TUMKUR DISTRICT PU COLLEGES' PRINCIPAL'S ASSOCIATION (R.)

Sub Code: 35 | **II PUC : MODEL QUESTION PAPER – 1** | **No. of Questions: 52**

TIME : 3 hours 15min. | **SUB : MATHEMATICS** | **MAX.MARKS:80**

Instructions:

1) *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*
2) *Part A has 15 Multiple choice questions, 5 Fill in the blanks of 1 mark each.*
3) *Use the graph sheet for question on linear programming in PART E.*

PARTA

I. Answer ALL the Multiple Choice Questions **15 ×1 = 15**

1. A relation R in the set A={1,2,3} given by {(2,2)} is

   (A) reflexive relation (B) not transitive relation (C) symmetric relation (D) none

2. Let $f:R_* \rightarrow R_*$ be defined as $f(x) = \frac{1}{x}$, $x \in R_*$, where R_* is a non-zero real number, then f is

   (A) surjective (B) injective (C) bijective (D) f is not defined

3. The domain of $\cot^{-1}x=y$ is

   (A) $-\frac{\pi}{2} < x < \frac{\pi}{2}$ (B) $0 \le x < \pi$ (C) $-\infty < x < \infty$ (D) $0 \le x \le \pi$

4. If A=[a_{ij}] is a square matrix of order m x n, then

   (A) m < n (B) m > n (C) m = n (D) none of these

5. If |AB|=16, |A|=8 where A and B are square matrices of same order , then |B| is

   (A) 8 (B) 4 (C) 2 (D) 1

6. The continuity of the function f(x)=cos2x+5 at x = π is

   (A) -6 (B) 5 (C) 6 (D) cos π

7. If $y=e^{\log x}$, then $\frac{dy}{dx}$ is

   (A) $\frac{1}{x}$ (B) 1 (C) -1 (D) $\frac{-1}{x}$

8. The total revenue rupees received from the sale of x units of a product is given by $R(x)=13x^2+26x-15$, then the marginal revenue when x=1 is ______ rupees.

   (A) 13 (B) 26 (C) 104 (D) 52

9. Anti-derivative of cos2x is

   (A) –sin2x+C (B) sin2x+C (C) ½ (sin2x)+C (D) -½ (sin2x)+C

10. $\int e^X \left[\frac{1}{1+x^2} + \tan^{-1} x \right].dx$ is

   (A) $\frac{e^x}{1+x^2}$ (B) $e^x .\tan x$ (C) $e^x .\tan^{-1}x + C$ (D) $\frac{e^x}{\tan^{-1}x} + C$

11. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a} \times \vec{b}| = \vec{a}.\vec{b}$, when $\tan\theta =$

   (A) $\sqrt{3}$ (B) 0 (C) $1/\sqrt{3}$ (D) 1

12. A line joining two points A(4,6,0) and B(3,-6,8), then the components of $\overrightarrow{AB}$=

(A) -1,8,-12 (B) 1,8,-12 (C) -1,-12,-8 (D) 1,12,8

13. If a line makes angle 90^o,60^o and 30^o with the positive direction of X, Y and Z-axis , then its direction ratios are

(A) 1, ½, $\sqrt{3}/2$ (B) 0, ½ , $\sqrt{3}/2$ (C) 1, $\sqrt{3}/2$, ½ (D) 0, $\sqrt{3}/2$, ½

14. In a linear programming problem, non-negative constraints are

(A) $x \leq 0,\ y \leq 0$ (B) $x \leq 0,\ y \geq 0$ (C) $x \geq 0,\ y \geq 0$ (D) none of these

15. If A and B are events such that P(A|B)=P(B|A), then

(A) A⊆ B but A≠B (B) A=B (C) A□B=ϕ (D) P(A)=P(B)

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket.

[4, 25, ½ , 1, -π/3 , 0.12] **5 x 1 = 5**

16. The $\tan^{-1}\left(-\sqrt{3}\right)$ is equal to ________

17. If A is a 3 x 3 matrix and |A|=5, then | adj A| = ________

18. The number of arbitrary constants in the general solution of a differential equation of fourth order are _____

19. If l, m, n are direction cosines of a line, then $l^2+m^2+n^2$= _______

20. If A and B are independent events such that P(A)=0.3, P(B)=0.4 then P(A∩B)= _________

PART B

Answer any SIX questions: **6×2 =12**

21. Prove that $2\sin^{-1}(\frac{3}{5})= \tan^{-1}(\frac{24}{7})$

22. Find the area of a triangle with the vertices are (3,8),(-4,2) and (5,1).

23. Find $\frac{dy}{dx}$ if y+siny=cosx.

24. Find the rate of change of the area of a circle with respect to its radius r when r=5cm.

25. Find the local minimum value of the function f given by f(x)=|x|+3, x∈R

26. Evaluate $\int \frac{1}{x-x^2}.dx$

27. Evaluate $\int_0^{\frac{\pi}{2}} \frac{sinx}{1+cos^2x}.dx$

28. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})$=15, then find $|\vec{x}|$

29. Find the angle between the pair of lines $\vec{r} = 3\hat{i}+\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}-2\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-56\hat{k})+\mu(3\hat{i}-5\hat{j}-4\hat{k})$

30. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

31. Given that the events A and B are mutually exclusive events such that P(A)=1/2, P(AUB)=3/5and P(B)=k, then find k.

PARTC

Answer any SIX questions: **6×3 = 18**

32. Show that the relation R in R defined as $R = \{(x,y): 3x - y = 0\}$, is neither reflexive, nor symmetric nor transitive.

33. Write in simplest form $\tan^{-1}\left(\frac{(\cos x - \sin x)}{(\cos x + \sin x)}\right)$, $-\frac{\pi}{4} < x < \frac{3\pi}{4}$

34. If $A = \begin{pmatrix} 3 & -2 \\ 5 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 1 \\ -1 & 0 \end{pmatrix}$, then find $(A - 2B)^T$

35. Find $\frac{dy}{dx}$ if $x^2+xy+y^2=100$.

36. Differentiate $(x)^{\sin x}$ wrt to x

37. Find the interval in which the function f is strictly increasing or decreasing , $f(x) = 10-6x-2x^2$.

38. Evaluate $\int (x^2+1).\log x.dx$

39. Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$, given y=1 when x=0

40. If $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

41. Find the area of the triangle formed by the vertices (1,1,2),(2,3,5) and (1,5,5) using vector method.

42. Bag I contains 3 red and 4 black balls and bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the bag II.

PART D

Answer any FOUR questions: **4×5 = 20**

43. Consider the function f: R→R defined by f(x)=4x+3. Show that f is invertible. Find the inverse of f.

44. If $A = \begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 8 \\ 11 & 21 \end{pmatrix}$ and $C = \begin{pmatrix} 7 & 13 \\ 5 & 19 \end{pmatrix}$. Verify (B+C)A=BA+CA.

45. Solve the system of equations by matrix method
x-y+z=4, 2x+y-3z=0 and x+y+z=2

46. If $y=\sin^{-1}x$ show that $(1-x^2).\frac{d^2y}{dx^2} - x.\frac{dy}{dx} = 0$

47. Find the integral of $\frac{1}{x^2-a^2}$ wrt x and hence evaluate $\int \frac{1}{x^2-3}.dx$

48. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by using integration method.

49. Find the general solution of the differential equation $(1+x^2).dy+2xy.dx=\cot x.dx$

50. Derive the equation of a line in space through a given point and parallel to a given vector both in the vector form and Cartesian form.

PART E

Answer the following question:

51. Prove that $\int_0^a f(x).dx = \int_0^a f(a-x).dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}}.dx$ **6**

OR

Solve the following Linear Programming Problem graphically, minimize and maximize z=x+2y subject to the constraints x+2y ≥100, 2x-y≤ 0, 2x+y≤ 200 and x,y ≥ 0

52. Show that the matrix $A=\begin{pmatrix} 5 & 3 \\ -1 & 2 \end{pmatrix}$ satisfy the equation $A^2-3A-7I=O$, where I is 2x2 identity matrix and O is 2x 2 zero matrix. Using this equation find A^{-1}. **4**

OR

Find the relationship between a and b so that

$f(x)= \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at x=3?

* * * * *

TUMKUR DISTRICT PU COLLEGES' PRINCIPAL'S ASSOCIATION (R.)

Sub Code: 35 | **II PUC : MODEL QUESTION PAPER – 2** | **No. of Questions: 52**

TIME : 3 hours 15min. | **SUB : MATHEMATICS** | **MAX.MARKS:80**

Instructions:

1. *The question paper has five parts namely A, B, C, D and E. Answer all the parts.*
2. *Part A has 15 Multiple choice questions, 5 Fill in the blanks of 1 mark each.*
3. *Use the graph sheet for question on linear programming in PART E.*

PARTA

Answer ALL the Multiple Choice Questions **15 ×1 = 15**

1. Let R be the relation in the set N given by R={(a,b):a=b-2, b > 6}. Choose the correct answer.

   (A) $(2,4)\in R$ (B) $(3,8)\in R$ (C) $(6,8)\in R$ (D) $(8,7)\in R$

2. Let f:R→R be defined as f(x)=| x |, then f is

   (A) one-one and onto (B) one-one but not onto

   (C) onto but not one-one (D) neither one-one nor onto

3. Principal value of $\cos^{-1}(-\frac{1}{2})$ is

   (A) $-\pi/6$ (B) $-\pi/3$ (C) $5\pi/6$ (D) $2\pi/3$

4. If a matrix has 18 elements, then the number of matrices having all possible orders is

   (A) 4 (B) 6 (C) 2 (D) 8

5. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x =

   (A) 6 (B) -6 (C) 0 (D) ±6

6. f(x)=|x| is continuous

   (A) at zero only (B) at all negative real numbers

   (C) at all positive real numbers (D) at every real numbers

7. If $y=\log_7 2x$, then dy/dx is

   (A) 1/(x.log7) (B) 1/(7logx) (C) logx/7 (D) 7/logx

8. The minimum value of the function $y=2x^2+x-1$ at x is

   (A) -1/4 (B) 3/2 (C) -9/8 (D) 9/4

9. Anti-derivative of tanx is

   (A) Log|secx|+C (B) -Log|cosx|+C (C) Log|cosx|+C (D) Both (A) and (B)

10. $\int e^x.\sec x[1+\tan x].dx$ is

    (A) $e^x.\cos x+C$ (B) $e^x.\sec x+C$ (C) $e^x.\sin x+C$ (D) $e^x.\tan x+C$

11. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following is correct

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

12. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then λa is a unit vector if

(A) $\lambda=1$ (B) $\lambda=-1$ (C) $a=|\lambda|$ (D) $a=1/|\lambda|$

13. The angle between the lines whose direction ratios are a,b,c and b-c, c-a, a-b is

(A) 0^o (B) 30^o (C) 60^o (D) 90^o

14. The optimal value of the objective function is attained at the

(A) Corner points of feasible region (B) points on the Y-axis

(C) points on the X-axis (D) none of these

15. If P(A)=0.8, P(B)=0.5 and P(B|A)=0.4, then what is the value of $P(A\cap B)$?

(A) 0.32 (B) 0.25 (C) 0.1 (D) 0.4

Fill in the blanks by choosing the appropriate answer from those given in the bracket.

[3, 64, -10/7, 36, $\pi/3$, $2\pi/3$] **5 x 1 = 5**

16. The value of $\sin^{-1}(\sin 2\pi/3)$ is …………

17. If |A|=8, then |A.A′|= ………………..

18. Order of $(y''')^2+(y'')^3+(y')^4+y^5=0$ is ……………..

19. The lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, then k = ………..

20. The probability of obtaining an even number on each die, when a pair of dice is rolled is …………

PART B

Answer any SIX questions: **6×2 =12**

21. Show that $\sin^{-1}(2x.\sqrt{1-x^2})=2\cos^{-1}x$, $\frac{1}{\sqrt{2}} \leq x \leq 1$

22. Find the area of a triangle whose vertices are (-2,-3),(3,2) and (-1,-8).

23. Find $\frac{dy}{dx}$ if $\sin^2x+\cos^2y=K$, where K is constant.

24. The radius of a circle is increasing at the rate of 0.7cm/s. What is the rate of increase of its circumference.

25. Find the intervals in which the function $f(x)=x^2+2x-5$ is strictly increasing.

26. Evaluate $\int x.\sec^2x.dx$

27. Evaluate $\int \cos 6x.\sqrt{1+\sin 6x}.dx$

28. Find the projection of the vector $2\hat{i}+3\hat{j}+2\hat{k}$ on $\hat{i}+2\hat{j}+\hat{k}$

29. Find the angle between pair of lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

30. Assume that each born child is equally likely to be a boy or girl. If a family has two children. What is the conditional probability of both are girls given that the youngest is a girl.

31. Given two independent events A and B such that P(A)=0.3, P(B)=0.6. Find P(A and not B).

PARTC

Answer any SIX questions: **6×3 = 18**

32. Show that the relation R in R defined as $R=\{(a,b): a \le b^2\}$, is neither reflexive, nor symmetric nor transitive.

33. Prove that $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

34. Express the matrix $\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$ as sum of symmetric and skew symmetric matrix

35. Differentiate $(x)^{\sin x}$ wrt x , x > 0

36. If x=10(t-sint), y=12(1-cost), find $\frac{dy}{dx}$

37. Find the two positive numbers whose sum is 15 and the sum of whose squares is minimum

38. Evaluate $\int \frac{\cos x}{(1-\sin x).(2-\sin x)}.dx$

39. Find the general solution of the differential equation y.logy.dx - x.dy=0

40. Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}=\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$

41. Find the vector perpendicular to each of the vectors $\vec{a}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$ and $\vec{b}=3\hat{\imath}+5\hat{\jmath}-2\hat{k}$ which has magnitude 10 units.

42. A man is known to speak truth 3 out of 4 times. He through a die and reports that it is a six. Find the probability that it is actually a six.

PART D

Answer any FOUR questions: **4×5 = 20**

43. If f:R→R defined by $f(x)=1+x^2$. Verify whether the function is one-one, onto or both.

44. If $A=\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 3 \\ -1 & 4 \end{pmatrix}$ and $C=\begin{pmatrix} 2 & -2 \\ 3 & 0 \end{pmatrix}$ Show that A(BC)=(AB).C

45. Solve the system of equations by matrix method
2x+3y+3z=5, x-2y+z=-4 and 3x-y-2z=3

46. If $y=(\tan^{-1}x)^2$ show that $(x^2+1)^2.y_2+2x(x^2+1).y_1=2$

47. Find the integral of $\sqrt{a^2-x^2}$ wrt x and hence evaluate $\int(5-x^2+2x).dx$

48. Find the area of the region bounded by the circle $x^2+y^2=4$ using integration method.

49. Find the particular solution of the differential equation $\frac{dy}{dx}+2y.\tan x=\sin x$, y=0 when $x=\frac{\pi}{3}$

50. Derive the equation of a line in space through a given point and parallel to a given vector both in the vector form and Cartesian form.

PART E

Answer the following question:

51. Prove that $\int_{-a}^{a} f(x).dx = 2\int_{0}^{a} f(x).dx$, if f(x) is an even function

$\int_{-a}^{a} f(x).dx = 0,$ if f(x) is an odd function

and hence evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sinx^7 x$ .dx

OR

Solve the following Linear Programming Problem graphically, minimize z=-3x+4y subject to the constraints x+2y≤8, 3x+2y≤12, x,y≥0

52. Show that the matrix $A=\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ satisfy the equation $A^2-4A+I=O$, where I is 2x2 identity matrix and O is 2x 2 zero matrix. Using this equation find A^{-1}.

OR

Find the value of k if a function $f(x)=\begin{cases} kx+1, & x\leq 5 \\ 3x-5, & x>5 \end{cases}$ is continuous at x=5

* * *