energy per nucleon shows that the strong attractive force is required to bind the nucleons.

Note: The binding energy per nucleon is maximum for Iron-56 (Fe)

For ^{56}Fe, E_{bn} = 8.75MeV.

Nuclear force: The strong attractive force between the nucleons in the nucleus is called nuclear force.

Characteristic of nuclear force:

1. Nuclear force is a strongest and attractive force in nature. i.e., nuclear force is stronger than coulomb force or gravitational force..
2. Nuclear force is charge independent force. i.e., Nuclear force between P and P=nuclear force between N and N = nuclear force between P and N. Where, P=proton, N=neutron.
3. Nuclear force is a short range force. i.e., if the distance between two nucleons is more than 10 fermi, then the nuclear force become zero.
4. If the distance between nucleons is less than 0.8fermi, then nuclear force becomes repulsive.
5. If the distance between nucleons is greater than 0.8fm, then the nuclear force becomes attractive.

Note:

1. Nuclear force doesnot depend on mass and charge i.e, nuclear force is non gravitational force and non-electrostatic force.
2. Nuclear force > electromagnetic force > weak nuclear force > gravitational force

Radioactivity: Radioactivity is a nuclear phenomenon in which unstable nucleus undergoes decay.

Radioactivity was discovered by 'Henry Becquerel'

Types of radioactive decay:

There are 3 types 1) alpha decay (α), 2) Beta decay (β), 3) Gamma decay (γ)

Alpha decay (α): The decay in which helium nucleus is emitted from the unstable nucleus is called α - decay.

Ex: $^{238}_{92}U$ is decayed into $^{234}_{90}Th$ by emitting α - particle $\left(^{4}_{2}He\right)$

$$^{238}_{92}U \longrightarrow ^{234}_{90}Th + ^{4}_{2}He$$

Note:

1. During α - decay, mass number is decreased by 4 and atomic number is decreased by 2.
2. General equation for α - decay is given by,

   $${}^{A}_{Z}X \xrightarrow{\alpha-decay} {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

   Where, ${}^{A}_{Z}X$ = parent element, ${}^{A-4}_{Z-2}Y$ =daughter element. ${}^{4}_{2}He$ =α-particle.
3. For α- particle mass number A=4, Z=2. The charge on α- particle is +2e.
4. From Einstein's mass-energy equivalence relation and energy conservation. It is clear that this spontaneous decay is possible only when the total mass of the decay products is less than the mass of the initial nucleus. This difference is mass appears as kinetic energy of the products. By referring to the table of nuclear masses, one can check that the total mass of ${}^{234}_{90}Th$ and ${}^{4}_{2}He$ is indeed less than that of ${}^{238}_{92}U$.
5. **The disintegration energy or the Q-values of a nuclear reaction:**
   It is the difference between the initial mass energy and the total mass energy of the decay products.
   Let, M_x=Mass of parent element X, M_y =Mass of daughter element Y,
   M_α=Mass of α- particle $({}^{4}_{2}He)$

   ∴ Mass defect, $\Delta m = M_x - (M_y + M\alpha)$

   The mass defect is converted into energy

   i.e., Q = Δmc^2

   Q = $\left[M_x - (M_y + M_{\alpha)}\right]C^2$

   Q>0, i.e., energy is liberated during α- decay

   ∴ α-decay is an 'Exothermic reaction'.

Beta–decay (β -decay) : The decay in which electron or positron is emitted from the unstable nucleus. There are 2 types of β -decay: i) β^- decay , ii) β^+ decay

β - decay: The decay in which electron is emitted from the unstable nucleus is called β^- decay. Ex 1): ${}^{60}_{27}Co \xrightarrow{\beta^- decay} {}^{60}_{28}Ni + {}^{0}_{-1}e + \overline{\gamma}$

Where, $\overline{\gamma}$ → Anti-neutrino, β^- particle = electron $= {}^{0}_{-1}e = e^-$

Ex 2): ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \overline{\gamma}$

Note:

1. During β^- decay, in addition to electron one more particle is emitted called Anti-neutrino ($\overline{\gamma}$)
2. General equation for β^-decay:

   $${}^{A}_{z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \overline{\gamma}$$

   In β^+-decay, mass number remain same, atomic number is increased by one.

β^+ **decay:** The decay in which positron is emitted from the unstable nucleus is called β^+decay.

Ex: ${}^{22}_{11}Na \xrightarrow{\beta^+} {}^{22}_{10}Ne + {}^{0}_{+1}e + \gamma$

Where, $\gamma \rightarrow$ Neutrino , β^+particle = positron$({}^{0}_{+1}e)$ or(e^+)

Note:

1. General equation for β^+ decay

   $${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{-1}e + \overline{\gamma}$$

2. During β^+-decay, in addition to positron one more particle is emitted called Neutrino (γ).
3. In β^+-decay, mass number remain same, atomic no is decreased by one.
4. During β^- decay, neutron is converted into proton as follows.

   $$p \rightarrow n + {}^{0}_{-1}e + \gamma$$

5. During β^+ decay proton is converted into neutron as follows.

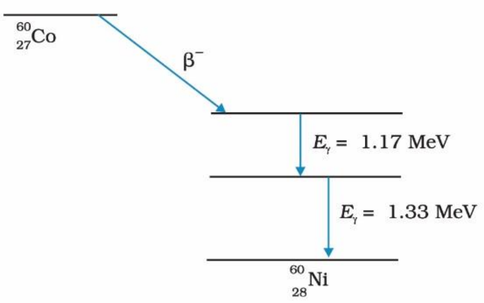
   $$p \rightarrow n + {}^{0}_{+1}e + \gamma$$

6. Neutrino was discovered by 'Pauli'

   Mass of Neutrino = 0, Charge on Neutrino = 0. But Neutrino has spin and energy.
7. Positron is the anti- particle of electron charge of positron = +e

   Mass on positron = Mass of electron.

Gamma-decay (γ - decay):



The decay in which high energy photon is emitted from unstable nucleus is called γ - decay.

Usually Gamma decay takes place after the emission of α and β particles:

Ex: when $^{60}_{27}CO$ emits β^- particle then it becomes $^{60}_{28}Ni$. The daughter Nuclei $(^{60}_{28}Ni)$ emits two gamma rays & then it comes to ground state.

Note:

1. General equation for γ-decay is, $\underset{\text{(Excited state)}}{(^A_Z X)} \xrightarrow{\gamma-\text{ray}} \underset{\text{(Ground state)}}{^A_Z X}$
2. During γ - decay mass number and atomic number remains same.

Activity or Rate of decay: Activity of a radioactive sample is defined as the number of nuclei disintegrated per unit time.

$$\text{Activity (or) Rate of decay} = \frac{\text{Number of nuclei disintegrated}}{\text{Time taken}}$$

S.I unit of Activity is becquerel (Bq)

Define one Becquerel: The activity of a radioactive sample is said to be one becquerel if one disintegration takes place in one second.

i.e., 1 becquerel. (Bq) = 1desintegration/sec.

Note:

1. Curie (ci) is the practical unit of activity.

   1 curie = 3.7×10^{10} Bq

2. Ruther ford (Rd) is the practical unit for activity.

   1 Ruther ford = 10^6 Bq

Law of radioactive decay: "It states that, the rate of decay is directly proportional to the number of nuclei present in the sample at that time.

i.e. Rate of decay $\propto$ Number of nuclei in the radioactive sample.

$$\frac{dN}{dt}\alpha N.$$

Derive the equation : N= N_0 e^{-xt}:

Consider a radio active sample.

Let,

N_0=initial number of nuclei present in the radioactive sample at time t_0 = 0sec.

N=number of nuclei present in the sample after a time 't' sec. (undecayed).

N_0 – N=dN = number of nuclei decayed in a time interval 'dt'.

$$\therefore \text{ Rate of decay} = \frac{dN}{dt}.$$

According to decay law,

Rate of decay $\propto$ Number of nuclei present in the sample at time 't'.

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

Where, $\lambda =$ Decay constant,

Here, negative sign indicates that number of nuclei goes on decreases with time.

$$\therefore \frac{dN}{N} = -\lambda dt$$

Integrating on both sides

$$\int_{N_0}^{N} \frac{dN}{N} = \int_{t_0}^{t} \lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt$$

$$\left[\log_e N\right]_{N_0}^{N} = -\lambda \left[t\right]_{t_0}^{t}$$

$$\log_e N - \log_e N_0 = -\lambda\left[t - t_0\right]$$

$$\log_e\left(\frac{N}{N_0}\right) = -\lambda\left[t - 0\right] \qquad (\because t_0 = 0)$$

$$\log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

$$e^{-\lambda t} = \frac{N}{N_0}$$

or $$\frac{N}{N_0} = e^{\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

$\int \frac{1}{x} dx = \ln x$ $\quad \int dn = x$

$\log_e a - \log_e b = \log_e\left(\frac{a}{b}\right)$

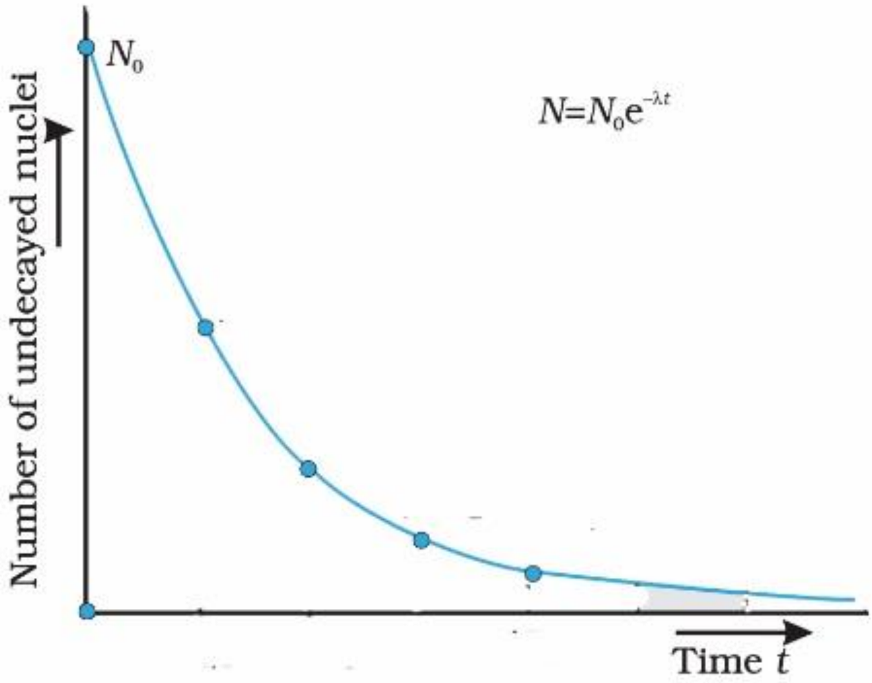
$\log_e^x = l_n^x$

$\log_e^x = y$

$e^y = x$

Note:

1. The number of nuclei present in the radioactive sample decreases exponentially with time as shown



2. We can have

   $M = M_0e^{-\lambda t}$

   Where,

   M_0 = Initial mass of a radioactive sample,

   M = Final mass after a time 't' sec, λ = Decay constant

3. Rate of decay

   $R = R_0e^{-\lambda t}$

Where, R_0 =Initial activity, R =Final activity after a time 't' sec

4. $R = -\frac{dN}{dt}$
5. $R = \lambda N$ at time 't'
6. $R_0 = \lambda N_0$ is the decay rate at time t=0.

Half life of a radioactive sample ($T_{1/2}$):

The time in which the number of nuclei present in the sample reduces to half of its initial value

i.e., if $t = T_{1/2}$

Then, $N = \frac{N_0}{2}$

Derive an expression for half - life:

We have, $N = N_0 e^{-\lambda t}$ ⟶ (1)

Where, N_0 = initial number of nuclei,

N = No. of nuclei present after a time't'

$T_{1/2}$ = Half life

If, $t = T_{1/2}$

Then, $N = \frac{N_0}{2}$

∴ Equation (1) becomes,

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = \frac{1}{e^{\lambda T_{1/2}}}$$

$$e^{-\lambda T_{1/2}} = 2$$

Take $\log_e$ on both sides

$$\log_e\left(e^{\lambda T_{1/2}}\right) = \log_e(2)$$

$$(\log_2)e^x = x$$

$$\lambda T_{1/2} = \log_e(2)$$

$$\lambda T_{1/2} = 2.303 \times \log_{10}(2)$$

$$\lambda T_{1/2} = 2.303 \times 0.3010$$

$$\lambda T_{1/2} = 0.693$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

$\text{Log}_e\, e^x = x$ $\therefore \log_e\left(e^{\lambda T_{1/2}}\right)$ $= \lambda T_{1/2}$

Note:

We have, $N = \frac{N_0}{2^n}$

Where, $n = \frac{t}{T_{1/2}}$ = Number of half lives

Mean life (T): The time in which the number of nuclei present in the sample reduces to 37% of its initial value

S.I. unit is second

Decay constant (λ): The reciprocal of mean life is called decay constant.

$$\text{Decay constant} = \frac{1}{\text{mean life}}$$

$$\lambda = \frac{1}{\tau}$$

S.I. unit is s^{-1}

Expression for mean life

We have, $T_{1/2} = \frac{0.693}{\lambda}$

But, $\lambda = \frac{1}{\tau}$

or $\tau = \frac{1}{\lambda}$

$$T_{1/2} = 0.693 \times \tau$$

Where, $T_{1/2}$ = Half life, τ = Mean life

This is the relation between mean life and half life

Nuclear energy: The energy obtained from the conversion of nuclear mass is known as nuclear energy.

Nuclear fission: The process of splitting of heavy nucleus into two nuclear fragment of intermediate mass with the release of neutrons and energy.

Ex: When a slow neutron is bombarded with ${}^{235}_{92}U$, then it is splitted in ${}^{144}_{56}Ba$ and ${}^{89}_{36}Kr$ with the release of three fast neutrons and energy.

$${}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{236}_{92}U \longrightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n + Q$$

The energy released per fission is about 200 MeV. The fragment products are radioactive nuclei. They emit β particles and they become stable.

The released neutrons are usually fast neutrons. Nuclear fission reaction was first observed by Otto Hahn and Straussman.

To find Q value (energy value):

Q =[Initial mass of reactant – Final mass of product]C^2 in Joule

or Q =[Initial mass of reactant – Final mass of product]931 MeV

Note: The neutrons having kinetic energy 2MeV are called fast neutrons. The neutrons having kinetic energy less than 1MeV are called slow neutrons.

Nuclear Fusion: The process of combining two lighter nuclei to form a single larger nucleus is called nuclear fusion.

Ex: 1) ${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \gamma + 0.42MeV$.

2) ${}_1^2H + {}_1^2H \rightarrow {}_2^3H + n + 3.27MeV$

3) ${}_1^2H + {}_1^2H \rightarrow {}_1^3H + {}_1^1H + 4.03MeV$

Thermonuclear fusion:

The fusion reaction requires very high temperature is of the order of $10^8 K$, So it is also called thermo nuclear fusion.

The energy produced in the interior of star is due to thermo nuclear fusion.

The fusion reaction in sun is multistep process. In this process, hydrogen is burned in to helium. This process is called proton-proton cycle.

According to proton – proton cycle,

$${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \gamma + 0.42MeV \text{ ---------- } (1)$$

$$e^+ + e^- \rightarrow \gamma + \gamma + 1.02MeV \text{ ----------- } (2)$$

$${}_1^2H + {}_1^1H \rightarrow {}_2^3He + \gamma + 1.02 MeV \text{ ------------- } (3)$$

$${}_2^3He + {}_2^3H \rightarrow {}_2^4He + {}_1^1H + {}_1^1H + 12.86 MeV \text{ -------------- } (4)$$

For the fourth reaction to occur, the first three reactions must occur twice, in which case two light helium nuclei unite to form ordinary helium nucleus. If we consider the combination. $2(1)+2(2)+2(3)+4$, the net effect is

$$4\,{}_1^1H + 2e^- \rightarrow {}_2^4He + 2\gamma + 6\gamma + 26.7MeV$$

Or $$\left(4\,{}_1^1H + 4e^-\right) \rightarrow \left({}_2^4H + 2e^-\right) + 2\gamma + 6\gamma + 26.7MeV$$

Thus, four hydrogen atoms combine to form an ${}_2^4He$ atom with a release of 26.7MeV of energy.

Note:

1) **Fertile material:** The material which cannot undergo fission reaction directly is called fertile material. Ex: ${}^{238}_{92}U$
2) **Fissile material:** The material which can undergo fission reaction very easily is called fissile material. Ex: ${}^{235}_{92}U$ and ${}^{239}_{93}Pu$
3) Slow neutrons can cause fission reaction.
4) If the fission reaction takes place continuously, then it is called nuclear chain reaction.
5) There are two types of chain reaction.
   i) Controlled chain reaction
   ii) Uncontrolled chain reaction.
6) Controlled chain reaction: The chain reaction in which energy and fission causing neutrons are released at constant rate. Nuclear power reactor works on the principle of controlled chain reaction.
7) Uncontrolled chain reaction: The chain reaction in which energy and fission causing neutrons increases rapidly.

   In uncontrolled chain reaction, enormous amount of energy released with in short interval of time.

Atom bomb (nuclear bomb) works on the principle of uncontrolled chain reaction.

Nuclear reactor: It is a device in which controlled chain reaction is achieved.
It gives the energy at constant rate

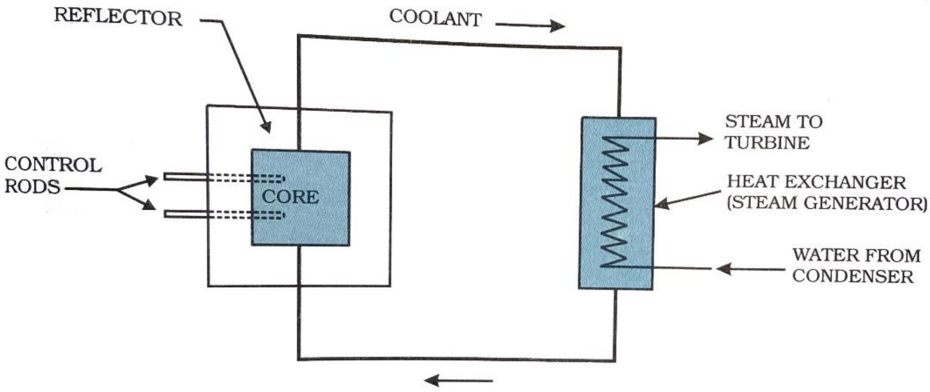
Types of nuclear reactors

There are three types

1) **Research reactor**: The nuclear reactor used to produce radio isotopes for research purpose is called research reactor.
2) **Breeder reactor**: The reactor used to convert fertile material into fissile material is called breeder reactor.
3) **Nuclear power reactor**: The reactor used to convert nuclear energy in to electrical energy is called nuclear power reactor.

Note: Nuclear reactor was first constructed by Enrichofermi.

Schematic diagram of nuclear power reactor:



Parts of nuclear reactor:

Nuclear fuel: It is the fissile material used in power reactor.

Ex: ${}^{235}_{92}U$, ${}^{239}_{93}Pu$

Moderator: It is a material used to slow down the fast neutrons.

Ex: Water, Heavy water, graphite.

Control rods: It is a material used to absorb exess fission causing neutrons.

Ex: Cadmium.

Coolant: It is a material used to transfer heat from reactor to steam chamber.

Protective shield: It is in the form of a concrete thick wall surrounds the core to save the persons working around the reactor from the hazardous radiations.

Reproduction factor or Multiplication factor (K):

It is defined as the ratio of rate of production of neutron to the rate of loss of neutron.

i.e. $K = \frac{\text{Rate of production of neutron}}{\text{Rate of loss of neutron}}$

if, K=1, the mass of fissionable material is said to be critical and the chain reaction is sustained.

If , K<1, the chain reaction stops

K>1, chain reaction accelerated the reactor, power increases exponentially and reactor will become supercritical and can even explode.

Note:

1) The first nuclear reactor in India is Apsara at BARC, Mumbai.
   In this reactor, water is used as moderator
2) Nuclear power reactor in Karnataka is located at Kaiga.
   In this reactor, heavy water is used as moderator.
3) **Thermo nuclear fusion device:** It is device in which controlled thermo nuclear fusion reaction takes place. Aim of this device is to generate steady power by heating the mixture of positive and negative ions at a temperature 10^8K. Drawback of this device is that no container can withstand such high temperature.

One Mark Questions

1. What is nuclear physics?
2. Mention the composition of nucleus.
3. Define atomic mass unit.
4. What is nuclear charge?
5. What is nuclear mass?
6. What is nuclear density?
7. What is nuclear force?
8. What is nuclear binding energy?
9. What is binding energy per nucleon?
10. What is binding energy curve?
11. What are fossile materials?
12. What is radioactivity?
13. Define activity of a radioactive substance.
14. Define Becquerel.
15. Mention the relation between curie and Becquerel.
16. Define half life of a radioactive substance. (2014-S)
17. Define mean life of a radioactive substance.
18. Mention the relation between halflife and mean life. (2014-S)
19. What is disintegration energy? Or Define Q-value of nuclear reaction.
20. Mention the expression for Q-value of nuclear reaction.
21. What is nuclear energy?
22. On what principle nuclear power reactor works?
23. On Which principle atom bomb works?
24. Define reproduction factor or multiplication factor.
25. IN the following nuclear reaction. Identifies the particle X. $n \rightarrow P + e^{-} + X$.

26. What is the ratio of the nuclear densities of two nuclei have mass number in the ratio 1:3.
27. Define specific binding energy (2015-A)
28. Write the SI unit of radioactivity. (2015-S, 2017-A)
29. How many neutrons will be there in the nucleus of an element with mass number A and atomic number Z?
30. Mention the commonly used unit to measure the nuclear mass.
31. Which type of radioactive emission produces a daughter nucleus which is an isobar of the parent?
32. Mention the SI unit of activity.
33. What are isotones?
34. How does the radius of the nucleus vary with respect to mass number?

Two Mark Questions

1. What are isotopes? Give one example. (2016-A, 2014-S)
2. What are isobar? Give one example. (2016-A)
3. What are isotones? Give one example.
4. Explain nuclear size.
5. Derive the relation between mass number and radius of the nucleus.
6. Mention Einstein's mass-energy relation and explain the symbols.
7. What is mass defect? Mention the expression for it.
8. What is α-decay? Give an example.
9. What is β-decay? Give an example.
10. What is β^+-decay? Give an example.
11. What is Gamma-decay? Explain with example
12. State and explain radioactive decay law. (2014-A)
13. What is nuclear fission? Give an example.
14. What is nuclear fission? Give an example.
15. What is controlled chain reaction? Give an example.
16. What is uncontrolled chain reaction? Give an example.
17. What is mass defect? Write the formula for the mass defect for the nucleus of an element $^{Z}_{A}X$.
18. Mention any two characteristics of nuclear forces.
19. Mention the order of nuclear density. How does the nuclear density vary as we move from the centre to the surface?
20. Define nuclear fission and give an example for it.
21. Define half-life and mean-life of a radioactive nucleus.

Three or Five Mark Questions

1. Mention any three / four properties of nuclear forces. (2016-A)
2. Draw the binding energy curve and mention its significances.
3. Give any three / five difference between nuclear fission and fusion.
4. Derive $N = N_0 e^{-\lambda t}$. (2017-A)
5. Define half life. Obtain an expression for half life.
6. Explain different parts of nuclear reactor?
7. Draw the graph of binding energy per nucleon with respect to mass number. What is the significance of the graph?
8. Write the equation representing nuclear reaction corresponding to α, β and γ emission.
9. What is Q-value of a nuclear reaction? Write the formula for Q-value for β-emission and explain the terms.
10. Define Atomic Mass Unit. Mention Einstein's mass energy relation.
11. Prove that $\mathrm{N} = \mathrm{N_0 e^{-\lambda t}}$ where the symbols have their usual meaning.
12. Define mean-life. Write the expression for mean-life in terms of decay constant.
13. Obtain the relation between half-life and decay constant.

Problems:

1. Calculate the binding energy and binding energy per nucleon (in MeV) of a nitrogen nucleus $\left({}^{14}_{7}N\right)$ from the following data.
   Mass of proton= 1.00783u
   Mass of neutron=1.00867u
   Mass of nitrogen nucleus=14.00307u. (2014-A)
2. The activity of a radioactive substance is 4700 per minute. Five minute later the activity is 2700 per minute. Find a) decay constant and b) half-life of the radioactive substance. (2016-S)
3. Determine the mass of Na^{22} which has an activity of 5mCi. Half life of Na^{22} is 2.6 years. Avagadro number=6.023×10^{23} atoms. (2015-A)
4. Calculate the half life and mean life of radium-226 of activity 1Ci: Given the mass of radium=226 is 1 gm and 226 gram of radium consists of 6.023×10^{23} atoms. (2015-S)
5. Calculate the binding energy and binding energy per nucleon in MeV for carbo-12 nucleus. Given that mass of the proton is 1.00727amu while the mass of the neutron is 1.00866amu.
6. Half-life of ${}^{90}_{38}\mathrm{Sr}$ is 28years. Calculate the activity in Ci of 30mg of ${}^{90}_{38}\mathrm{Sr}$.
7. Calculate the Q-value of the emitted α-particle in the α-decay of ${}^{220}_{86}\mathrm{Rn}$.

~*~*~*~*~*~

Chapter 14:

SEMICONDUCTOR ELECTRONICS

The branch of physics which deals about semiconductors and its devices is called **solid state electronics**. The electron mechanics is called **electronics**.

Explain briefly the concepts of energy bands:

According Neils Bohr atomic model, in an isolated atom the energy of any of its electrons is decided by the orbit in which it revolves. In solids the atoms are closely packed. There is an interaction between neighbouring atoms. So the outer energy levels of electrons from neighbouring atoms would come very close or could even overlap.

The group of energy levels with continuous energy variation is called **energy band**.

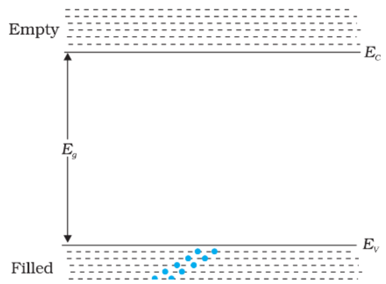
The energy band which includes the energy levels of valence electrons is called the **valence band**.

Valence band is occupied by valence electrons.

The energy band above the valence band is called the **conduction band**.

Normally the conduction band is empty or occupied by free electrons.

The lowest energy level in the conduction band is shown as 'E_c' and the highest energy level in the valence band is show as 'E_v'.



The gap between the top of the valence band and bottom of the conduction band is called **energy band gap**.

'E_g' (Energy band gap) may be large small or zero depending upon the material.

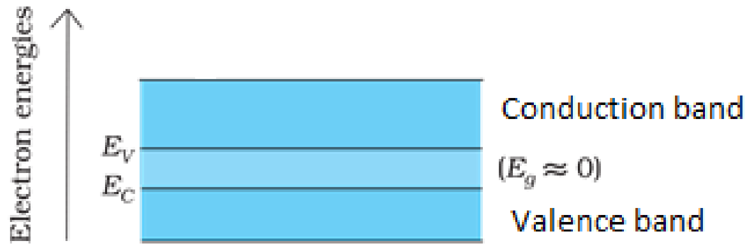
Note: When the valence electron gets sufficient energy, they get excited to conduction band.

Explain with a sketch classification of solids on the basis of energy bands (Distinguish between conductors, semiconductors and insulators on the basis of band theory):

There are three types.

i) Conductors, ii) Semiconductors, iii) Insulators

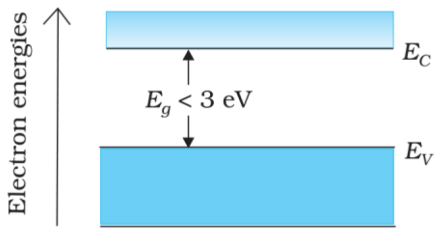
i) Conductors:



In a conductor, the valence band and conduction band are overlapped, i.e. energy gap, E_g=0. Therefore electrons from valence band can easily move into the conduction band. Therefore the conduction band is completely filled by free electrons. As a result the resistance of conductor is low and the conductivity is high. The resistance and resistivity increases with increase in temperature.

Ex: All metals (Al, Cu, Fe, Au, etc)

ii) Semiconductors:

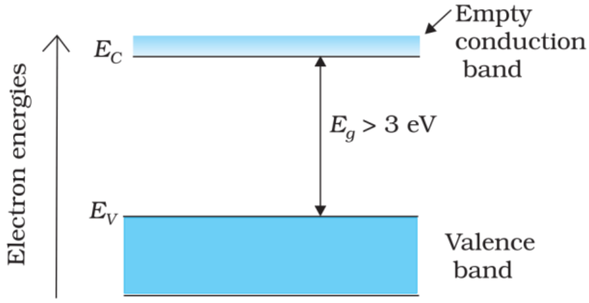


In semiconductor, the valence band conduction band are separated by a small energy gap. i.e., Eg<3eV. Because of the small band gap, at room temperature some electrons from valence band can acquire enough energy to cross the energy gap and enter the conduction band. Therefore conduction band is partially filled with free electrons at room temperature. As a result the semiconductor conducts the current at room temperature.

The resistance and resistivity decreases with increase in temperature.

Ex: silicon, germanium, carbon. For silicon Eg=1.1eV, for germanium Eg=0.7eV.

iii) Insulators:



In insulators, the valence band and conduction band are separated by a large energy gap, i.e. E_g>3eV. Note that the energy gap is so large, the electrons cannot be excited from the valence band to the conduction band by thermal excitation. Therefore the conduction band is completely empty, so no electrical conduction is possible at room temperature.
Ex: plastic, wood, glass.

Classification of solids (Conductors, Semiconductors and Insulators) on the basis of resitivity and conductivity:

On the basis of the relative values of electrical conductivity (σ) or resistivity (ρ) $\left(\text{i.e., } \rho = \frac{1}{\sigma}\right)$, the solids are broadly classified as

i) Metals (Conductors): Metals are substances which easily allow the passage of electric current through them. These are having large number of free electrons. They possess very low resistivity (ρ) and high conductivity (σ).

$\rho \sim 10^{-2}$ to 10^{-8} Ω m, $\sigma \sim 10^{2}$ to 10^{8} S m^{-1} or Siemens/m. Ex: Copper, Aluminium etc.

ii) Semiconductors: It is a substance which has few number of electrons at room temperatrure and the resistivity (ρ) of a semiconductor is less than insulator, but more than conductors. It has negative temperature co-efficient of resistance. i.e., $R \propto \frac{1}{T}$.

Ex: Germanium (Ge), Silicon (Si) etc.

$\rho \sim 10^{-5}$ to 10^{6} Ω m

$\sigma \sim 10^{5}$ to 10^{-6} Sm^{-1}

iii) Insulators: It is also a substance which has practically no free electrons and it does not allow the electric current through it and it has high resistivity or low conductivity.

Ex: Glass, Rubber, PVC etc.

$\rho \sim 10^{11}$ to 10^{19} Ω m

$\sigma \sim 10^{-11}$ to 10^{-19} Sm^{-1}

Note-1:

i) Elemental semiconductors : Si and Ge

ii) Compound semiconductor: Example:

a) Inorganic: CdS, GaAS, CdSe, InP, etc

b) Organic : Anthracene, doped pthalocyanines, etc

c) Organic polymers: Polypyrrole, polyaniline, polythiophene, etc.

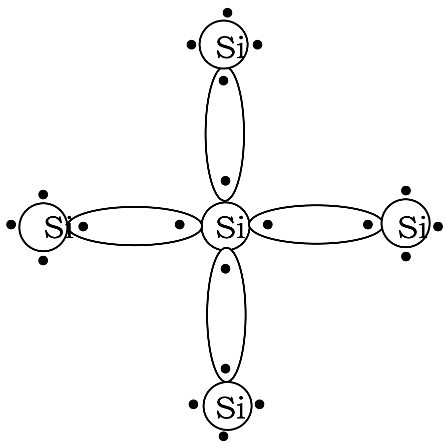
Note-2: Structure of Silicon (Si) and Germanium (Ge):

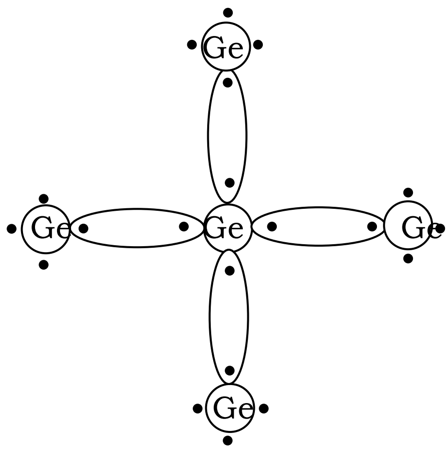
Atomic Number of Si= 14

$1s^2, 2s^2, 2p^6,$ $\mathbf{3s^2, 3p^2}$

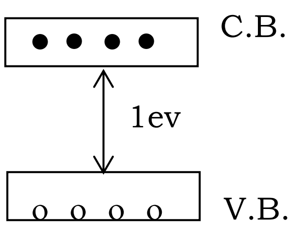
Atomic Number of Ge= 32

$1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10},$ $\mathbf{4s^2, 4p^2}$





Note-3: Concepts of holes:

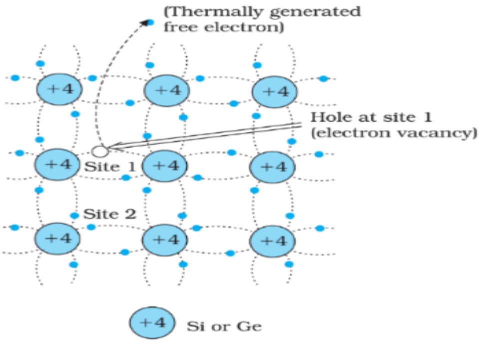


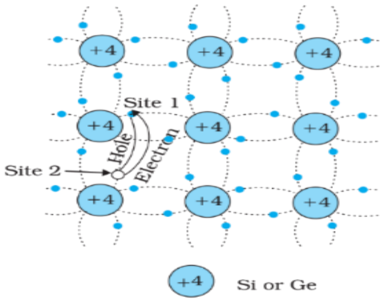
● = Free electrons

o = holes

The Si and Ge are tetravalent. Each Si and Ge atom can form four covalent bonds with the neighbour atoms. At absolute zero, (0K) all the valence electrons form covalent bonds. The conduction band is completely empty. Thus semiconductors behave as an insulator at 0K. If the temperature increases, the electrons get sufficient energy and jump to conduction band creating a vacancy for electron in the valence band. The vacancy can be filled by free electrons. The vacancy is called *a hole*.

Note-4:





Note-5: The creation of electron-hole pair due to thermal energy is called thermal generation.

Note-6: In thermal generation number of holes is equal to number of free electrons.

Note-7: In semiconductor both electrons and holes are charge carriers.

Define hole: The vacancy in the valance band which can be occupied by electron is called hole. The hole carries positive charge.

Types of semiconductor: There are two types 1) Intrinsic semiconductor and 2) Extrensic semiconductor.

Intrinsic semiconductor: A semiconductor in a extremely pure form is known as an instrinsic semiconductor. **Ex:** pure silicon and germanium.

In instrinsic semiconductors, the number of free electrons (n_e) is equal to the number of holes (n_h). That is $n_e=n_h=n_i$.

Where 'n_i' is called instrinsic carrier concentration.

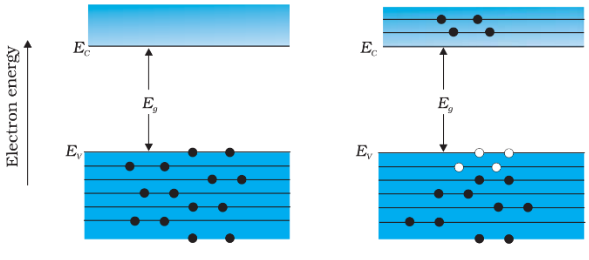
Expression for total current in an intrinsic semiconductor:

1) In intrinsic semiconductor, free electron moves completely independently as conduction electron and gives rise to an electron current 'Ie' under an applied electric field. Remember that the motion of hole is only a convenient way of describing the actual motion of bound electrons, Whenever there is an empty bond anywhere in the crystal. Under the action of an electric field, these holes moves towards the negative potential giving the hole current (I_h). The total current 'I' is thus the sum of electron current 'I_e' and the hole current 'I_h'

$$I=I_e+I_h$$

It may be noted that apart from the process of generation of conduction of electrons and holes, a simultaneous process of recombination occurs in which the electrons recombine with holes. The recombination occurs due to an electron colliding with a hole. At equilibrium the rate of generation is equal to the rate of recombination of charge carriers.

Write a note on Instrinsic Semiconductor with variation in temperature shown with energy band diagram:



(a) (b)

An intrinsic semiconductor will behave like an insulator at T=OK as shown in figure (a). It is the thermal energy at higher temperatures (T>OK), which excites some electrons from the valence band to the conduction band. These thermally excited electrons at T>OK, partially occupy conduction band, therefore the energy band diagram of a intrinsic semiconductor as shown in figure (b). Here, some electrons shown in the conduction band, these have come from the valence band leaving equal number of holes there.

Doping: The process of adding impurity to pure semiconductor is called *Doping.*

Dopant: The impurity added to pure semiconductor is called *Dopant or Doping agent.*

There are two types of dopants used in doping the tetravalent Si or Ge

Ex: **Pentavalent impurities**: Phosphorous (p), Antimony (Sb) and Arsenic (As).

Trivalent impurities: Aluminium(Al), Boron(B), Indium(In), Gallium(Ga)

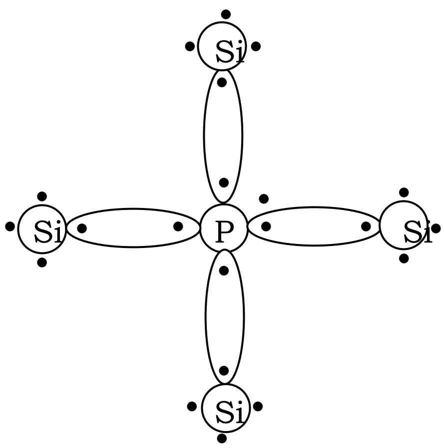
Note: We shall now discuss how the doping changes the number of charge carriers (and hence the conductivity) of semiconductors Si or Ge belongs to fourth group in the periodic table and therefore we choose the dopant element from nearby fifth (5^{th}) or third (3^{rd}) group. Expecting and taking care that the size of the dopant atom is nearly the same as that of Si or Ge. Interestingly the pentavalent and trivalent dopants in Si or Ge give two entirely different types of semiconductors as discussed below.

Extrinsic Semiconductor: The doped semiconductors are called *extrinsic semiconductors.*

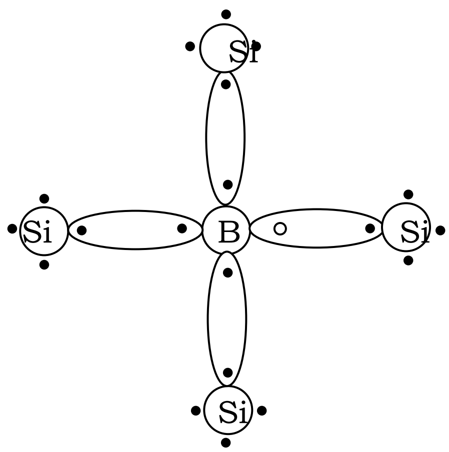
Types of extrinsic semiconductors:

Depending upon the type of impurity added, extrinsic semiconductors are classified into
i) n-type semiconductor and ii) p-type semiconductor

n-type semiconductor: The n-type semiconductor is obtained when a pentavalent impurity is added to pure semiconductor. The pentavalent impurity like phosphorus has five valence electrons. Four of them form covalent bonds with four neighbouring atoms leaving a free electron. Thus every pentavalent impurity atom donates one free electron for conduction. Hence pentavalent impurity atom is called *donar impurity.* The free electrons donated by donar impurity are available for conduction even at 0K. In n-type, number of free electrons are greater than number of holes. Therefore majority charge carries are free electrons and minority charge carriers are holes.



n-type

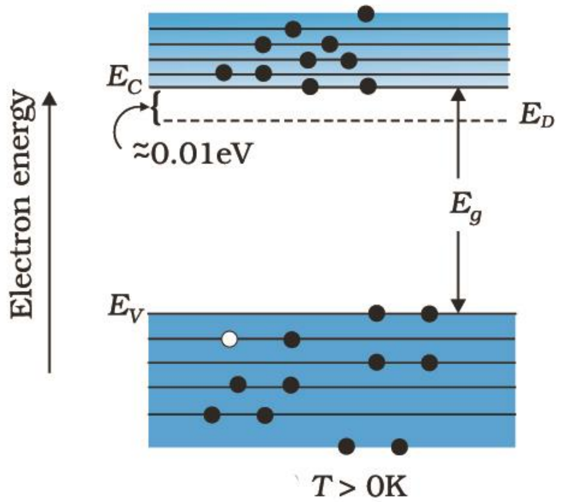


p-type

p -type semiconductor: The p-type semiconductor is obtained when a trivalent impurity is added to pure semiconductor. The trivalent impurity like aluminium has three valence electrons. The three valence electrons form covalent bonds with the neighbouring atoms leaving a hole. Thus every trivalent impurity atom donates one hole for conduction. Hence trivalent impurity atom is called *acceptor impurity.* The holes created by acceptor impurity are available for the conduction even at 0K. In p-type, number of holes are greater than number of free electrons. Therefore majority charge carries are holes and minority charge carriers are free electrons.

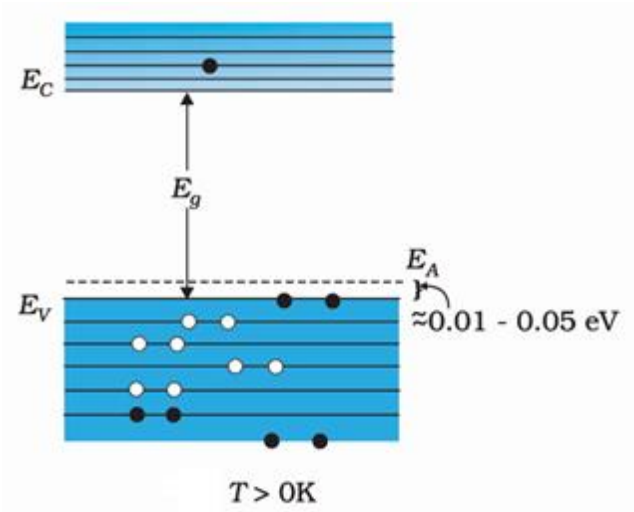
Explain n-type semiconductor on the basis of energy band diagram:

In the energy band diagram of n-type Si semiconductor, the donor energy level E_D is slightly below the bottom E_C of the conduction band and electrons from this level move into the conduction band with very small supply of energy. At room temperature, most of the donor atoms get ionized but very few ($\sim10^{-12}$) atoms of Si get ionized. So the conduction band will have most electrons coming from the donor impurities as shown in fig.



Explain p-type semiconductor on the basis of energy band diagram:

In the energy band diagram of p-type semiconductor, the acceptor energy level E_A is slightly above the top E_V of the valence band as shown in fig.



With very small supply of energy an electron from the valence band can jump to the level E_n and ionize the acceptor negatively (i.e. hole from level E_a sinks down into the valence band. Electrons rise up and holes fall down when they gain external energy).

At room temperature, most of the acceptor atoms get ionized leaving holes in the valence band. Thus the density of holes in the valence band is predominantly due to impurity in the extrinsic semiconductor. The electron and hole concentration in a semiconductor in thermal equilibrium is given by

$$n_e n_h = n_i^2$$

Note-1: for n-type semiconductors, we have

$n_e >> n_h$

Electron density $n=N_D-N_A$, so electrons are majority carriers

Hole density $P = \frac{n_i^2}{n}$, so holes are minority carriers

Where, N_D = Donor density, N_A= Acceptor density

Note-2: For p-type semiconducotrs $n_h >> n_e$.
The electron hole concentration in a semiconductor in thermal equilibrium is given by $n_e n_h = n_i^2$

Hole density, $P=N_A-N_D$, so holes are the majority carriers.

Electron density, $n = \frac{n_i^2}{P}$, so electrons are the minority carriers.

Note-3:

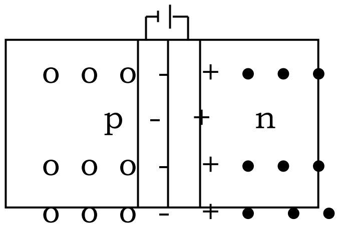
- Intrinsic and extrinsic semiconductors are electrically neutral.
- In p-type, the 'p' means positive, i.e. the majority charge carriers are positive charge carriers i.e. **holes.**
- In n-type, the 'n' means negative, i.e. the majority charge carriers are negative charge carriers i.e. **electrons.**

What is p-n junction?

A single piece of pure semiconductor with one half doped with trivalent impurity to obtain the p-type and other half doped with pentavalent impurity to obtain the n-type. This is called **p-n junction**.

Explain the formation of p-n junction:

Junction potential difference



Depletion region

A single piece of pure semiconductor with one half doped with trivalent impurity to obtain the p-type and other half doped with pentavalent impurity to obtain the n-type. This is called **p-n junction**.

The holes are majority charge carriers in p-type and free electrons are majority charge carriers in n-type. The holes carry positive charge and electrons carry negative charge. Some holes diffuse from p-type to n-type through the junction and some free electrons diffuse from n-type to p-type. A layer of positive and negative immobile charges is formed across the junction. As a result the potential difference is set up across the junction called potential barrier. This stops the further diffusion of immobile charges. The free electrons and holes are absent around the junction. The region around the junction free from mobile charge carrier is called **depletion region or depletion layer**. The thickness of depletion region is about 1 micron or 10^{-6}m. The thickness depends on the concentration of doping. The thickness of depletion layer decreases with increase in impurity concentration. The junction potential difference increases with increase in impurity concentration.

Important terms related to semiconductor diode:

Potential barrier (Junction potential difference): The potential difference across the junction which prevents the further diffusion of holes and free electrons is called potential barrier.

Depletion region (depletion layer): The region around the junction free from mobile charge carrier is called **depletion region or depletion layer**.

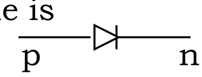
Diffusion current: It is the current due to the concentration difference in the majority charge carriers when the p-n junction is formed.

Drift current: It is the current due to minority charge carriers when electric field is setup by the immobile ions in the depletion region.

Drift : The motion of charge carriers due to the electric field is called drift.

Note-1: The p-n junction is commercially called **semiconductor diode**. For silicon diode junction potential difference is 0.7V. For germanium diode is 0.38V at room temperature.

Note-2: The symbol of semiconducting diode is



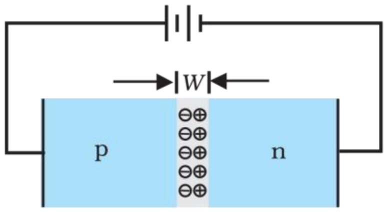
The arrow points from p-side to n-side. The arrow indicates the direction of conventional current.

Biasing: When no external potential difference is applied across the p-n junction, it is said to be **unbiased**. When the p-n junction is connected to an external potential difference, it is said to be **Biased**. The p-n junction can be biased in 2 ways. The Forward Bias and Reverse Bias

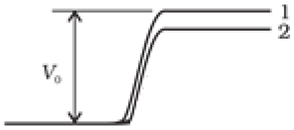
What is forward bias?

A p-n junction is said to be forward Biased, if p-type is connected to positive terminal and n-type is connected to negative terminal of the battery.

Explain how a P-n junction diode behaves under 'FORWARD BIAS" condition.



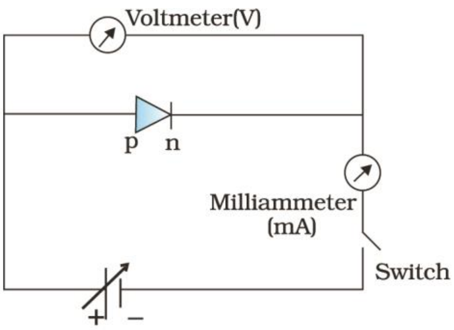
(a)

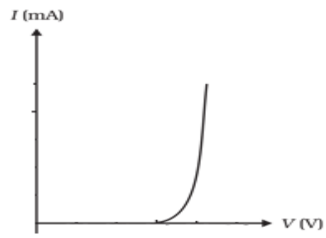


Barrier potential
(1) without battery, (2) battery voltage.

In forward bias P-side is connected to positive terminal of the battery and n-side to the negative terminal. The direction of applied voltage (V) is opposite to the built in potential 'V_0'. As a result, the depletion layer width decreases and the barrier height is reduced as shown in (b). The effective barrier height under bias is (V_0 – V). Now junction resistance becomes almost negligible and thus diode permits the current (order of mA) to flow through it.

Explain with the circuit diagram the working of diode in forward bias characteristic:





In forward bias, positive terminal of a battery is connected to p-type and negative terminal is connected to n-type. During forward Bias the applied voltage opposes the junction voltage, i.e. applied voltage cancels the potential barrier. The junction resistance decreases. The majority charge carries holes and free electrons can cross the junction easily. The current flows through the junction. This current is called **forward current.** The forward current increases with increases in applied voltage.

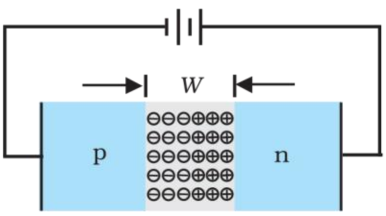
The applied voltage at which the forward current increase rapidly is called **Knee voltage (V_K)**. After Knee voltage the forward bias characteristic is almost linear. The forward bias resistance is very low. For an ideal diode, the forward bias resistance is zero ($R_f = 0$). During forward bias the diode conducts the current to flow through it. The graph of forward current verses forward voltage is called **forward bias characteristic**.

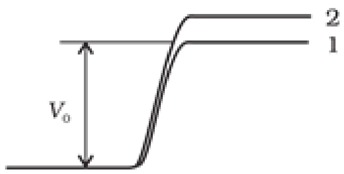
Define Cut in voltage or threshold voltage (knee voltage): It is the minimum forward voltage in which the diode goes to on state (conducting state) with increase in current.
Cut in voltage: For Ge→0.2 to 0.3V, Si→0.6 to 0.7V.

What is reverse bias?

A p-n junction is said to be reverse biased, if p-type is connected to negative terminal and n-type is connected to positive terminal of the battery.

Explain how p-n junction diode behaves under "REVERSE BIAS" condition:



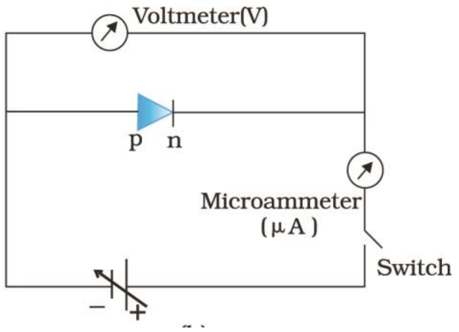


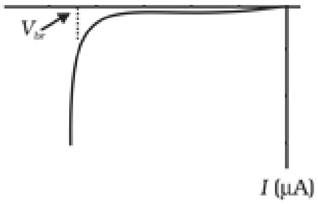
Barrier potential under reverse bias.

In reverse bias n-side is connected to positive terminal and P – side is connected to negative terminal of the battery. The direction of applied voltage is same as the direction of barrier potential. As a result, the barrier height increases and the depletion region widens, a high resistance path is established. The effective barrier height under reverse bias is (V_0 + V) as shown in fig (b). This suppresses the flow of electrons from n→p and holes from p→n. Thus, diffusion current, decreases enormously compared to the diode under forward bias.

The current under reverse bias is essentially voltage independent up to a critical reverse bias voltage, known as breakdown voltage (V_{br}) when $V = V_{br}$, the diode current increases sharply is of the order of micro – amperes (few μA).

Explain with circuit diagram the working of reverse biased characteristic:





In reverse bias positive terminal of a battery is connected to n-type and negative terminal is connected to p-type. During reverse bias the applied voltage adds to the junction voltage. The majority charge carries holes and free electrons cannot cross the junction. However a small current flows through the junction due to minority charge carriers. This current is called **reverse current or leakage current.** At a particular applied voltage the reverse current suddenly becomes maximum, this reverse voltage is called **break down voltage (V_b)**. The reverse resistance is very high. For an ideal diode, the reverse bias resistance is infinity. ($R_r = \infty$) The diode does not conducts during reverse bias. The graph of reverse current verses reverse voltage is called **reverse bias characteristic**.

Reverse saturation current: It is the reverse current which remains constant with increase in reverse bias voltage. It is of the order of 10^{-6}A.

Break down voltage (V_B) : It is the reverse bias voltage at which the reverse current increases sharply.

Dynamic resistance: It is the ratio of small change in voltage ΔV to a small change in current ΔI. i.e. $r_d = \frac{\Delta V}{\Delta I}$.

Note:

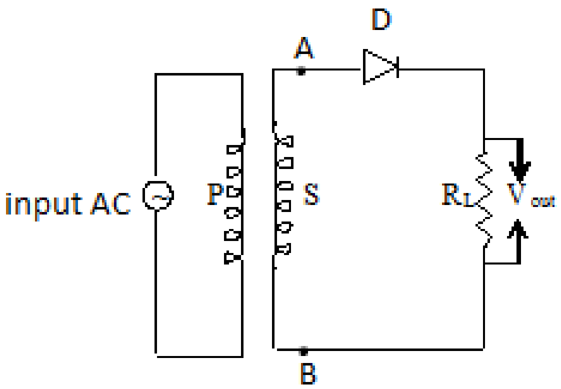
1. The diode characteristic is not a straight line, therefore diode is a non-ohmic device.
2. At breakdown voltage the current is maximum is due to breakdown of covalent bonds. A large number of electron-hole pairs are formed. The high reverse current may damage the junction.
3. The reverse bias resistance is very much greater than forward bias resistance.

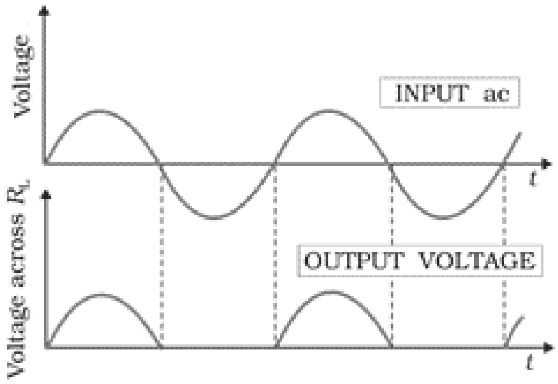
Note-4: Junction diode as a rectifier: From the V-I characteristics of a junction diode we see that it allows current to pass only when it is forward biased. So, if an alternating voltage is applied across a diode the current flows only in the part of the cycle when the diode is forward biased. This property is used to rectify alternating voltages and the circuit used for this purpose is called a rectifier. A device that passes current only in the forward direction and can therefore be used as an a.c. to d.c. converter.

Rectification: The process of converting AC to DC is called rectification.
The device which converts AC to DC is called *rectifier*.

Principle of rectification: Semiconductor diode conducts the current during forward bias and did not conduct the current during reverse bias. i.e. diode has unidirectional conducting property. This is the principle of rectification.

What is half wave rectifier?
The rectifier which converts only half cycles of AC into DC is called half wave rectifier.
Explain with circuit diagram of half-wave rectifier:



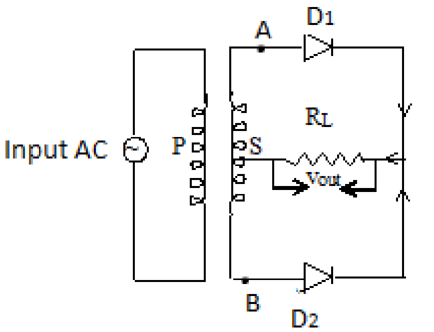


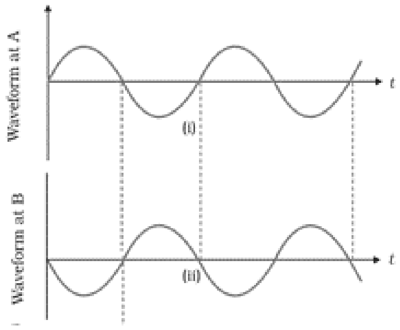
The circuit diagram is as shown in the figure. The AC to be converted in to DC is fed to the primary (p) of the transformer. A diode (D) and load resistor (R_L) are connected to secondary (S) of the transformer. During positive half cycle of AC, the point A is positive potential with respect to point B. then diode is forward biased and it conducts the current, the output is obtained across the load resistance. During the negative half cycles of AC, the point A is negative potential with respect to B. then the diode is reveres biased and it does not conduct the current, the output is not obtained across the load resistance. We get the output only during one-half cycles. Thus it is called *half wave rectifier*. The graph of input AC and output DC is shown in the figure.

What is full wave rectifier?

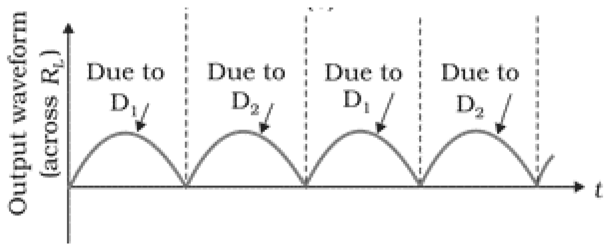
The rectifier which converts both the half cycles of AC into DC is called full wave rectifier.

Explain with a circuit diagram the working of full wave rectifier:





Input AC



The circuit diagram is as shown in the figure. The AC to be converted into DC is fed to the primary (p) of the transformer. Two diodes D_1, D_2 and load resistance R_L are connected to secondary (S) of the centre tapped transformer.

During positive half cycles of AC, the point A is positive w.r.t. B, then the diode D_1 is forward biased and diode D_2 is reverse biased. The diode D_1 conducts the current and diode D_2 does not conducts the current. The output is obtained across the load resistance R_L due to diode D_1. During negative half cycles of AC, the point A is negative w.r.t. B. then the diode D_1 is reverse biased and diode D_2 is forward biased. The diode D_1 doesn't conduct and the diode D_2 conducts the current. The output is obtained across the load resistance R_L due to diode D_2. The output is obtained during both half cycles of AC. Thus it is called *full wave rectifier*. A graph of input 'AC' and output 'DC' is as shown in the figure.

What is the necessity of using filter circuits in a rectifiers?

The rectified voltage is in the form of pulses of the shape of harmonics. Though it is unidirectional it does not have a steady value. To get steady d.c. output from the pulsating voltage normally a capacitor is connected across the output terminals (parallel to the load R_L) one can also use an inductor in series with 'R_L' for the same purpose. Since these addition circuits appear to filter out the a.c. ripple and give a pure d.c. voltage, so they are called filters.

Note-1: Ripple factor: An alternating current component superimposed on a direct current (d.c.) component resulting in the instantaneous value of a unidirectional current or voltage. The term is particularly applied to the output of a rectifier.

The frequency of the a.c. component is the ripple frequency. For a full wave rectifier it is twice the frequency of the input signal.

Note-2: Ripple factor (r) is defined as :

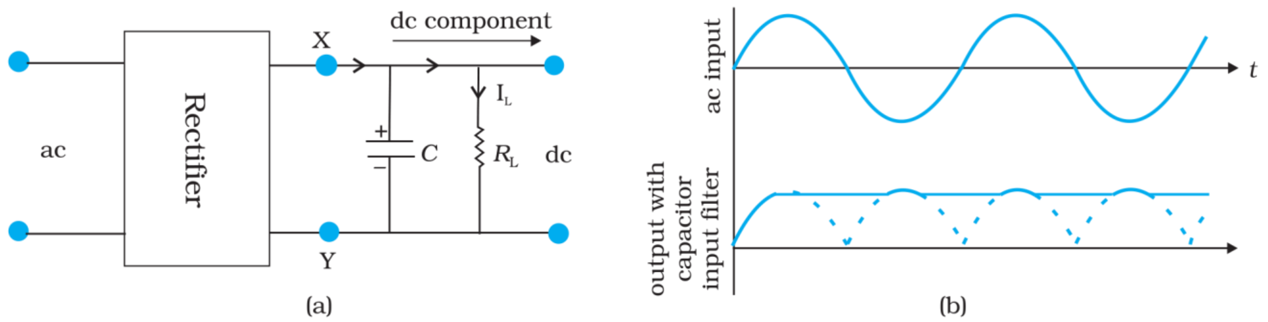
$$r = \frac{\text{r.m.s. value of a.c. components}}{\text{d.c. value of the wave}}$$

$$r = \frac{V_{r(rms)}}{V_{d.c.}} \quad \text{OR} \quad r = \frac{I_{r(rms)}}{I_{d.c.}}$$

For a H.W.R., ripple factor is 1.21

For a FWR, Ripple factor is 0.48

Explain with a neat sketch of an full wave rectifier with capacitor filter:



(a) A full-wave rectifier with capacitor filter, (b) Input and output voltage of rectifier in (a).

The above circuit shall discuss the role of capacitor in filtering. When the voltage across the capacitor is rising, it gets charged. If there is no external load, it remains charged to the peak voltage of the rectified output. When there is a load, it gets discharged through the load and the voltage across it begins to fall. In the next half cycle of rectified output it again gets charged to the peak value. This concept of charging and discharging depends on the values of 'RC' elements. i.e. τ=RC seconds.

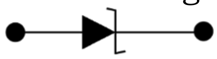
Note: Capacitor blocks DC and allows AC.

What is Zener diode?

A zener diode is a silicon junction diode which is operated under reverse bias and arranged to breakdown when a specific reverse bias voltage applied to it.

It is a special purpose semiconductor diode, named after its inventor C-Zener. Due to its reverse breakdown it is used as a voltage regulator.

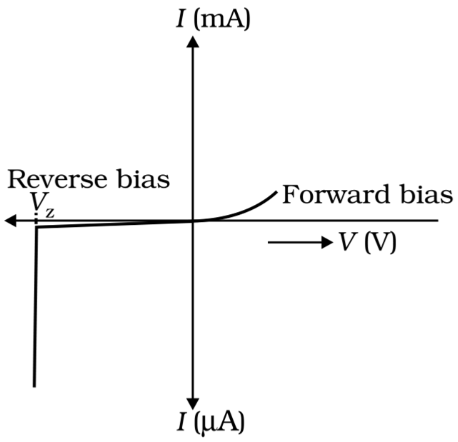
The symbol of zener diode is as shown.



Note: Zener diode is fabricated by heavily doping the both p and n sides of the junction. Due to this, depletion region is very thin ($<10^{-6}$m) and consequently the breakdown of the junction will occur at very low reverse voltage on the other hand, a lightly doped diode has a higher breakdown voltage.

Draw I-V characteristics of zener diode:

The I-V characteristics of a zener diode is shown in figure (b). It is seen that when the applied reverse bias voltage (V) reaches the breakdown voltage 'V_z' of the zener diode, there is a large change in current. In otherwards, zener voltage remain constant, even though the current through the zener diode varies over a wide range. This property of the zener diode is used for regulating supply voltages so that they are constant.



Internal field emission or field ionisation:

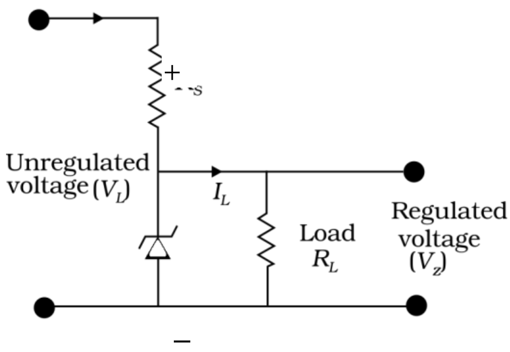
The emission of electrons from the host atoms due to the high electric field is known as internal field emission or field ionisation.

The Electric field required for field ionisation is of the order of 10^6 Vm^{-1}

Note: As the reverse bias voltage is increased, the electric field at the junction becomes significant. When the reverse bias voltage $V=V_z$, then the electric field strength is high enough to pull valence electrons from the host atoms on the p-side which are accelerated to n-side., These account for high current observed at the breakdown.

Explain with a circuit diagram how zener diode acts as a voltage regulator?

The circuit diagram of a voltage regulator using a zener diode as shown in figure.



Zener diode as DC voltage regulator (to be corrected).

The unregulated d.c. voltage is connected to the zener diode through a series resistance 'R_s' such that the zener diode is revere biased. If the input voltage increases, the current through 'R_s' and zener diode also increases. This increases the voltage drop across 'R_s' but voltage drop across the zener diode remains constant. (This is because in the breakdown region, zener voltage remains constant even though the current through the zener diode changes.) Similarly if the input voltage decreases, the current through the 'R_s' and zener diode also decreases. This decreases voltage drop across 'R_s' but the voltage across the zener diode remains constant. Thus any increase or decrease in the input voltage results in, increase or decrease of voltage drop across 'R_s' without any change in the voltage across the zener diode. Thus zener diode acts as a voltage regulator.

Note: We have to select the Zener diode according to the required output voltage and according the series resistance 'R_s'.

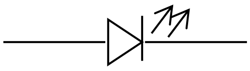
Optoelectronics Junction Devices:

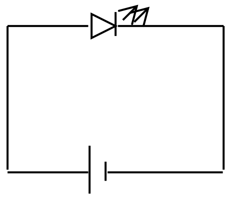
The semiconductor diodes in which charge carriers are generated by photons (photo-excitation) are called optoelectronic devices.

We shall study the functioning of the following optoelectronic devices.

i) Light Emitting Diode (LED).
ii) Photodiode.
iii) Solar Cells (Photovoltaic devices).

Light emitting diode(LED): It is a semiconductor diode which emits light when it is forward biased. It is represented as





Function (Working principle of LED): When the diode is forward biased, electrons are sent from n→p and holes are sent from p→n. At the junction boundary the concentration of minority carriers increases compared to the equilibrium concentration and these carriers recombine with majority carriers. On recombination, the energy is released in the form of photons. Photons with energy equal to or slightly less than the band gap are emitted. When the forward current of the diode is small, the intensity of light emitted is small. As the forward current increases, intensity of light increases and reaches a maximum. Further increase in the forward current results in decrease of light intensity.

LED's are biased such that the light emitting efficiency is maximum.

Note: Energy gap= light energy emitted.

$$E_g = h\nu = \frac{hC}{\lambda}$$

$\therefore \lambda = \frac{hC}{E_g}$ Where, h= Planck's constant, C= Velocity of light, λ= wavelength of emitted light

Applications of LED:

1. It is used in indicator lamp.
2. It is used in remote control for T.V.
3. It is used in traffic signals.
4. It is used in optical fibre communication devices.
5. It is used in calculators, watches.
6. Burglar alarm system.
7. White LED can be used for lighting purpose.

Note-1: LED's that can emit red, yellow, orange, green and blue light are commercially available.

The semiconductor used for fabrication of visible LED must at least have a band gap of 1.8eV.

Spectral range of visible light is from about 0.4μm to 0.7μm i.e., from about 3eV to 1.8eV.

The semiconductor materials used for making LED's are Ga As (Gallium Arsenide), Gallium arsenide phosphide (Ga As P), or Gallium Phosphide (Ga P)

LED's made from Ga As emit infrared (invisible) radiation (Eg ~1.4eV).

LED's made from Ga As P provides either red light or yellow light (Eg ~1.9eV).

While red or green emission can be produced by using Ga P.

Note-2: To make LED's, Ge and Si semiconductor material are not used because they are poor emitter of light

Note-3: Reverse breakdown voltages of LED's are very low, typically around 5V.

Advantages of LED over incandescent lamps:

LED's have the following advantages over conventional incandescent low power lamps:

i) Low operational voltage or less power.
ii) Fast action and no warm-up time required.
iii) Long life and ruggedness.
iv) Fast on-off switching capability
v) Emitted light is monochromatic (band width is 100Å to 500 Å)

Note: Semiconductor Laser diode (SLD): It is specific type of LED which emits coherent light as in laser. It works on the principle of laser.

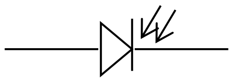
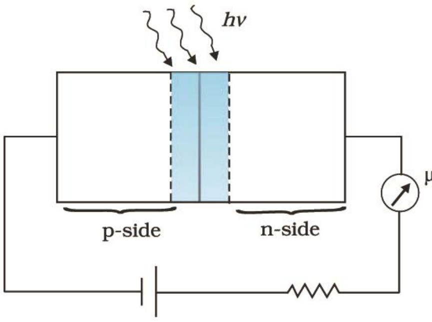
Photo diode: It is a semiconductor diode which produce electric current when light is incident on it. It is represented as 

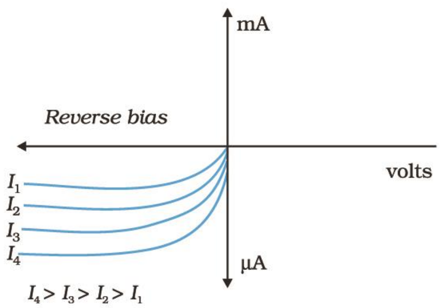
Photo diode is operated in reverse biased condition.

Function (Working principle of photodiode):

When the photodiode is illuminated with light (photons) with energy (hγ) greater than the energy gap (Eg) of the semiconductor, then electron – hole pairs are generated due to the absorption of photons. The diode is fabricated such that the generation of electron-hole pairs takes place in or near the depletion region of the diode. Due to electric filed of the junction, electrons and holes are separated before they recombine. Electrons are collected on n-side and holes are collected on p-side giving rise to an emf. When an external load is connected, current flows. The magnitude of the photocurrent depends on the intensity of incident light. I-V characteristics of a photo diode for illumination intensity is as shown.



(a)



(b)

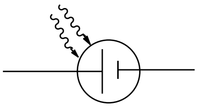
Note: Photo electric current is directly proportional to intensity of light.

Applications:

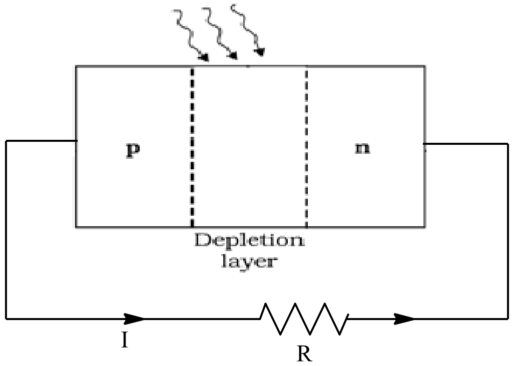
1) It is used to detect both visible, invisible light and detect optical signals.
2) It is used in logic circuits.
3) It is used to measure intensity of light.
4) It is used as light meter in camera.
5) It is used in optical fibre communication.

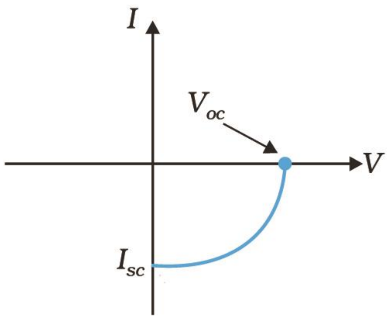
Solar cell:

i) **Solar Cell:** A solar cell is basically a P-n junction which generates e.m.f. when solar radiation falls on the P-n junction.

A circuit symbol of solar cell  is

Working principle of solar cell:





It works on the principle of photovoltaic effect and bias is not applied. When radiation of suitable energy falls on the diode, the electron-hole pairs are generated. The free electrons reaching the n-side are collected by the front contact and holes reaching P-side are collected by the back contact. Thus P-side becomes positive and n-side becomes negative giving rise to photo voltage. When an external load is connected as shown in figure a photo current I flows through the load resistance R. I-V characteristics of solar cell is a shown.

Uses: Solar cells are used in satellites and space vehicles and also power supply to some calculators, other domestic purposes, used to charging storage batteries, etc.

The important criteria for the selection of a material for solar cell fabrication:

(i) Band gap (~1.0 to 1.8eV) (ii) high optical absorption (~$10^4 cm^{-1}$) (iii) electrical conductivity, (iv) Availability of the raw material and (v) cost.

Note: Sunlight is not always required for a solar cell. Any light with photon energies greater than the bandgap will do.

Note: Solar cells are made with semiconductors like Si (Eg ≈ 1.1eV), Ga As (Eg ≈ 1.43eV), Cd Te (Eg : ≈ 1.45eV), $CuInSe_2$ (Eg=1.04eV) etc.

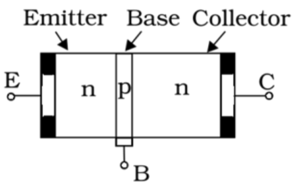
What is a transistor?

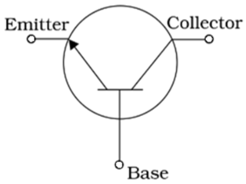
Transistor is a three terminal (Emitter base and collector), three layer and two p-n junctions semiconductor device.

Classification of transistor into two types:

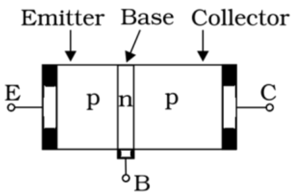
1) N-P-N transistor and 2) P-N-P transistor

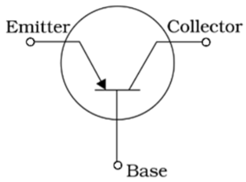
1) N-P-N transistor: Here two segments of n-type semiconductor (emitter and collector) are separated by a segment of p-type semiconductor (termed as base) as shown in figure.





2) P-N-P transistor: Here two segments p-type semiconductor (termed as emitter and collector) are separated by a segment of n-type semiconductor (termed as base) as shown in figure.





How in a transistor i.e., E, B, and C are doped and also differs in their size of the segments?

a) Emitter: It has a moderate size and heavily doped. It supplies a large number of majority carriers for the current flow through the transistor.

b) Base: This is the central segment. It is very thin and lightly doped. It transfers majority charge carriers from emitter to collector.

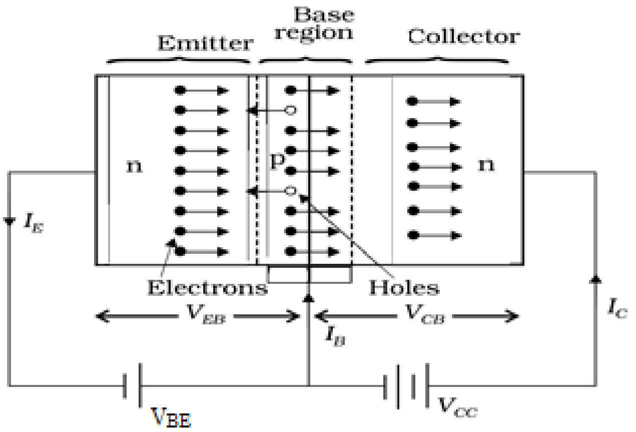
c) Collector: The collector is moderately doped and larger in size as compared to the emitter. The collector collects a major portion of a majority carriers supplied by the emitter.

Doping level: Emitter > collector >Base.
Thickness: Collector > Emitter > Base

Note: In normal mode of operation of transistors, emitter-base region is forward bias and collector-base region is reverse biased.

Explain with a neat sketch the working principle of an n-p-n transistor?

In normal mode of operation of transistors, emitter-base region is forward bias and collector-base region is reverse biased. The electrons are majority charge carriers in n-type and holes are majority charge carriers in p-type. The emitter injects free electrons into the base. This constitutes emitter current (I_E). A few free electrons (about 5%) undergo recombination with holes in the base region. This constitutes base current (I_B). The remaining free electrons (about 95%) move towards collector and they are collected by collector.



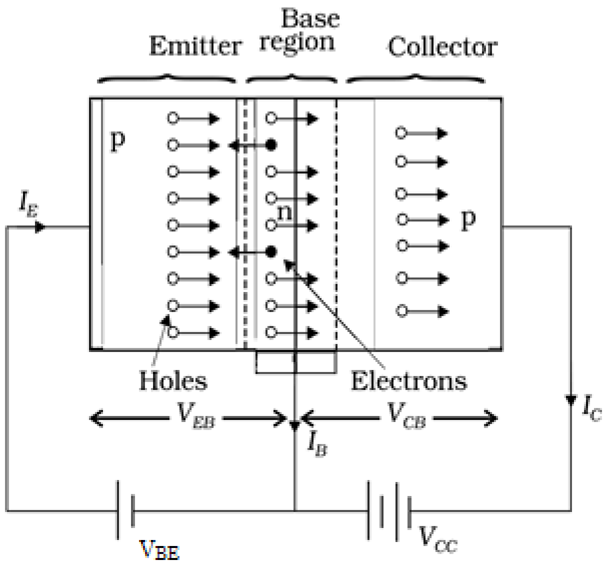
This constitutes collector current (I_C).

$$\therefore \mathbf{I_E = I_B + I_C}$$

But I_B is very small because base is lightly doped. Therefore I_E is nearly equal to I_C.

Thus a transistor transfers current from forward biased region to reverse bias region without much difference, i.e from low resistance region to high resistance region without much difference. This is the action of transistor

Explain with a neat sketch the working principle of p-n-p transistor:



In normal mode of operation of transistors, emitter-base region is forward biased and collector-base region is reverse biased. The electrons are majority charge carriers in n-type and holes are majority charge carriers in p-type. The emitter injects holes into the base. This constitutes emitter current (I_E). A few holes (about 5%) undergo recombination with free electrons in the base region. This constitutes base current (I_B). The remaining holes (about 95%) move towards collector and they are collected by collector. This constitutes collector current (I_C).

$$\therefore \mathbf{I_E = I_B + I_C}$$

But I_B is very small because base is lightly doped. Therefore I_E is nearly equal to I_C.

Thus a transistor transfers current from forward biased region to reverse bias region without much difference, i.e from low resistance region to high resistance region without much difference. This is the action of transistor.

Note:

1. In npn-transistor free electrons are majority charge carrier
2. In pnp-transistor holes are majority charge carriers.

List out the basic transistor circuit configurations:

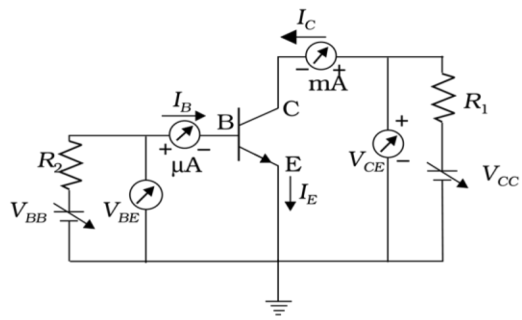
In a transistor, only three terminals are available, i.e. emitter (E), base (B) and collector (C), therefore in a circuit the input and output connections have to be such that one of these (E, B or C) is common to both input and the output. Accordingly, the transistor can be connected in either of the following three configurations.

1) Common emitter configuration (CE-mode)
2) Common base configuration (CB-mode) and
3) Common collector configuration (CC-mode)

Note: The transistor is most used in the 'CE' configuration and we are restricted our discussion to only this configuration. Since more commonly used transistors are n-p-n transistors, we shall confine our discussion to such transistors only. With p-n-p transistors the polarities of the external power supplies are to be inverted.

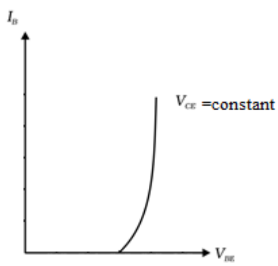
Explain with circuit diagram to plot input and output characteristics in common emitter (CE) mode configuration:

When a transistor is used in CE configuration the input is between the base and the emitter and the output is between the collector and the emitter. The variation of the base current 'I_B' with the base emitter voltage 'V_{BE}' is called the input characteristic.



Similarly, the variation of the collector current 'I_C' with the collector –emitter voltage 'V_{CE}' is called the output characteristic.

Input characteristic of a transistor in CE-mode: It is a graph between base current 'I_B' and base to emitter voltage (V_{BE}) at constant collector to emitter voltage 'V_{CE}'.

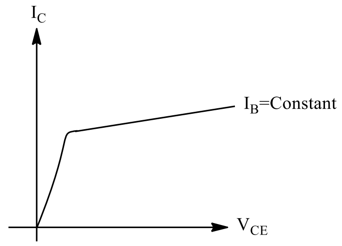


Note:

1. For Si-transistor 'V_{BE}' is 0.6 to 0.7 volts, 'V_{CE}' must be sufficiently larger than 0.7 volts.
2. In 'CE' arrangement the emitter is common to both input and output circuits.

The output characteristics of a transistor in CE mode:

It is graph of collector current (I_C) and collector to emitter voltage (V_{CE}) at constant base current (I_B).



Define the terms w.r.t. CE-mode:

i) Input resistance (r_i): It is the ratio of change in base to emitter voltage (ΔV_{BE}) to the change in base current (ΔI_B) at constant collector to emitter voltage (V_{CE}).

$$\therefore \quad r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right) V_{CE} \text{ constant}$$

The value of 'r_i' can be anything from a few hundreds to a few thousand ohms.

ii) Output resistance (r_o): It is the ratio change in collector to emitter voltage ΔV_{CE} to the change in collector current 'ΔI_C' at constant base current 'I_B'.

$$\therefore \quad r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B \text{ constant}}$$

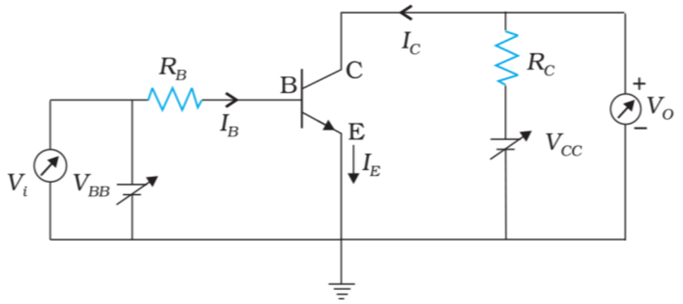
iii) Current amplification factor (β) [Current gain]: The ratio of change in the collector current (ΔI_C) to the change in the base current (ΔI_B) at a constant 'V_{CE}', when the transistor is in active state.

$$\therefore \quad \beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right) V_{CE} \text{ Constant}$$

$$\& \quad \beta_{dc} = \frac{I_C}{I_B}$$

Note: β_{ac} is called small signal current gain.

Explain with a circuit diagram how transistor works as a switching device?



We shall try to understand the operation of the transistor as a switch by analysing the behaviour of the base biased transistor in CE configuration as shown in figure.

Applying Kirchoff's voltage rule to the input and output sides of this circuit, we get

$V_{BB} = I_B R_B + V_{BE}$ ---------- (1) (input side)

And

$V_{CC} = I_C R_C + V_{CE}$

$V_{CE} = V_{CC} - I_C R_C$ ---------- (2) (output side)

We shall treat 'V_{BB}' as the d.c. input voltage 'V_i' and 'V_{CE}' as the d.c. output voltage 'V_o'

i.e. $V_i = V_{BB}$ and $V_o = V_{CE}$

then, equation (1) and (2) becomes

$V_i = I_B R_B + V_{BE}$ and

$V_o = V_{CC} - I_C R_C$

Let us see how 'V_o' changes with 'V_i' increases from zero onwards. In the cases of silicon transistor, as long as input 'V_i' is less than 0.6 volts, the transistor will be in cut off state and current 'I_C' will be zero.

i.e. $V_o = V_{CC} - I_C R_C$ $\quad [V_i << 0.6V,\ I_C = 0]$

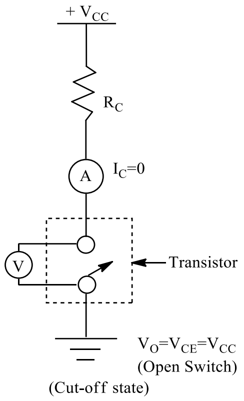
then $V_o = V_{CC} \rightarrow$ in this condition the transistor is switched off,

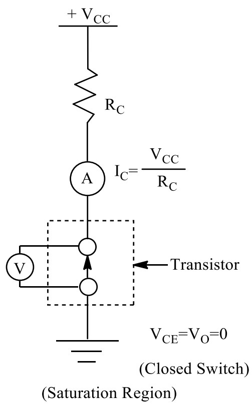
If, $V_i > 0.6V$, $I_C > 0$, then

$V_o = 0 \rightarrow$ in this condition the transistor is in on state. i.e. saturation region.

Note-1: A transistor can be used as a switch if it is operated between cut off state and saturation region.

Note-2: Let us see now how the transistor is operated as a switch. As long as 'V_i' is low and unable to forward bias the transistor, 'V_0' is high (at V_{CC}). If 'V_i' is high enough to drive the transistor into saturation, then 'V_0' is low, very near to zero. When the transistor is non-conducting it is said to be switched off and when it is driven into saturation it is said to be switched 'ON'. Thus low input switches the transistor off and a high input switches it on.

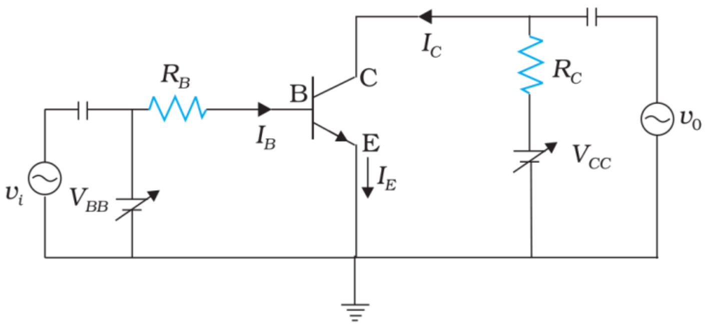


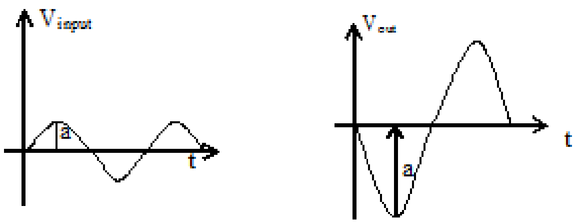


Note-3: A low input to the transistor gives a high output and a high input gives a low output.

Explain with circuit diagram the working of transistor as an Amplifier in CE-configuration:

In general, amplifiers are used to amplify alternating signals. Now let us superimpose an a.c. input signal (V_i) to be amplified on the bias V_{BB} (d.c.) as shown in figure. The output is taken between the collector and the ground.





Input wave form

Output wave form

The working of an amplifier can be easily understood, if we first assume that $V_i = 0$.

Then applying kirchoff's law to the output loop, we get.

$$V_{CC} = V_{CE} + I_C R_C \quad \text{------------} \; (1)$$

Like that, the input loop gives

$$V_{BB} = V_{BE} + I_B R_B$$

When V_i is not zero, we get

$$V_{BB} + \Delta V_i = V_{BE} + I_B R_B + \Delta I_B (R_B + r_i)$$

$$\Delta V_i = \Delta I_B (R_B + r_i)$$

$$\Delta V_i = r \Delta I_B \qquad (r = R_B + r_i)$$

$$\therefore \quad \Delta I_B = \frac{\Delta V_i}{r}$$

The change in I_B causes a change in I_C, we define a parameter β_{ac}, which is similar to the β_{dC}, defined in previous section as

$$\beta_{ac} = \frac{\Delta i_c}{\Delta i_b}$$

$$\therefore \Delta i_c = \beta_{ac} \Delta i_b$$

The change in 'I_C' due to a change in I_B causes a change in V_{CE}

These changes can be given by equation (1) as

$$\Delta V_{CC} = \Delta V_{CE} + R_C \Delta i_c$$

But $\Delta V_{CC}=0$ Because V_{CC} is fixed

$\therefore$ $0=\Delta V_{CE} + R_C \Delta i_c$

$$\Delta V_{CE} = - R_C \Delta i_c$$

But $\Delta V_{CE}=\Delta V_0$

Then $\Delta V_0 = -R_C \beta_{ac} \Delta i_b$.

The voltage gain of the amplifier is

$$A_V = \frac{\Delta V_O}{\Delta V_i}$$

$$A_V = \frac{-\beta ac \Delta I_B R_C}{r \Delta I_B}$$

$$A_V = \frac{-\beta ac R_C}{r}$$

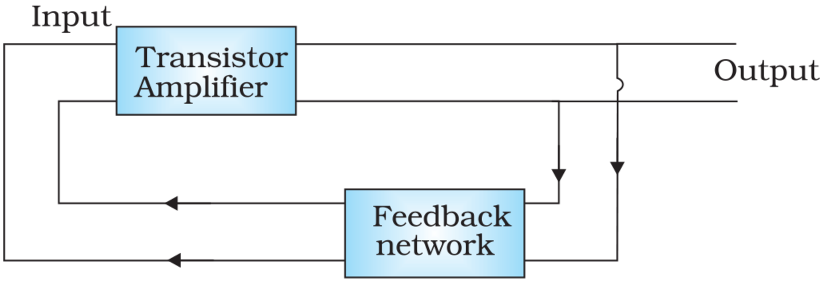
The negative sign represents that output voltage is opposite with phase with the input voltage.

We know that current gain (β_{ac}) and voltage gain (A_V). Therefore power gain 'A_P' can be expressed as the product of the current gain and voltage gain. Mathematically $A_P = \beta_{ac} \times A_V$

Since β_{ac} and A_V are greater than '1', we get a.c. power gain.

Oscillator: It is a device used to reduce undamped electromagnetic oscillations.

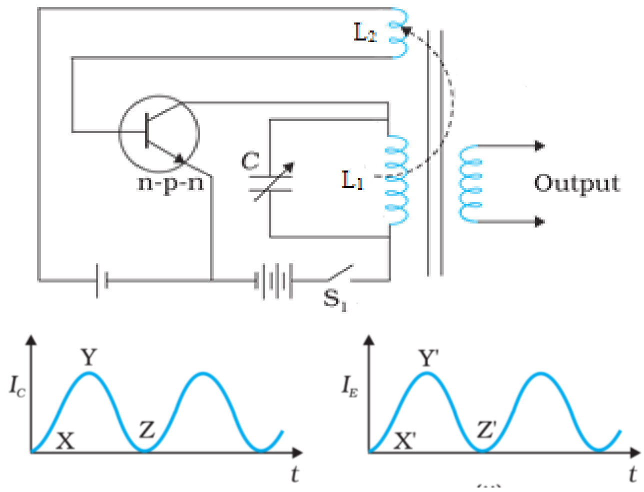
The block diagram of principle of transistor amplifier with positive feedback as an oscillator:



Explain the principle of transistor amplifier with positive feedback as an oscillator:

In transistor oscillator, a portion of the output is returned back to the input in phase with the input power. This process is called as positive feedback. Thus a transistor is regarded as a self sustained transistor amplifier oscillator with a positive feedback.

Explain with a circuit diagram of transistor oscillator using feedback amplifier or Tuned collector oscillator?



The figure shows the circuit of tuned collector oscillator. It contains L_1 and C in the collector. The frequency of oscillations depends upon the values of L_1 and C and is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \longrightarrow (1)$$

The feedback coil L_2 in the base circuit is magnetically coupled to the tank circuit coil L_1. In practice L_1 and L_2 form the primary and secondary of the transformer.

Circuit operation: When the switch 'S_1' is closed, collector current starts increasing and charges the capacitor 'C'. When this capacitor is fully charged, it discharges through coil L_1, setting up oscillations of frequency given by equation (1). These oscillations induce some voltage in the coil L_2 by mutual induction. The frequency of voltage in the coil L_2 is same as that of tank circuit but its magnitude depends up on the number of turns L_2 and coupling between L_2 and L_1.

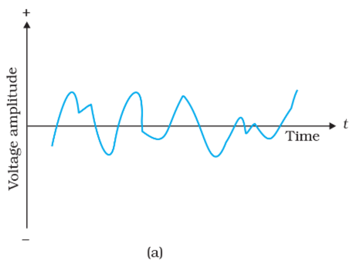
A phase shift of 180^0 is created between voltages of $L_1(T_2)$ and $L_2(T_1)$ due to transformer action. A further phase shift of 180^0 takes place between base-emitter and collector circuit due to transistor properties. As a result, energy feedback to the tank circuit is in phase with generated oscillations.

Note:

1) An Amplifier uses negative feedback
2) An Oscillator uses positive feedback
3) LC circuit is called tank or tuned circuit.
4) Tuned circuit is connected in the collector side, hence it is called tuned collector oscillator.

Digital electronics and Logic gates:

Analogue signal: The signal which has continuous, time varying VOLTAGE or Current is called analogue signal.



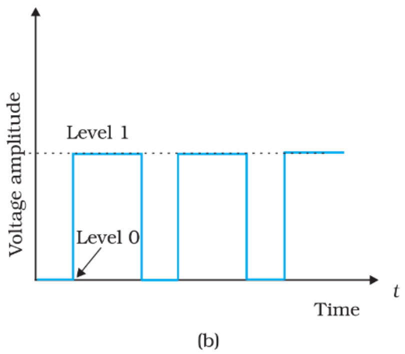
(a) Analogue signal,

Ex: Voice Signal.

Digital Signal: The signal which has discrete values of voltage or current w.r.t. time is called digital signal.

Ex: Square wave or Pulse wave form.

The figure below shows pulse wave form in which only discrete values of voltage is possible.



(b) Digital signal.

It is convenient to use binary numbers to represent such signals. A binary number has only two digits '0' (say '0' volts) and '1' (say +5volts). In digital electronics, we use only two levels of voltage as shown in figure (b) such signals are called digital signals. In digital circuits only two values (represented by '0' or '1') of the input and output voltage are permissible.

Digital electronics: The branch of physics which deals about digital circuits and digital signals is called **digital electronics**.

Note: There are two levels in digital circuit. They are 0 and 1 or OFF and ON or low and high. George Boole developed the algebra for the working of digital circuits called **Boolean algebra**. It is based on binary system. It uses two digits 0 and 1. The three basic operations in Boolean algebra are OR, AND and NOT.

Logic Gates: A logic gate is a digital circuit that follows certain logical relationship between the input and output voltages.

Logic gates are used in calculators, digital watches, computers, robots, Industrial control systems, in telecommunications etc.,

Types of Basic gates: There are three basic logic gates. They are
1) OR-gate, 2) AND-gate and 3) NOT-gate.

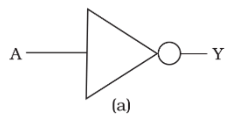
Note: Logic gate has two or more inputs and only one output (except for a NOT-GATE- it has only one input and a output). Therefore, they are generally known as logic gates.

Truth table: It is a list of all the possible input logic level combinations with their respective output logic levels.

i) **NOT-GATE (INVERTER)**: The logic gate which produces the output 1 if the input is 0 and vice-versa.

That is, it produces an inverted version of the input at its output. This is why it is also known as an inverter. This is the most basic gate, with one input and output.

Symbol of NOT-GATE:



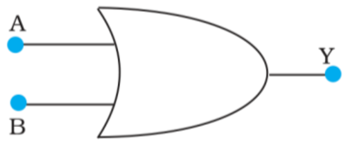
Boolean equation for NOT-gate is $Y = \bar{A}$,
(it is read as Y= NOT A)
where, A=input, Y=output.

Input	Output
A	Y
0	1
1	0

Truth table

ii) **OR-GATE**: The logic gate in which the output is 1when any one of the input or both the inputs are 1's is called OR-Gate.
An OR-Gate has two or more inputs with one output.

Symbol of OR-GATE:



Truth table:

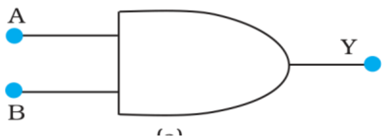
Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Boolean equation of OR-Gate is Y=A+B, (it is read as, Y=A OR B)
where, A & B=inputs, Y=output.

iii) **AND GATE**: The logic gate in which output is 1 when all the inputs are 1.

An AND-GATE has two or more inputs and one output.

Symbol of AND-GATE:



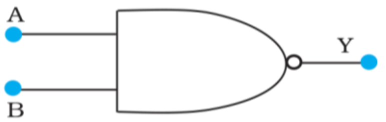
Truth table:

Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Boolean equation of AND gate is $Y=A \cdot B$, (it is read as, Y=A AND B) where, A & B=inputs, Y=output.

iv) **NAND-GATE:** The NAND-Gate is an AND-gate followed by a NOT-gate. If inputs 'A' and 'B' are both 1 (high) the output is 0 (low)

Symbol of NAND-GATE:



Truth table:

Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

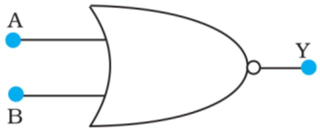
Boolean equation of NAND-Gate is $Y=\overline{A \cdot B}$, (it is read as Y=Not A and B)

where, A & B=inputs, Y=output.

NAND- gates are also called universal gates since by using these gates you can realise other basic gates like AND, OR, NOT gates etc.

v) **NOR-gate**: NOR-Gate is an OR-gate followed by NOT-gate. Its output is 1(high) only when both inputs 'A' and 'B' are 0 (low) and when any one of the input is 1 (high) then output is 0 (low). It has two or more inputs and one output.

Symbol of NOR-GATE:



Truth table:

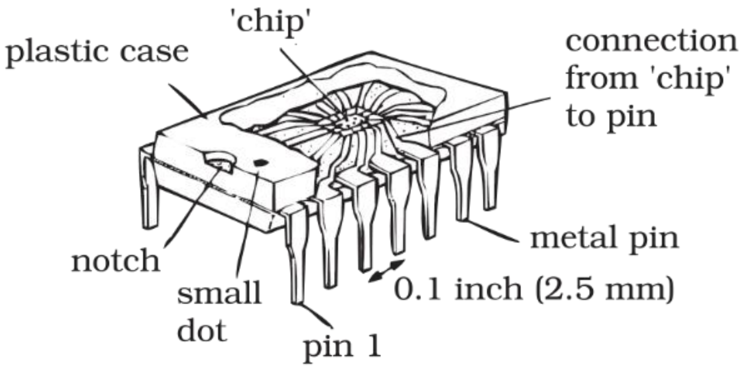
Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Boolean equation of NOR-Gate is $Y=\overline{A+B}$, (it is read as Y=Not A or B)

NOR-gates are considered as universal gates because you can obtain all the gates like AND, OR, NOT gates etc using only NOR-gates.

INTEGRATED CIRCUITS ('IC'):

The fabrication of an entire circuit (Consisting of many passive components like 'R' and 'C' and active devices like diodes and transistors) on a small single block or chip of a semiconductor using photographic techniques is known as integrated circuit (IC).



The most widely used technology is the Monolithic IC. The word monolithic is a combination of two Greek words, monos means single and lithos means stone. This in effect means that the entire circuit is formed on a single silicon crystal (or chip). The chip dimensions are as small as 1mm × 1mm or it could even be smaller.

IC's can be grouped in two categories:

i) **Linear or Analogue IC's** : The linear IC's process analogue singals i.e., it varies linearly with the input.
   One of the most linear IC's the operational amplifier (OPAMP)
ii) The digital IC's process the digital signals that have only two values. They contain circuits such as logic gates.

Classification of Integrations on the basis of number of circuit components or Logic gates:

a) Small scale Integration (SSI) ≤ 10 logic gates per chip.
b) Medium scale integration (MSI) ≤ 100 gates per chip
c) Large scale integration (LSI) ≤ 1000 gates per chip and
d) Very large scale integration (VLSI) > 1000 gates per chip.

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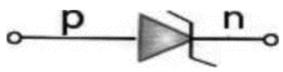
ONE MARK QUESTIONS

1. What is an electronic device?
   Ans. It is a device in which controlled flow of electrons takes place either in vacuum or in semiconductors.
2. What is an energy band in a solid?
   Ans. Energy band is a group of close by energy levels with continuous energy variation.
3. What is a valence band?
   Ans. Valence band is the energy band which includes the energy levels of the valence electrons. It is the range of energies possessed by valence electrons.
4. What is conduction band?
   Ans. Conduction band is the energy band which includes the energy levels of conduction electrons or free electrons.
5. What is energy gap or energy band gap?
   Ans. The gap (spacing) between the top of the valence band (E_V) and the bottom of the conduction band (E_c) is called the energy band gap (E_g) or energy gap.
6. What is the order of energy gap in a semiconductor?
   Ans. 1eV
7. At what temperature would an intrinsic semiconductor behave like a perfect insulator?
   Ans. 0 K (absolute zero temperature)
8. What is an intrinsic semiconductor?
   Ans. It is a pure semiconductor in which electrical conductivity is solely due to the thermally generated electrons and holes.
9. What is doping?
   Ans. The process of adding suitable impurity atoms to the crystal structure of pure semiconductor like Ge or Si to enhance their electrical conductivity is called doping.
10. What is a hole?
    Ans. The vacancy of an electron(of charge -e) in the covalent bond with an effective positive charge +e is called a hole.
11. What is an extrinsic semiconductor?
    Ans. The semiconductor obtained by doping a pure semiconductor like silicon with impurity atoms to enhance its conductivity is called an extrinsic or doped semiconductor.
12. Name one dopant which can be used with germanium to form an n-type semiconductor.
    Ans. Phosphorus.
13. What are dopants?
    Ans. The impurity atoms added to pure semiconductors like germanium to increase their electrical conductivity are called dopants.
14. Name the majority charge carriers in p-type semiconductors.
    Ans. Holes.

15. What is depletion region in a p-n junction?
    Ans. The space charge region at the p-n junction which consists only of immobile ions and is depleted of mobile charge carriers is called depletion region.
16. How does the width of the depletion region of a p-n junction change when it is reverse biased? Ans. The depletion region width increases.
17. What is the forward resistance of an ideal p-n junction diode?
    Ans. Zero.
18. Draw the circuit symbol of a semiconductor diode.
19. Name any one optoelectronic device.
    Ans. Photodiode / Light emitting diode / photovoltaic cell or solar cell.
20. Draw the circuit symbol of a Zener diode.
    Ans.
    

21. What is rectification?
    Ans. The process of converting AC (alternating current) to pulsating DC is called rectification.
22. How does the conductivity of a semiconductor change with rise in its temperature?
    Ans. The conductivity increases exponentially with temperature.
23. Is the ionisation energy of an isolated free atom different from the ionization energy E_g for the
    atoms in a crystalline lattice?
    Ans. Yes. It is different since in a periodic crystal lattice each bound electron is influenced by many
    neighbouring atoms.
24. Which process causes depletion region in a p-n junction?
    Ans. The diffusion of majority charge carriers i.e., free electrons and holes across the p-n junction
    causes the depletion region.
25. What is the order of the thickness of the depletion layer in an unbiased p-n junction?
    Ans. micrometer (10^{-6} m).
26. What is a photodiode?
    Ans. It is a special purpose p-n junction diode whose reverse current strength varies with the
    intensity of incident light.
27. Under which bias condition a Zener diode is used as a voltage regulator?
    Ans. Reverse bias.
28. Draw the circuit symbol of an npn transistor.
    change in base current at constant collector-emitter voltage.
29. What kind of biasing will be required to the emitter and collector junctions when a transistor is used as an amplifier?
    Ans. Emitter-base junction is forward biased while collector-base junction is reverse biased.

30. Which region of the transistor is made thin and is lightly doped?
    Ans. Base.
31. Define current gain or current amplification factor of transistor in CE mode.
    Ans. The current gain (P) is defined as the ratio of change in collector current to corresponding

32. What is an oscillator?
Ans. It is an electronic device which is used to produce sustained electrical oscillations of constant frequency and amplitude without any external input.
33. What type of feedback is used in an oscillator?
Ans. Positive feedback.
34. What is an analogue signal?
Ans. An electrical signal (current or voltage) which varies continuously with time is called an analogue signal.
35. What is a digital signal?
Ans. A signal (current or voltage) which takes only discrete values is called digital signal.
36. What is a logic gate?
Ans. A logic gate is a digital circuit that follows certain logical relationship between the input and output signals and works on the principles of Boolean algebra.
37. Draw the logic symbol of an OR gate.
38. Write the truth table for a NOT gate.

TWO MARKS QUESTIONS

1. Name the charge carriers in the following at room temperature: (i) conductor (ii) semiconductor.
2. Mention the necessary conditions for doping.
3. Name one impurity each, which when added to pure Si produces (i) n-type and (ii) p-type semiconductor.
4. Give two differences between intrinsic and extrinsic semiconductors.
5. Give two differences between n-type and p-type semiconductors. Ans.
6. What happens to the width of the depletion layer of a p-n junction when it is (i) forward biased? (ii) reverse biased?
7. Draw a labelled diagram of a half wave rectifier. Draw the input and output waveforms.
8. What is photodiode? Mention its one use. (2015-A)
9. Draw a labelled diagram of a full wave rectifier. Draw the input and output waveforms. Ans.
10. Zener diodes have higher dopant densities as compared to ordinary p-n junction diodes.
11. How does it effect: (i) the width of the depletion layer? (ii) the junction field?
12. Explain why a photodiode is usually operated under reverse bias.
13. What is an LED? Mention two advantages of LED over conventional incandescent lamps.

14. Mention the factor which determines the (i) frequency and (ii) intensity of light emitted by LED. Ans. (i)
15. Give two operational differences between light emitting diode (LED) and photodiode.
16. What is a transistor? Draw the circuit symbol of pnp transistor.
17. Draw input characteristics of a transistor in CE mode and define input resistance.
18. Draw output characteristics of a transistor in CE mode and define output resistance.
19. Draw the transfer characteristics of base-biased transistor in CE configuration and indicate the regions of operation when transistor is used as (i) an amplifier (ii) a switch.
20. Draw the logic symbol of AND gate and write its truth table.
21. Draw the logic symbol of NOR gate and write its truth table.
22. Draw the logic symbol of NAND gate and write its truth table.

THREE MARKS QUESTIONS

1. What is intrinsic semiconductor? Explain the formation of a hole in the covalent bond structure of a Ge crystal.
2. Which electrical conductivity is solely dependent on thermally generated charge carriers. Example: pure crystals.
3. How is an n-type semiconductor formed? Name the majority charge carriers in it. Draw the energy band diagram of an n-type semiconductor.
4. How is a p-type semiconductor formed? Name the majority charge carriers in it. Draw the energy band diagram of a p-type semiconductor.
5. Distinguish between n-type and p-type semiconductors on the basis of energy band diagrams.
6. Explain the formation of the depletion region in a p-n junction. How does the width of this region change when the junction is (i) forward biased? and (ii) reverse biased?
7. When an electron diffuses from n to p-region, it leaves behind an immobile positive ion (donor ion) on the n-side. As electrons continue to diffuse a layer of positive charge (positive space charge region) develops on the n-side of the junction.
8. What is forward bias? Draw a circuit diagram for the forward biased p-n junction and sketch the voltage-current graph for the same.
9. What is reverse bias? Draw a circuit diagram for the reverse biased p-n junction and sketch the voltage-current graph for the same.
10. With the help of a circuit diagram explain the use of Zener diode as a voltage regulator.
11. What is a transistor? Describe the various regions of a transistor.
12. Describe briefly with the help of a circuit diagram, the paths of current carriers in an npn transistor with emitter-base junction forward biased and collector-base junction reverse biased.

13. Draw a circuit diagram of a transistor amplifier in the common-emitter configuration. Briefly explain, how the input and output signals differ in phase by 180°.
14. When an AC signal is fed to the input circuit, its positive half cycle increases the forward bias of the circuit which, in turn, increases the emitter current and hence the collector current.
15. Draw the transfer characteristic curve of a base biased transistor CE configuration. Explain how the active region of the V_o versus V_i curve is used for amplification.
16. What is a logic gate? Draw the symbol of a NOT gate and write its truth table.

FIVE MARKS QUESTIONS

1. What is energy band? On the basis of energy band diagrams, distinguish between metals, insulators and semiconductors.
2. Distinction between metals, insulators and semiconductors:
3. What are intrinsic semiconductors? Explain the formation of a hole in an intrinsic semiconductor.
4. What is extrinsic semiconductor?
5. What is a p-n junction? Explain the formation of the depletion region in a p-n junction. How does the width of this region change when the junction is (i) forward biased? (ii) reverse biased?
6. Draw the circuit diagrams of a p-n junction diode in (i) forward bias and (ii) reverse bias. Draw the I-V characteristics for the same and discuss the resistance of the junction in both the cases.
7. With a neat circuit diagram, explain the working of a half wave rectifier employing a semiconductor diode. Draw the relevant waveforms.
8. With a neat circuit diagram, explain the working of a full wave centre-tap rectifier using junction diodes. Draw the input and output waveforms.
9. Draw the circuit arrangement for studying the input and output characteristics of an npn transistor in CE configuration. Draw these characteristics and define input resistance and output resistance.
10. With the help of a circuit diagram explain the action of a npn transistor in CE configuration as a switch. Draw the transfer characteristics and indicate the relevant regions of operation.
11. Describe with a circuit diagram the working of an amplifier using an npn transistor in CE configuration. Draw relevant waveforms and obtain an expression for the voltage gain.