SAMPLE CHAPTER

mtG

2024 EDITION

OBJECTIVE

KARNATAKA CET

Karnataka Common Entrance Test

MCQs Extracted from Each Line of Latest I & II PUC Textbooks

6500+ MCQs

3 MOCK TEST PAPERS
AS PER LATEST PATTERN

KEY FEATURES...

- Topicwise 10 Years' Trend Analysis
- Comprehensive Theory with Illustrations, Self Tests and Concept Map
- Three Types of Exercises with Detailed Solutions
  - Topicwise KCET Connect
  - KCET Ready
  - KCET Exam Archive : Previous 10 Years' (2014-2023) Questions

PHYSICS

3 Motion in a Plane

10 Years' KCET Topicwise Trend at a Glance

NCERT Topic	No. of Questions									
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
3.2 Scalars and Vectors	1	–	1	–	1	–	–	–	1	–
3.6 Vector Addition - Analytical Method	–	–	–	1	–	–	–	–	–	–
3.9 Projectile Motion	1	1	1	–	–	1	–	3	1	–
3.10 Uniform Circular Motion	–	1	–	1	–	–	1	–	–	1

3.1 Introduction

In one dimensional motion, only two directions are possible. So the directional aspect of the quantities like position, displacement, velocity and acceleration can be taken care of by using positive and negative signs. But in order to describe motion of an object in two dimensions (a plane) or three dimensions (space), an object can have a large number of directions. In order to deal with such situations effectively, we need to introduce the concept of new physical quantities, called vectors, in which we take care of both magnitude and direction. Here we discuss about, how to add, subtract and multiply vectors?

3.2 Scalars and Vectors

Physical quantities can be classified as scalars or vectors.

Scalars : Scalars are those quantities which have magnitude only. It is specified completely by a single number, along with proper unit. *e.g.*, distance, speed, temperature, etc. Some important points regarding scalars are:

Scalars have only magnitude but no direction.

The rules for combining scalars are the rules of ordinary algebra.

Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers. But addition and subtraction of scalars make sense only for quantities with same units however multiplication and division are valid for quantities with different units.

For example, perimeter of a rectangle

= 2 × (length + breadth)

Density = mass / volume

Vectors : Vectors are those quantities which have both magnitude and direction and obey the triangle law of addition. *e.g.*, velocity, acceleration, etc.

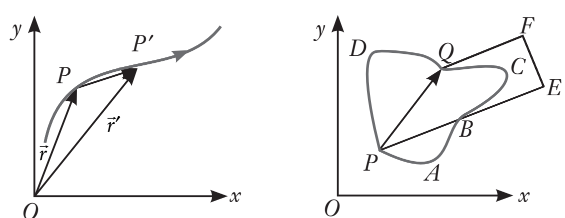
Representation of a vector : A vector is represented by a straight line with an arrow head. The length of the line represents the magnitude of a vector. In writing, a vector can be represented by a single letter with arrow head on it. *e.g.*, force is a vector quantity and it is represented by $\vec{F}$. In books, vectors are represented by bold faced letters.

The magnitude of the vector is called the modulus of the vector. The modulus of vector $\vec{A}$ is represented by $|\vec{A}|$ or A.

Position and displacement vectors

Position vector : A vector which gives position of an object with reference to the origin of a co-ordinate system is called position vector.

Let P and P' be the position of the object at time t and t' respectively moving in x-y plane. Here $\overline{OP}$ and $\overline{OP'}$ are the position vectors of the object as shown in figure (a).



(a) (b)

The position vector provides two information :

It tells the straight line distance of the object from the origin O.

It tells the direction of the object with respect to the origin.

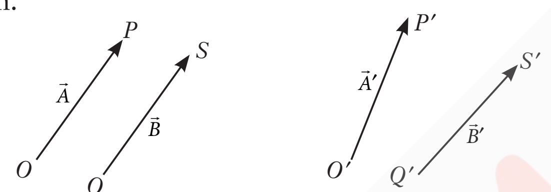
Displacement vector : It is that vector which tells how much and in which direction an object has changed its position in a given time interval.

If the object moves from P to P', the vector PP' is called the displacement vector corresponding to motion from point P to P' as shown in figure (a).

Displacement vector is the straight line joining the initial and final position and does not depend on the actual path under taken by the object between the two positions.

In figure (b) displacement vector $\overline{PQ}$ is same for different paths of journey say $PABCQ$, PDQ and $PBEFQ$. So magnitude of displacement is either less or equal to the path length of an object between two points.

Equality of vectors : Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal if and only if, they have the same magnitude and the same direction. Figure (a) shows two equal vectors $\vec{A}$ and $\vec{B}$ and figure (b) shows two unequal vectors $\vec{A}'$ and $\vec{B}'$ even though they are of the same length.



(a) (b)

***Illustration* :** State, for each of the following physical quantities, if it is a scalar or a vector : volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Soln.: Scalars : volume, mass, speed, density, number of moles, angular frequency.

Vectors : Acceleration, velocity, displacement, angular velocity.

***Illustration* :** State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :

(a) adding any two scalars.

(b) multiplying any vector by any scalar.

(c) adding a component of a vector to the same vector.

Soln.: (a) No, adding any two scalars is not meaningful because only the scalars of same dimensions (*i.e.*, of same nature) can be added.

(b) Yes, multiplying any vector by any scalar is meaningful algebraic operation. It is because when any vector is multiplied by any scalar, then we get a vector having magnitude equal to scalar number times the magnitude of the given vector. *e.g.*, when acceleration $\vec{a}$ is multiplied by mass m, we get force $\vec{F} = m\vec{a}$ which is a meaningful operation.

(c) No, a component of a vector can be added to the same vector only by using the law of vector addition. So, the addition of a vector to the same vector is not a meaningful operation.

Self Test - 1

1. Which of the following is not a vector quantity ?
(a) Momentum (b) Weight
(c) Potential energy (d) Nuclear spin

2. In Latin, the word vector means
(a) magnitude (b) direction
(c) carrier (d) cap

3. Which of the following is not a scalar quantity?
(a) Temperature
(b) Coefficient of friction
(c) Charge (d) Impulse

4. A vector is not changed if
(a) it is displaced parallel to itself.
(b) it is rotated through an arbitrary angle.
(c) it is cross-multiplied by a unit vector.
(d) it is multiplied by an arbitrary scalar.

5. The component of a vector $\vec{r}$ along x-axis will have a maximum value if
(a) $\vec{r}$ is along +ve x-axis
(b) $\vec{r}$ is along +ve y-axis
(c) $\vec{r}$ is along –ve y-axis
(d) $\vec{r}$ makes an angle of 45° with the x-axis

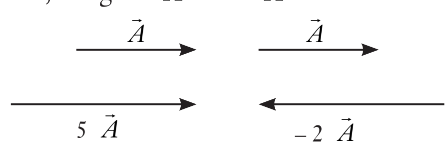
3.3 Multiplication of Vectors by Real Numbers

When we multiply a vector $\vec{A}$ by a positive real number λ, then we get a new vector along the direction of vector $\vec{A}$ Its magnitude becomes λ times the magnitude of the given vector.

Similarly, if we multiply a vector $\vec{A}$ with a negative real number $-\lambda$, then we get a vector whose magnitude is λ times the magnitude of vector $\vec{A}$ but direction is opposite to that of vector $\vec{A}$.

Hence, $\lambda(\vec{A}) = \lambda\vec{A}$ and $-\lambda(\vec{A}) = -\lambda\vec{A}$

(i) Consider a vector $\vec{A}$ is multiplied by a real number, $\lambda = 5$ or -2, we get $5\vec{A}$ or $-2\vec{A}$

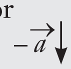


If we multiply a constant velocity vector by time, we will get a displacement vector in the direction of velocity vector.

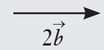
Illustration **:** If $\vec{a}, \vec{b}, \vec{c}$ are as given, then find

(i) $-\vec{a}$ (ii) $2\vec{b}$ (iii) $-3\vec{c}$

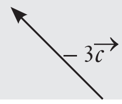
Soln.: (i) : Negative of a vector means the vector of same magnitude but opposite in direction.



(ii) Multiplying a vector with a positive scalar means the vector's magnitude gets multiplied but the vector retains its direction.



(iii) Multiplying a vector by a negative scalar means the direction of the vector gets reversed whereas its magnitude gets multiplied by the magnitude of the scalar.



Self Test - 2

1. If $\vec{A}$ is a vector of magnitude 5 units due east. What is the magnitude and direction of a vector $-5\vec{A}$?

(a) 5 units due east (b) 25 units due west
(c) 5 units due west (d) 25 units due east

2. If $\vec{A}$ is a vector of magnitude 4 units due east. What is the magnitude and direction of a vector $-4\vec{A}$?

(a) 4 units due east (b) 8 units due east
(c) 16 units due east (d) 16 units due west

3. Multiplication of a vector $\vec{A}$ with a negative number - k gives a vector

(a) $-k\vec{A}$ in the same direction
(b) $k\vec{A}$ in the opposite direction
(c) $-k\vec{A}$ in the opposite direction
(d) $k\vec{A}$ in the same direction

4. Multiplication of a vector $\vec{A}$ with a positive number k gives a vector

(a) $k\vec{A}$ in the same direction
(b) $-k\vec{A}$ in the opposite direction
(c) $k\vec{A}$ in the opposite direction
(d) none of these

5. If $\vec{A}$ is a vector of magnitude 7 units due west. What is the magnitude and direction of a vector $-7\vec{A}$?

(a) 7 units due west (b) 49 units due east
(c) 7 units due east (d) 49 units due west

3.4 Addition and Subtraction of Vectors – Graphical Method

General rule for addition of vectors : It states that the vectors to be added are arranged in such a way so that the head of first vector coincides with the tail of second vector, whose head coincides with the tail of third vector and so on, then the single vector drawn from the tail of the first vector to the head of the last vector represents their resultant vector.

Properties of vector addition : The important properties of vector addition are as follows :

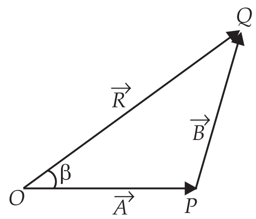
Vector addition is commutative : $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Vector addition is distributive : $n(\vec{A}+\vec{B})=n\vec{A}+n\vec{B}$

Vector addition is associative :

$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$

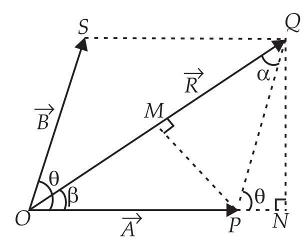
Triangle law of vector addition : It states that if two vectors acting simultaneously at a point are represented by the two sides of a triangle taken in the same order, their resultant vector is represented by the third side of the triangle taken in the opposite order.



$\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$ as shown in the figure.

Then, $\vec{R}=\vec{A}+\vec{B} \Rightarrow R=|\vec{A}+\vec{B}|$

Parallelogram law of vector addition : It states that if two vectors acting simultaneously at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from that point. $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$ as shown in the figure. Then,



$$R=\sqrt{A^2+B^2+2AB\cos\theta} \text{ and } \tan\beta=\frac{B\sin\theta}{A+B\cos\theta}$$

Polygon law of vector addition : It states that if a number of vectors are represented by the sides of an open polygon taken in the same order, then their resultant is represented by the closing side of the polygon taken in the opposite order.

Special Cases

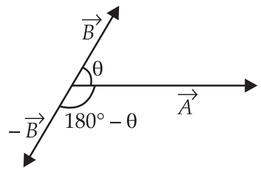
When two vectors are acting in the same direction. Then, $R = A + B$, $\beta = 0°$

When two vectors are acting in the opposite direction. Then, $R = A - B$, $\beta = 180°$

When two vectors are mutually perpendicular to each other. Then

$$R = \sqrt{A^2 + B^2}, \beta = \tan^{-1}\left(\frac{B}{A}\right)$$

Subtraction of vectors : Subtraction of vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$. Thus $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.



If the angle between $\vec{A}$ and $\vec{B}$ is θ, then the angle between $\vec{A}$ and $-\vec{B}$ is (180° – θ).

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Properties of vector subtraction : The important properties of vector subtraction are as follows :

Vector subtraction does not follow commutative law : $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$

Vector subtraction does not follow associative law : $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$

Zero vector or null vector

It is the vector which has zero magnitude and an arbitrary direction. A zero vector is represented by $\vec{0}$ (arrow over the number zero). It is also called null vector.

For example : (i) The velocity vector of a stationary particle is a zero vector.

(ii) The acceleration vector of an object moving with a uniform velocity is a zero vector.

Properties of zero vector.

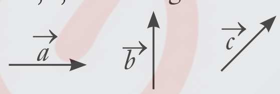
(i) The addition or subtraction of zero vector from given vector does not alter the given vector

$$\vec{A} + \vec{0} = \vec{A};\ \vec{A} - \vec{0} = \vec{A}$$

(ii) The multiplication of non-zero real number n with a zero vector is again zero vector. Thus, $n\vec{0} = \vec{0}$

(iii) If n_1 and n_2 are two different non-zero real numbers, where $n_1 \neq n_2$, then the relation, $n_1\vec{A} = n_2\vec{B}$ can hold only if $\vec{A}$ and $\vec{B}$ are zero vectors and are not parallel or antiparallel vectors.

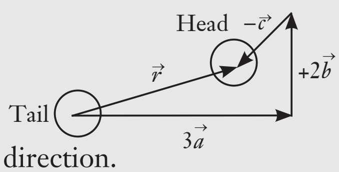
***Illustration* :** If $\vec{a}, \vec{b}, \vec{c}$ are as given:



Then find

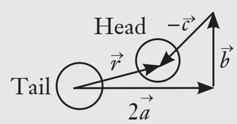
(i) $3\vec{a} + 2\vec{b} - \vec{c}$. (ii) $2\vec{a} + \vec{b} - \vec{c}$

Soln.: (i) Multiplying a vector with a positive scalar means the vector's magnitude gets multiplied but the vector retains its direction.



$\Rightarrow\ 3\vec{a} + 2\vec{b} - \vec{c} = \vec{r}$

(ii) Similarly, $2\vec{a} + \vec{b} - \vec{c} = \vec{r}$

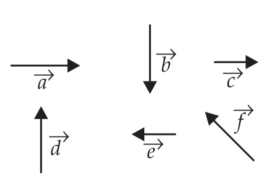


Self Test - 3

1. Which of the following is not a property of a null vector?
   (a) $\vec{A} + \vec{0} = \vec{A}$
   (b) $\lambda\vec{0} = \vec{0}$, where λ is a scalar
   (c) $0\vec{A} = \vec{A}$ (d) $\vec{A} - \vec{A} = \vec{0}$

2. Two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ have a resultant $\vec{R}$ which makes an angle α with $\vec{A}$. If the directions of $\vec{A}$ and $\vec{B}$ are interchanged, the resultant will have the same
   (a) direction (b) magnitude
   (c) direction as well as magnitude
   (d) none of these

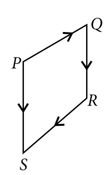
3. Six vectors, $\vec{a}$ to $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true?



   (a) $\vec{b} + \vec{c} = \vec{f}$ (b) $\vec{d} + \vec{c} = \vec{f}$
   (c) $\vec{d} + \vec{e} = \vec{f}$ (d) $\vec{b} + \vec{e} = \vec{f}$

4. If $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right angled triangle. Then out of the following which one is satisfied?
   (a) $\vec{A} = \vec{B} + \vec{C}$ and $A^2 = B^2 + C^2$
   (b) $\vec{A} = \vec{B} + \vec{C}$ and $B^2 = A^2 + C^2$
   (c) $\vec{B} = \vec{A} + \vec{C}$ and $B^2 = A^2 + C^2$
   (d) $\vec{B} = \vec{A} + \vec{C}$ and $A^2 = B^2 + C^2$

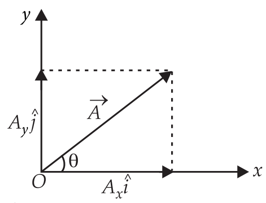
5. In the given diagram, if $\overline{PQ} = \vec{A}$, $\overline{QR} = \vec{B}$ and $\overline{RS} = \vec{C}$ then $\overline{PS}$ equals



   (a) $\vec{A} - \vec{B} + \vec{C}$ (b) $\vec{A} + \vec{B} - \vec{C}$
   (c) $\vec{A} + \vec{B} + \vec{C}$ (d) $\vec{A} - \vec{B} - \vec{C}$

3.5 Resolution of Vectors

- The process of splitting up a vector into two or more vectors is known as resolution of a vector. The vectors into which a given vector is splitted are known as component vectors. When a vector is splitted into two or three component vectors at right angles to each other, the component vectors are called rectangular components of a vector.
- If $\vec{A}$ makes an angle θ with x-axis, and A_x and A_y be the rectangular components of $\vec{A}$ along x-axis and y-axis respectively, then



$\vec{A} = A_x\hat{i} + A_y\hat{j}$

Here $A_x = A\cos\theta$

and $A_y = A\sin\theta$.

$\therefore \quad A_x^2 + A_y^2 = A^2(\cos^2\theta + \sin^2\theta)$

or $\quad A = (A_x^2 + A_y^2)^{1/2}$

and $\tan\theta = A_y / A_x$.

Resolution of a vector in space : Let α, β, and γ are the angles between $\vec{A}$ and the x, y and z-axis, respectively as shown in the figure, then

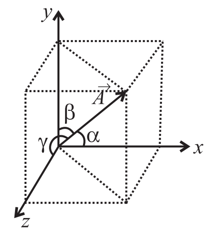
$A_x = A\cos\alpha,\ A_y = A\cos\beta,\ A_z = A\cos\gamma$

In general, we have

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

The magnitude of $\vec{A}$ is

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



$\cos\alpha = \dfrac{A_x}{A};\ \cos\beta = \dfrac{A_y}{A};\ \cos\gamma = \dfrac{A_z}{A}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, where cos α, cos β and cos γ are called the direction cosines of the vector $\vec{A}$.

A vector can have infinite component vectors but the maximum number of rectangular component vectors are three.

Illustration : A force is inclined at 50° to the horizontal. If its rectangular component in the horizontal direction be 50 N, find the magnitude of the force and its vertical component.

Soln.: Let $\vec{F}$ be the force and let $\vec{F}_x$ and $\vec{F}_y$ denote its horizontal and vertical components. Since $\vec{F}$ is inclined at 50° to the horizontal,

$F_x = F\cos 50°$ and $F_y = F\sin 50°$

Thus, $F = \dfrac{F_x}{\cos 50°} = \dfrac{50}{0.6428}$ N (as F_x = 50 N)

or F = 77.78 N

Also, $F_y = F\sin 50°$ = (77.78 N) (0.7660) = 59.58 N

Thus, F = 77.78 N, F_y = 59.58 N

Self Test - 4

1. Two vectors are given by $\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{i} + 6\hat{j} + 2\hat{k}$. Another vector $\vec{C}$ has the same magnitude as $\vec{B}$ but has the same direction as $\vec{A}$. Then which of the following vectors represents $\vec{C}$?
   (a) $\frac{7}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$ (b) $\frac{3}{7}(\hat{i} - 2\hat{j} + 2\hat{k})$
   (c) $\frac{7}{9}(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $\frac{9}{7}(\hat{i} + 2\hat{j} + 2\hat{k})$
2. The x and y components of a force F acting at 30° to x-axis are respectively
   (a) $\frac{F}{\sqrt{2}}, F$ (b) $\frac{F}{2}, \frac{\sqrt{3}}{2}F$
   (c) $\frac{\sqrt{3}}{2}F, \frac{1}{2}F$ (d) $F, \frac{F}{\sqrt{2}}$
3. The angle subtended by the vector $\vec{A} = (4\hat{i} + 3\hat{j} + 12\hat{k})$ with the x-axis is
   (a) $\sin^{-1}\left(\frac{3}{13}\right)$ (b) $\sin^{-1}\left(\frac{4}{13}\right)$
   (c) $\cos^{-1}\left(\frac{4}{13}\right)$ (d) $\cos^{-1}\left(\frac{3}{13}\right)$
4. If $\hat{n}$ is a unit vector in the direction of the vector $\vec{A}$, then
   (a) $\hat{n} = \dfrac{\vec{A}}{|\vec{A}|}$ (b) $\hat{n} = \dfrac{|\vec{A}|}{\vec{A}}$
   (c) $\hat{n} = |\vec{A}|\vec{A}$ (d) $\hat{n} = \vec{A}$
5. The magnitude of the x-component of vector $\vec{A}$ is 3 and the magnitude of vector $\vec{A}$ is 5. What is the magnitude of the y-component of vector $\vec{A}$?
   (a) 3 (b) 4 (c) 5 (d) 8

3.6 Vector Addition - Analytical Method

- It is easier to add vectors by combining their respective components. Consider two vector $\vec{A}$ and $\vec{B}$ in x-y plane with components A_x, A_y and B_x, B_y.

  $\vec{A} = A_x\hat{i} + A_y\hat{j};\ \vec{B} = B_x\hat{i} + B_y\hat{j}$

  Let $\vec{R}$ be their sum. Then

$\vec{R} = \vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j}) = R_x\hat{i} + R_y\hat{j}$

$\therefore \quad R_x = A_x + B_x; \quad R_y = A_y + B_y$

Thus, each components of the resultant vector $\vec{R}$, is the sum of the corresponding components of $\vec{A}$ and $\vec{B}$.

- In three dimensions, we have
  $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$; $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$
  $\vec{R} = \vec{A} + \vec{B} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$
  where $R_x = A_x + B_x$, $R_y = A_y + B_y$, $R_z = A_z + B_z$
  This method can be extended to addition and subtraction of any number of vectors.

Illustration : The magnitudes of the X and Y components of $\vec{A}$ are 3 and 8. Also the magnitudes of the X and Y components of $\vec{A} + \vec{B}$ are 6 and 12 respectively. Show that the magnitude of $\vec{B}$ is 5.

Soln.: Given $A_x = 3$, $A_y = 8$.

Also $R_x = 6$, $R_y = 12$ where $\vec{R} = \vec{A} + \vec{B}$.

$\therefore \quad A_x + B_x = R_x$ or $B_x = R_x - A_x = 6 - 3 = 3$

Similarly $B_y = R_y - A_y = 12 - 8 = 4$

$\therefore \quad B^2 = B_x^2 + B_y^2 = (3)^2 + (4)^2 = 9 + 16 = (5)^2$

$\therefore \quad B = 5$

Self Test - 5

1. The components of the sum of two vectors $2\hat{i} + 3\hat{j}$ and $2\hat{j} + 3\hat{k}$ along x and y directions respectively are
   (a) 2 and 5 (b) 4 and 6
   (c) 2 and 6 (d) 4 and 3

2. Two vectors $\vec{A}$ and $\vec{B}$ have components A_x, A_y, A_z and B_x, B_y, B_z respectively. If $\vec{A} + \vec{B} = 0$, then
   (a) $A_x = B_x, A_y = -B_y, A_z = -B_z$
   (b) $A_x = B_x, A_y = B_y, A_z = -B_z$
   (c) $A_x = B_x, A_y = B_y, A_z = B_z$
   (d) $A_x = -B_x, A_y = -B_y, A_z = -B_z$

3. Two vectors are given by $\vec{A} = (3\hat{i} + \hat{j} + 3\hat{k})$ and $\vec{B} = (3\hat{i} + 5\hat{j} - 2\hat{k})$. Find the third vector $\vec{C}$ if $\vec{A} + 3\vec{B} - \vec{C} = 0$.
   (a) $(12\hat{i} + 14\hat{j} + 12\hat{k})$ (b) $(13\hat{i} + 17\hat{j} + 12\hat{k})$
   (c) $(12\hat{i} + 16\hat{j} - 3\hat{k})$ (d) $(15\hat{i} + 13\hat{j} + 4\hat{k})$

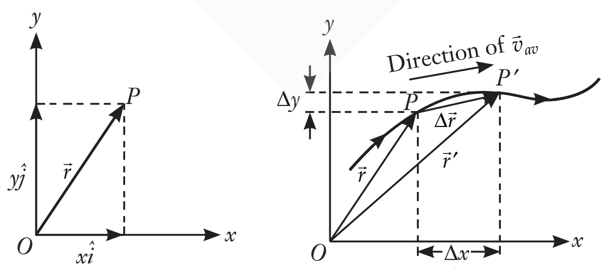
4. With respect to a rectangular cartesian co-ordinate system three vectors are expressed as $\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, along the x, y, z axes respectively. The unit vector along the direction of the sum of these vectors is
   (a) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (b) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j} - \hat{k})$
   (c) $\hat{r} = \frac{1}{3}(\hat{i} - \hat{j} + \hat{k})$ (d) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

5. The following four forces act simultaneously on a particle at rest at the origin of the co-ordinate system.
   $\vec{F}_1 = 2\hat{i} - 3\hat{j} - 2\hat{k}$, $\vec{F}_2 = 5\hat{i} + 8\hat{j} + 6\hat{k}$
   $\vec{F}_3 = -4\hat{i} - 5\hat{j} + 5\hat{k}$, $\vec{F}_4 = -3\hat{i} + 4\hat{j} - 7\hat{k}$
   The particle will move in
   (a) XY plane (b) YZ plane
   (c) ZX plane (d) Only X plane

3.7 Motion in a Plane

Motion in Two Dimensions: Motion in a Plane

Here, we will learn about describing motion in two dimensions using vectors.



(a) (b)

Position vector : The position vector of a particle P located in x-y plane with respect to origin O is as shown in figure (a) is $\vec{r} = x\hat{i} + y\hat{j}$

Displacement vector : Suppose particle moves from point P to P' along the curve. Then, the displacement is $\Delta\vec{r} = \vec{r}' - \vec{r}$

It is directed from P to P'.

In a component form,

$$\Delta\vec{r} = (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j}) = \hat{i}\Delta x + \hat{j}\Delta y$$

Velocity : The average velocity ($\vec{v}_{av}$) of a particle is ratio of the displacement and corresponding time interval :

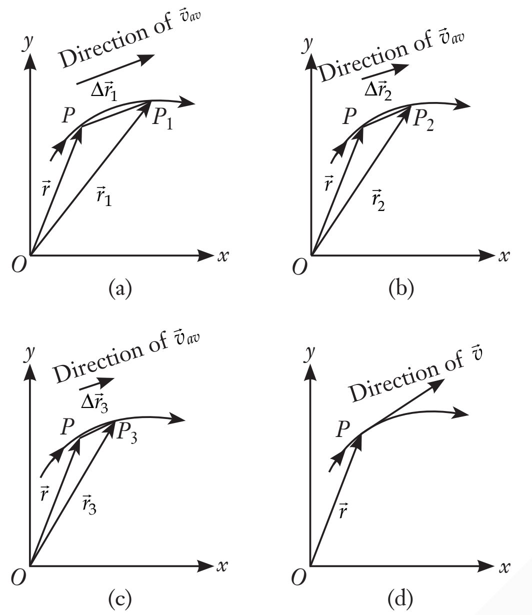
$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \hat{i}\frac{\Delta x}{\Delta t} + \hat{j}\frac{\Delta y}{\Delta t}; \quad \vec{v}_{av} = v_{x_{av}}\hat{i} + v_{y_{av}}\hat{j}$$

The direction of average velocity is the same as that of $\Delta\vec{r}$.

The velocity (instantaneous velocity) is given by the limiting value of the average velocity as time interval approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

This limiting process is shown in the figures. As the time interval Δt approaches zero, the average velocity approaches the velocity $\vec{v}$. The direction of $\vec{v}_{av}$ is parallel to the line tangent to the path.



P_1, P_2 and P_3 represent the positions of objects after time Δt_1, Δt_2 and Δt_3. $\Delta\vec{r}_1, \Delta\vec{r}_2$ and $\Delta\vec{r}_3$ are the displacement of the particle in time Δt_1, Δt_2 and Δt_3 respectively. As $\Delta t \to 0$, $\Delta\vec{r}_1 \to 0$ and is along the tangent to the path.

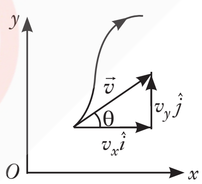
The direction of velocity at any point on the path of a particle is tangential to the path at that point and is in the direction of motion.

$\vec{v}$ in the component form,

$$\vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}\right); \ \vec{v} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} = v_x\hat{i} + v_y\hat{j}$$

Magnitude of $\vec{v} = v = \sqrt{v_x^2 + v_y^2}$ and direction of $\vec{v}$,

$$\tan\theta = \frac{v_y}{v_x}; \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$



Average acceleration : The average acceleration of a particle for a time interval (Δt) moving in x-y plane is the change in velocity divided by the time interval. The components v_x and v_y of velocity v and the angle θ it makes with x-axis as shown in the above figure. Then

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} \quad \vec{a}_{av} = (\vec{a}_x)_{av}\hat{i} + (\vec{a}_y)_{av}\hat{j}$$

Instantaneous acceleration : The average acceleration for three time intervals Δt_1, Δt_2, and Δt_3, ($\Delta t_1 > \Delta t_2 > \Delta t_3$) are shown in the figure (a), (b) and (c). In the limiting case as $\Delta t \to 0$, $\Delta v \to 0$ and average acceleration becomes instantaneous acceleration or acceleration as shown in figure (d).

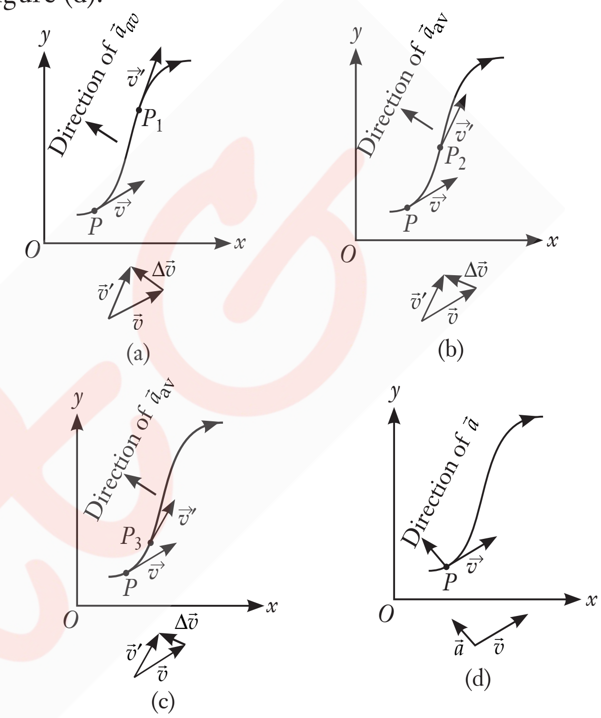


***Illustration* :** A body of mass 5 kg starts from the origin with an initial velocity $\vec{u} = (30\hat{i} + 40\hat{j})$ ms^{-1}. A constant force $\vec{F} = (-6\hat{i} - 5\hat{j})\,N$ acts on the body. Show that the time in which the Y-component of velocity becomes zero is 40 sec.

Soln.: Mass = 5 kg

Velocity along Y-direction = 0

$\vec{u} = 30\hat{i} + 40\hat{j}$

Force along Y-direction = $F_y = -5$

$\vec{F} = 6\hat{i} - 5\hat{j}$

Let the time taken = t, for which Y-component becomes zero

Force= $\dfrac{m \times \text{change in velocity}}{t} \Rightarrow 5 = \dfrac{5 \times 40}{t} \Rightarrow t = 40\,\text{sec}$

Self Test - 6

1. A bird flies at an angle of 60° to the horizontal. Its horizontal component of velocity is 10 m s^{-1}. Find the vertical component of velocity in m s^{-1}.

   (a) $10\sqrt{3}$ (b) $\dfrac{10}{\sqrt{3}}$ (c) 5 (d) 26

2. The coordinates of a particle moving in x-y plane at any instant of time t are $x = 4t^2$; $y = 3t^2$. The speed of the particle at that instant is

   (a) $10t$ (b) $5t$ (c) $3t$ (d) $2t$

3. The velocity vector of the motion described by the position vector of a particle $\vec{r} = (2t\hat{i} + t^2\hat{j})$ is

(a) $\vec{v} = (2\hat{i} + 2t\hat{j})$ (b) $\vec{v} = (2t\hat{i} + 2t\hat{j})$

(c) $\vec{v} = (t\hat{i} + t^2\hat{j})$ (d) $\vec{v} = (2\hat{i} + t^2\hat{j})$

4. If the velocity (in m s^{-1}) of a particle is given by $4.0\hat{i} + 5.0t\hat{j}$, then the magnitude of its acceleration (in m s^{-2}) is

(a) 4 (b) –5 (c) 0 (d) 5

5. The position of a particle moving in the *x-y* plane at any time *t* is given by $x = (3t^2 - 6t)$ metres; $y = (t^2 - 2t)$ metres. Select the correct statement.

(a) Acceleration is zero at $t = 0$.

(b) Velocity is zero at $t = 0$.

(c) Velocity is zero at $t = 1$ s.

(d) Velocity and acceleration of the particle are never zero.

3.8 Motion in a Plane with Constant Acceleration

- Motion in a plane (two dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.
- Velocity of a particle moving with constant acceleration $\vec{a}$ at any time *t*.

  $\vec{v} = \vec{v}_0 + \vec{a}t$ where $\vec{v}_0$ is the velocity of particle at $t = 0$.
- Average velocity over time interval *t*, $\vec{v}_{av} = \dfrac{\vec{v} + \vec{v}_0}{2}$

  In terms of components : $v_x = v_{0x} + a_x t$; $v_y = v_{0y} + a_y t$
- Displacement of the particle,

  $$\vec{r} - \vec{r}_0 = \left(\frac{\vec{v} + \vec{v}_0}{2}\right)t = \left(\frac{(\vec{v}_0 + \vec{a}t) + \vec{v}_0}{2}\right)t$$

  $$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

  In terms of components, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$

  and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

Illustration **:** For motion in a plane with constant acceleration $\vec{a}$, initial velocity $\vec{v}_0$ and final velocity $\vec{v}$ after a time *t*, show that $\vec{v}\cdot(\vec{v} - \vec{a}t) = \vec{v}_0\cdot(\vec{v}_0 + \vec{a}t)$

Soln.: L.H.S. = $\vec{v}\cdot(\vec{v} - \vec{a}t) = \vec{v}\cdot\vec{v} - \vec{v}\cdot\vec{a}t$

$= v^2 - \vec{v}\cdot\vec{a}t = \left(v_0^2 + 2\vec{a}\cdot\vec{s}\right) - \vec{v}\cdot\vec{a}t$

$= v_0^2 + 2\vec{a}\cdot\left(\dfrac{\vec{v} + \vec{v}_0}{2}\right)t - \vec{v}\cdot\vec{a}t$

$= \vec{v}_0\cdot\vec{v}_0 + \vec{a}\cdot\vec{v}t + \vec{a}\cdot\vec{v}_0 t - \vec{v}\cdot\vec{a}t$

$= \vec{v}_0\cdot\vec{v}_0 + \vec{v}_0\cdot\vec{a}t = \vec{v}_0\cdot(\vec{v}_0 + \vec{a}t)$ = RHS

Self Test - 7

1. A particle starts from origin at $t = 0$ with a velocity $5\hat{i}$ m s^{-1} and moves in *x-y* plane under the action of a force which produces a constant acceleration of $3\hat{i} + 2\hat{j}$ m s^{-2}. The *y*-coordinate of the particle at the instant when its *x*-coordinate is 84 m is

(a) 12 m (b) 24 m (c) 36 m (d) 48 m

2. In previous question, the speed of the particle at this time is

(a) 16 m s^{-1} (b) 26 m s^{-1}

(c) 36 m s^{-1} (d) 46 m s^{-1}

3. A particle crossing the origin of co-ordinates at time $t = 0$, moves in the *xy*-plane with a constant acceleration *a* in the *y*-direction. If its equation of motion is $y = bx^2$ (*b* is a constant), its velocity component in the *x*-direction is

(a) $\sqrt{\dfrac{2b}{a}}$ (b) $\sqrt{\dfrac{a}{2b}}$ (c) $\sqrt{\dfrac{a}{b}}$ (d) $\sqrt{\dfrac{b}{a}}$

4. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ m s^{-1}, a particle moves in the $x-y$ plane with a constant acceleration of $(10\hat{i} + 4\hat{j})$ m s^{-2}. At time *t*, its coordinates are (20 m, y_0 m). The values of *t* and y_0 are, respectively

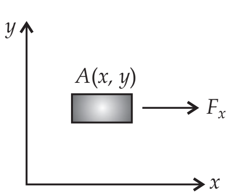
(a) 2 s and 24 m (b) 4 s and 52 m

(c) 5 s and 25 m (d) 2 s and 18 m

3.9 Projectile Motion

As we know, two dimensional motion can be resolved in two linear motions in two mutually perpendicular directions.

Characteristically, motion in one direction is independent of motion in other mutually perpendicular direction.

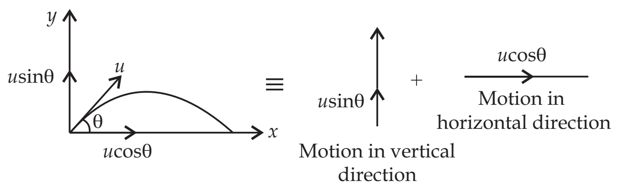


Say, a particle at rest is lying on a horizontal plane $(x - y)$ having coordinates (x, y) as shown. Now, if a force parallel to *x*-axis is acting on the particle. Which of the coordinates *x* or *y* will change?

It is easy to say that *x* coordinate will change and *y* will remain unchanged no matter whatever is the value of F_x along *x*-direction. Similarly a force parallel to *y*-axis does not affect motion along *x*-axis. When any object project near the surface of the earth, it performs two dimensional motion under gravity. Such a motion is called projectile motion. Here, due to gravity the horizontal motion will not be affected.

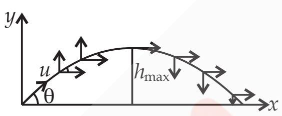
A Projectile Projected with Velocity *u* at an angle θ.

x-direction	y-direction
$u_x = u\cos\theta$; $a_x = 0$	$u_y = u\sin\theta$; $a_y = -g$



Velocities : Consider an object, thrown or projected with initial velocity u making an angle θ with the horizontal.
Along x-axis, initial velocity, $u_x = u\cos\theta$ (throughout constant)
Along y-axis, initial velocity $u_y = u\sin\theta$

Accelerations
Along x-axis, $a_x = 0$
$\therefore u_x = v_x = u\cos\theta$
(constant velocity)
Along y-axis, $a_y = -g$



Equation of trajectory : From kinematic equation along x-axis, $x = u_x t + \frac{1}{2}a_x t^2$; $x = u\cos\theta t$ $\quad (\because a_x = 0)$

$$\Rightarrow t = \frac{x}{u\cos\theta} \qquad \text{...(i)}$$

Now along y-axis,

$$y = u_y t + \frac{1}{2}a_y t^2 \; ; \; y = u\sin\theta t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow y = (\tan\theta)x - \frac{g}{(u^2\cos^2\theta)}x^2 \Rightarrow y = ax - bx^2 \qquad \text{...(ii)}$$

Here, $a = \tan\theta$ and $b = \frac{g}{u^2\cos^2\theta}$, both are constants.

Equation (ii) is parabolic in nature. Hence path taken or trajectory is parabolic for a projectile.

Time of maximum height (t_m) : The time at which the projectile reaches the maximum height.
At h_{max}, final velocity along y-axis becomes zero *i.e.*, $v_y = 0$.

$$\therefore v_y = u_y + a_y t_m \; ; \; 0 = u\sin\theta - gt_m \Rightarrow t_m = \frac{u\sin\theta}{g}$$

Time of flight (T_f) : Total time in which the projectile is in flight before touching the ground.
Here, $y = 0$
From $y = u_y t + \frac{1}{2}a_y t^2$; $0 = u\sin\theta T_f - \frac{1}{2}gT_f^2$

$$\Rightarrow T_f = \frac{2u\sin\theta}{g}$$

Maximum height (h_{max}) : It is the maximum height attained by the projectile during the journey.

At h_{max}, $y = h_{max}$ and $t = t_m$ $\quad \therefore y = u_y t + \frac{1}{2}a_y t^2$

$$\Rightarrow h_{max} = u\sin\theta t_m - \frac{1}{2}gt_m^2$$

$$h_{max} = \frac{u^2\sin^2\theta}{g} - \frac{1}{2}g\frac{u^2\sin^2\theta}{g^2} \qquad \left(\because t_m = \frac{u\sin\theta}{g}\right)$$

$$h_{max} = \frac{u^2\sin^2\theta}{2g}$$

Horizontal range (*R*) : It is the horizontal distance covered by the projectile before landing.

Since $v_x = \frac{x}{t}$ and along x-axis, $x = R$, $t = T_f$

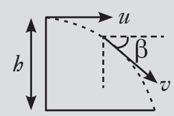
$$\therefore R = v_x \times T_f = u\cos\theta \times \frac{2u\sin\theta}{g} \Rightarrow R = \frac{u^2\sin 2\theta}{g}$$

Effects of air resistance : The air resistance decreases the maximum height attained and range of the projectile. It also decreases the speed with which the projectile strikes the ground.

Effect of variation of *g* : Acceleration due to gravity does not remain constant when the range exceeds say 1500 km or so. Then the direction of g changes because g always points towards the centre of earth. Due to this, shape of trajectory changes from parabolic to elliptical.

Info Shots

A Projectile Projected Horizontally from a Height *h* with Velocity *u*



- Time taken by the projectile to reach the ground is $\sqrt{2h/g}$.
- Time taken by the projectile to reach the ground does not depend upon the velocity of projection *i.e.*, u.
- Horizontal range, $x = ut = u\sqrt{\frac{2h}{g}}$.
- Equation of trajectory is $y = \frac{g}{2u^2}x^2$.
- Resultant velocity of the projectile at any time t is $v = \sqrt{u^2 + g^2t^2}$.
- Angle made by the resultant velocity with the horizontal is $\tan\beta = \frac{gt}{u}$.
- Velocity of the projectile on striking the ground $= \sqrt{u^2 + 2gh}$.

Projectile motion on an inclined plane

- Time of flight, $T = \frac{2u\sin(\alpha - \theta)}{g\cos\theta}$
- Range, $R = \frac{u^2}{g\cos^2\theta}$ [sin(2α – θ)– sinθ]

  Here, θ = inclination of the plane, α = angle of projection with the horizontal

Illustration **:** A projectile is given an initial velocity of $\hat{i}+2\hat{j}$. Find the cartesian equation of its path. (g = 10 m s^{-2})

Soln.: $\tan\theta = \dfrac{u\sin\theta}{u\cos\theta} = \dfrac{2}{1}$

The desired equation is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\times 2 - \frac{10x^2}{2\left(\sqrt{(1)^2+(2)^2}\right)^2\left(\frac{1}{\sqrt{5}}\right)^2}$$

or $y = 2x - 5x^2$

Illustration **:** The maximum range of a projectile fired with some initial velocity is found to be 1000 m, in the absence of wind and air resistance. Find the maximum height h reached by this projectile.

Soln.: $R_{max} = \dfrac{u^2}{g} = 1000$ m

(R is maximum when θ = 45°)

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2}{2g}\times\sin^2 45° = \frac{u^2}{4g} = \frac{1000}{4} = 250\text{ m}$$

Self Test - 8

1. A projectile is given an initial velocity of $\hat{i}+2\hat{j}$. The cartesian equation of its path is (Take g = 10 m s^{-2})

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
(c) $y = 2x - 15x^2$ (d) $y = 2x - 25x^2$

2. A particle is thrown at angle 45° with the horizontal. If it remains in the air for 1 s, what was its initial velocity?

(a) 9.8 m/s (b) 6.93 m/s
(c) 19.6 m/s (d) 8.49 m/s

3. The angle between the velocity and the acceleration at the highest point of a particle projected upwards at an angle of 30° with the horizontal is

(a) 0° (b) 45° (c) 90° (d) 180°

4. The horizontal range is four times the maximum height attained by a projectile. The angle of projection is

(a) 90° (b) 60° (c) 45° (d) 30°

5. An arrow is projected into air. Its time of flight is 5 s and range 200 m. What is the maximum height reached by it? (Take g = 10 m s^{-2})

(a) 31.25 m (b) 24.5 m
(c) 18.25 m (d) 46.75 m

3.10 Uniform Circular Motion

When an object is moving on a circular path with a constant speed then motion of the object is said to be uniform circular motion.

Time Period

In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path. It is generally denoted by symbol T and its unit is second.

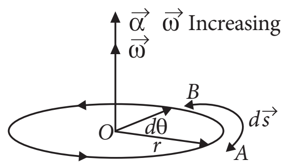
$$T = \frac{\text{Circumference of circle}}{\text{Linear velocity}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \quad (\because\ v = r\omega)$$

Frequency

- In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time. It is generally denoted by n. Its unit is s^{-1} or hertz (Hz).
- Relation between time period and frequency is $n = \dfrac{1}{T}$.
- Relation between angular velocity, frequency and time period, $\omega = \dfrac{2\pi}{T} = 2\pi n$

Angular Velocity

- It is defined as the time rate of change of angular displacement $(\vec{\theta})$ and is given by $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$
- It is directed along the axis of rotation. Angular velocity is a vector quantity. Its SI unit is rad s^{-1} and its dimensional formula is [M^{0}L^{0}T^{-1}].
- The direction of angular displacement and angular velocity is the same for the given particle performing circular motion.



Acceleration

The uniform circular motion (U.C.M.) is called accelerated motion, since the direction of velocity changes continuously but its magnitude remains constant.

Centripetal acceleration

- Acceleration acting on an object undergoing uniform circular motion is known as centripetal acceleration.
- Centripetal acceleration, $a_c = \dfrac{v^2}{r} = \omega^2 r$, where r is the radius of the circle.

- This equation can be expressed in vector form as,

$$\vec{a}_c = -\frac{v^2}{r}\hat{r} = -\omega^2\hat{r}$$

where $\hat{r}$ is unit vector in a direction of radius vector.
The negative sign in the expression for acceleration indicates that acceleration and radius vector are oppositely directed.
It always acts on an object along the radius towards the centre of the circular path.
Centripetal acceleration is not a constant vector.

Info Shots In projectile motion, both the magnitude and direction of acceleration remain constant throughout the motion, whereas in uniform circular motion, the magnitude of acceleration is constant its but direction changes continuously with time.

Illustration : An astronaut is rotating in a rotor of radius 4 m. If he can withstand upto acceleration of $10g$, then what is the maximum number of permissible revolutions? (g = 10 m s^{-2})

Soln.: In case of uniform circular motion

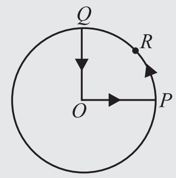
$$a_r = \frac{v^2}{r} = \omega^2 r \quad [\text{as } v = r\omega]$$

or $a_r = (2\pi f)^2 r$ [as $\omega = 2\pi f$]

or $f = \frac{1}{2\pi}\sqrt{\frac{a_r}{r}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{10\times 10}{4}} \quad i.e.\ f_{max} = \left[\frac{5}{2\pi}\right]\frac{\text{rev}}{\text{sec}}$$

Illustration : A cyclist starts from centre O of a circular park of radius 1 km and moves along the path $OPRQO$ as shown in figure. If he maintains constant speed of 10 m s^{-1}, what is his acceleration at point R ?



Soln.: The acceleration at point R is towards the centre of the circle.

Here, v = 10 m s^{-1}

r = 1 km = 1000 m

Centripetal acceleration,

$$a_c = \frac{v^2}{r} = \frac{(10\text{ m s}^{-1})^2}{1000\text{ m}} = 0.1\text{ m s}^{-2}$$

Info Shots In non-uniform circular motion, an object motion is along a circle, but the object speed is not constant. The object velocity vector is always tangent to the circle. The object has both normal as well as tangential acceleration

Resultant acceleration of the object, $a = \sqrt{a_r^2 + a_t^2}$

Also, $a_t = r\alpha$, $a_r = \frac{v^2}{r}$

Self Test - 9

1. Centripetal acceleration is
   (a) a constant vector
   (b) a constant scalar
   (c) a magnitude changing vector
   (d) not a constant vector

2. Velocity vector and acceleration vector in a uniform circular motion are related as
   (a) both in the same direction
   (b) perpendicular to each other
   (c) both in opposite direction
   (d) not related to each other

3. A body moving along a circular path of radius r with velocity v, has centripetal acceleration a. If its velocity is made equal to $2v$, then its centripetal acceleration is.
   (a) $4a$ (b) $2a$ (c) $a/4$ (d) $a/2$

4. A stone tied to the end of a string 100 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 22 s, then the acceleration of the stone is
   (a) 16 m s^{-2} (b) 4 m s^{-2}
   (c) 12 m s^{-2} (d) 8 m s^{-2}

5. In a uniform circular motion, the angle between the velocity and acceleration is
   (a) 0° (b) 45° (c) 60° (d) 90°

CONCEPT MAP

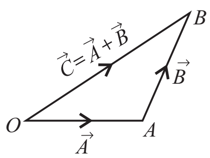
MOTION IN A PLANE

When a body moves in a plane, the motion is represented using any two axes of a co-ordinate system As the body moves both the dimensions change with passage of time.

Addition of Vectors

Triangle Law

If two vectors are represented by two sides of a triangle in same order, the resultant will be the third side but in opposite order, *i.e.*, $\vec{C} = \vec{A} + \vec{B}$



Multiplication by a Scalar Quantity

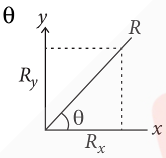
Multiplication of a vector by a scalar ϕ changes its magnitude by a factor of ϕ.

Resolution of Vectors

Horizontal component of $\vec{R}, R_x = R\cos\theta$

Vertical component of $\vec{R}, R_y = R\sin\theta$

$$R = \sqrt{R_x^2 + R_y^2}, \tan\theta = \left(\frac{R_y}{R_x}\right)$$



Rectangular components of 3-D vector

$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z \text{ or } \vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

$$\cos\alpha = \frac{R_x}{R} = \frac{R_x}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = l$$

$$\cos\beta = \frac{R_y}{R} = \frac{R_y}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = m$$

$$\cos\gamma = \frac{R_z}{R} = \frac{R_z}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = n$$

where l, m and n are called direction cosines of the vector $\vec{R}$.

$$\therefore\ l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{R_x^2 + R_y^2 + R_z^2}{R_x^2 + R_y^2 + R_z^2} = 1$$

Motion in Two Dimensions

Vector

Physical quantities having both magnitude and direction

Kinematics

During the rectilinear motion both the dimensions change simultaneously with passage of time.

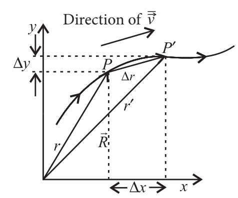
Position vector and displacement

Position vector of P, $\vec{r} = x\hat{i} + y\hat{j}$;

Position vector of P', $\vec{r}' = x'\hat{i} + y'\hat{j}$

Displacement $\overrightarrow{PP'}$,

$$\Delta\vec{r} = \vec{r}' - \vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$



Velocity

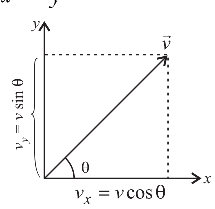
Average velocity, $\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$,

$\bar{\vec{v}} = \bar{v}_x\hat{i} + \bar{v}_y\hat{j}$

Instantaneous velocity, $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$,

$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$, $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Magnitude of $\vec{v}$ is $|\vec{v}| = v = \sqrt{v_x^2 + v_y^2}$ and direction of $\vec{v}$ is $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$



Acceleration

Average acceleration,

$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$, $\bar{\vec{a}} = \bar{a}_x\hat{i} + \bar{a}_y\hat{j}$

Instantaneous acceleration,

$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$, $\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$, $\vec{a} = a_x\hat{i} + a_y\hat{j}$

Kinematic Equations

$\vec{r} = \vec{r}_0 + \vec{u}_0 t + (1/2)\vec{a}t^2$

$\vec{v} = \vec{u} + \vec{a}t$

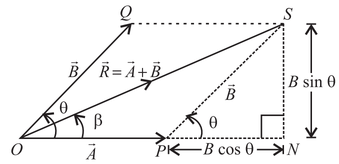
- $v_x = u_x + a_x t$ (along x-axis)
- $v_y = u_y + a_y t$ (along y-axis)
- $x = x_0 + u_x t + (1/2)a_x t^2$ (along x-axis)
- $y = y_0 + u_y t + (1/2)a_y t^2$ (along y-axis)

Parallelogram Law

For two co initial vectors represented by the two adjacent sides of a parallelogram, the diagonal of the parallelogram so formed will be the resultant.

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Direction of $\vec{R}$, $\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$



Non Uniform Circular Motion

A particle moves along a circular path with variable speed.

$a_r = \dfrac{v^2}{r}$, a_t = tangential acceleration

Resultant acceleration of the particle $a = \sqrt{a_t^2 + a_r^2}$

Tangential acceleration, $a_t = r\alpha$

Circular Motion

Uniform Circular Motion

A particle moves along a circular path with a constant speed.

Angular displacement, $\theta = \dfrac{s}{r}$

Angular velocity, $\omega = \dfrac{\theta}{t}$

Also, $\omega = \dfrac{2\pi}{T} = 2\pi\upsilon$

Linear velocity, $v = r\omega$

Centripetal acceleration, $a = \dfrac{v^2}{r} = r\omega^2$

Projectile Projected Horizontally

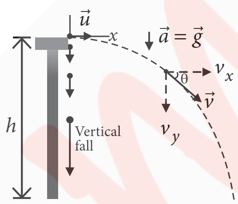
Equation of trajectory : $y = \dfrac{g}{2u^2} \cdot x^2$

Velocity after time t : $v = \sqrt{u^2 + g^2t^2}$

$\theta = \tan^{-1}\left(\dfrac{gt}{u}\right)$

Time of flight : $T = \sqrt{\dfrac{2h}{g}}$

Horizontal range : $R = u\sqrt{\dfrac{2h}{g}}$



Projectile Projected at an Angle with Horizontal

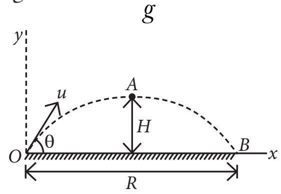
Position after time t : $x = (u\cos\theta)t$, $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$

Equation of trajectory : $y = x\tan\theta - \dfrac{g}{2u^2\cos^2\theta} \cdot x^2$

Maximum height : $H = \dfrac{u^2\sin^2\theta}{2g}$

Time of flight : $T = \dfrac{2u\sin\theta}{g}$

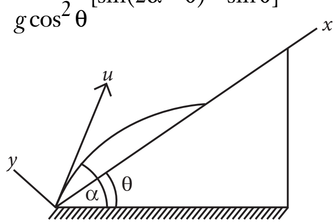
Horizontal range : $R = \dfrac{u^2\sin 2\theta}{g}$



Projectile Motion on an Inclined Plane

Time of flight, $T = \dfrac{2u\sin(\alpha - \theta)}{g\cos\theta}$

Range, $R = \dfrac{u^2}{g\cos^2\theta}[\sin(2\alpha - \theta) - \sin\theta]$



EXERCISE

KCET Connect

3.1 Introduction

1. Which among the following is not an example of two dimensional motion ?
(a) Movement of carrom coin in a carrom board
(b) Movement of mercury level in a thermometer
(c) A stone moving in a circular path
(d) All of these.

3.2 Scalars and Vectors

2. The scalar quantity among the following is
(a) weight of body
(b) temperature gradient
(c) elementary area
(d) electric potential.

3. Pick out the stranger in the group.
(a) Magnetic moment (b) Energy
(c) Moment of inertia (d) Magnetic potential

4. Which of the following is not a vector quantity ?
(a) Displacement (b) Velocity
(c) Potential energy (d) Acceleration

5. The magnitude of a vector cannot be
(a) positive (b) zero
(c) negative (d) unity.

6. Which of the following pairs of vectors are parallel?
(a) $\vec{A} = \hat{i} - 2\hat{j};\ \vec{B} = \hat{i} - 5\hat{j}$
(b) $\vec{A} = \hat{i} - 10\hat{j};\ \vec{B} = 2\hat{i} - 5\hat{j}$
(c) $\vec{A} = \hat{i} - 5\hat{j};\ \vec{B} = \hat{i} - 10\hat{j}$
(d) $\vec{A} = \hat{i} - 5\hat{j};\ \vec{B} = 2\hat{i} - 10\hat{j}$

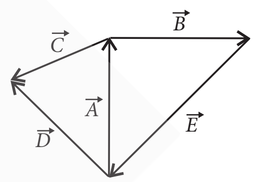
7. If vectors $\hat{i} - 3\hat{j} + 5\hat{k}$ and $\hat{i} - 3\hat{j} - a\hat{k}$ are equal vectors, then the value of a is
(a) 5 (b) 2 (c) – 3 (d) – 5.

3.3 Multiplication of Vectors by Real Numbers

8. If $\vec{A}$ is a vector of magnitude 8 units due north. What is the magnitude and direction of a vector $-8\vec{A}$?
(a) 64 units due east (b) 64 units due west
(c) 64 units due south (d) 8 units due east

3.4 Addition and Subtraction of Vectors – Gaphical Method

9. In figure, $\vec{E}$ equals

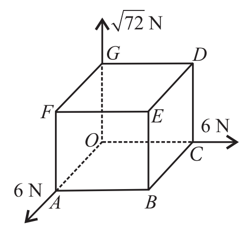


(a) $\vec{A}$
(b) $-\vec{A}$
(c) $\vec{A} + \vec{B}$
(d) $-(\vec{A} + \vec{B})$.

10. The maximum and minimum magnitude of the resultant of two given vectors are 17 units and 7 units respectively. Then, these two vectors have the magnitude.
(a) $P = 5, Q = 12$ (b) $P = 12, Q = 13$
(c) $P = 13, Q = 5$ (d) $P = 12, Q = 5$

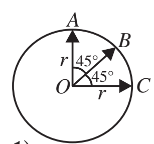
11. If the resultant of $\vec{A}$ and $\vec{B}$ makes angle α with $\vec{A}$ and β with $\vec{B}$, then
(a) $\alpha < \beta$ always (b) $\alpha < \beta$ if $A < B$
(c) $\alpha < \beta$ if $A > B$ (d) $\alpha < \beta$ if $A = B$.

12. Three forces of magnitudes 6 N, 6 N and $\sqrt{72}$ N act at corner of cube along three sides as shown in figure. Resultant of these forces is



(a) 12 N angle OB
(b) 18 N along OA
(c) 18 N along OC
(d) 12 N along OE.

13. The resultant of the three vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ shown in figure is



(a) r (b) $2r$
(c) $r(1+\sqrt{2})$ (d) $r(\sqrt{2}-1)$.

14. Find the direction of two forces, one of 6 N due east and other of 8 N due north.
(a) 53° 12′ (b) 55° 15′ (c) 45° (d) 60°

15. The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to the smaller force, then the forces are
(a) 6 N and 10 N (b) 8 N and 8 N
(c) 4 N and 12 N (d) 2 N and 14 N.

16. If vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ have magnitudes 5, 12 and 13 units and $\vec{P}+\vec{Q}=\vec{R}$, the angle between $\vec{Q}$ and $\vec{R}$ is

(a) $\cos^{-1}\left(\frac{5}{12}\right)$ (b) $\cos^{-1}\left(\frac{5}{13}\right)$

(c) $\cos^{-1}\left(\frac{12}{13}\right)$ (d) $\cos^{-1}\left(\frac{2}{13}\right)$.

17. The component of a vector is

(a) always less than its magnitude
(b) always greater than its magnitude
(c) always equal to its magnitude
(d) none of these.

3.5 Resolution of Vectors

18. $\vec{A}$ is a vector with magnitude A, then the unit vector $\hat{A}$ in the direction of $\vec{A}$ is

(a) $A\vec{A}$ (b) $\vec{A}\cdot\vec{A}$

(c) $\vec{A}\times\vec{A}$ (d) $\frac{\vec{A}}{A}$

19. The component of vector $\vec{A}=2\hat{i}+3\hat{j}$ along the direction of $(\hat{i}-\hat{j})$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$.

20. The angle made by $\vec{r}=3\hat{i}+3\hat{j}$ with x-axis is

(a) 30° (b) 60° (c) 45° (d) 90°

21. When unit vector $\hat{n}=a\hat{i}+b\hat{j}$ is perpendicular to $(\hat{i}+\hat{j})$, then a and b are

(a) 1, 0 (b) –2, 0

(c) 3, 0 (d) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$.

22. Let $\vec{A}=\frac{1}{\sqrt{2}}\cos\theta\,\hat{i}+\frac{1}{\sqrt{2}}\sin\theta\,\hat{j}$ be any vector. What will be the unit vector $\hat{n}$ in the direction of $\vec{A}$?

(a) $\cos\theta\,\hat{i}+\sin\theta\,\hat{j}$ (b) $-\cos\theta\,\hat{i}-\sin\theta\,\hat{j}$

(c) $\frac{1}{\sqrt{2}}(\cos\theta\,\hat{i}+\sin\theta\,\hat{j})$ (d) $\frac{1}{\sqrt{2}}(\cos\theta\,\hat{i}-\sin\theta\,\hat{j})$

23. Given : $\vec{A}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{B}=6\hat{i}+9\hat{j}+3\hat{k}$ which of the following statements is correct ?

(a) $\vec{A}$ and $\vec{B}$ are equal vectors.
(b) $\vec{A}$ and $\vec{B}$ are parallel vectors.
(c) $\vec{A}$ and $\vec{B}$ are perpendicular vectors.
(d) None of these.

24. The direction cosines of $\hat{i}+\hat{j}+\hat{k}$ are

(a) 1, 1, 1 (b) 2, 2, 2

(c) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

25. A unit vector perpendicular to both the vectors $2\hat{i}-2\hat{j}+\hat{k}$ and $3\hat{i}+4\hat{j}-5\hat{k}$, is

(a) $\frac{1}{\sqrt{401}}(6\hat{i}+13\hat{j}+14\hat{k})$

(b) $\frac{1}{\sqrt{401}}(6\hat{i}+13\hat{j}+14\hat{k})$

(c) $\frac{1}{\sqrt{401}}(6\hat{i}-13\hat{j}-14\hat{k})$

(d) $\frac{1}{\sqrt{405}}(6\hat{i}-13\hat{j}-4\hat{k})$

26. If a unit vector is represented by $(0.5\hat{i}+0.8\hat{j}+c\hat{k})$, then c^2 should be equal to

(a) 0.25 (b) 0.64
(c) 0.11 (d) 0.40.

27. If the vectors $\vec{a}=2\hat{i}-4\hat{j}-2\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}+x\hat{k}$ are at the right angles to each other, then the value of x should be

(a) 2 (b) –2 (c) 1 (d) –1.

28. What is the angle between $\hat{i}+\hat{j}+\hat{k}$ and $\hat{j}$?

(a) 0 (b) 45°
(c) 60° (d) None of these

29. If a body placed at the origin is acted upon by a force $\vec{F}=(\hat{i}+\hat{j}+\sqrt{2}\hat{k})$, then which of the following statements are correct.

(i) Magnitude of $\vec{F}$ is $(2+\sqrt{2})$.
(ii) Magnitude of $\vec{F}$ is 2.
(iii) $\vec{F}$ makes an angle of 45° with the Z-axis.
(iv) $\vec{F}$ makes an angle of 30° with the Z-axis.

(a) (i) and (ii) only (b) (ii) and (iii) only
(c) (iii) only (d) (i) and (iv) only

3.6 Vector Addition – Analytical Method

30. The components of the sum of two vectors $2\hat{i}+3\hat{j}$ and $2\hat{j}+3\hat{k}$ along x and y directions respectively are

(a) 2 and 5 (b) 4 and 6
(c) 2 and 6 (d) 4 and 3

31. Given $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -\hat{i} - \hat{j} - \hat{k}$. $(\vec{A} + \vec{B})$ will make angle with $\vec{A}$ as
(a) 0° (b) 180° (c) 90° (d) 60°

32. Which of the following quantities is dependent of the choice of orientation of the coordinate axes?
(a) $\vec{A} + \vec{B}$ (b) $A_x + B_y$
(c) $|\vec{A} + \vec{B}|$
(d) Angle between $\vec{A}$ and $\vec{B}$

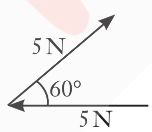
33. Let $|\vec{A_1}| = 3$, $|\vec{A_2}| = 5$ and $|\vec{A_1} + \vec{A_2}| = 5$. The value of $(2\vec{A_1} + 3\vec{A_2}) \cdot (3\vec{A_1} - 2\vec{A_2})$ is
(a) –106.5 (b) –99.5
(c) –118.5 (d) –112.5

34. What vector must be added to the sum of the two vectors $(2\hat{i} - 3\hat{j} + 4\hat{k})$ and $(-3\hat{i} - 2\hat{j} - 3\hat{k})$ so that the resultant is a unit vector along Z-axis?
(a) $5\hat{i} + \hat{j}$ (b) $-5\hat{i} - \hat{j}$
(c) $\hat{i} + 5\hat{j}$ (d) $-\hat{i} - 5\hat{j}$

35. For what angle between the two vectors, their resultant is maximum?
(a) 180° (b) zero (c) 90° (d) 45°

36. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is
(a) 0° (b) 60° (c) 90° (d) 120°.

37. Two forces, each numerically equal to 5 N, are acting as shown in the figure. Then the resultant is



(a) 2.5 N (b) 5 N
(c) $5\sqrt{3}$ N (d) 10 N.

38. Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $40\sqrt{3}$ N, the magnitude of each force is
(a) 40 N (b) 20 N (c) 80 N (d) 30 N.

39. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of $\vec{A}, \vec{B}$ and $\vec{C}$ are 5, 4 and 3 units respectively, the angle between $\vec{A}$ and $\vec{C}$ is
(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\cos^{-1}\left(\frac{4}{5}\right)$
(c) $\frac{\pi}{2}$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$.

40. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to the vector $\vec{A}$ and its magnitude is equal to half of the magnitude of vector $\vec{B}$. Then the angle between $\vec{A}$ and $\vec{B}$ is
(a) 30° (b) 45° (c) 150° (d) 120°.

41. Two forces in the ratio 1 : 2 act simultaneously on a particle. The resultant of these forces is three times the first force. The angle between them is
(a) 0° (b) 60° (c) 90° (d) 45°.

42. If $\sqrt{A^2 + B^2}$ represents the magnitude of resultant of two vectors $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$, then the angle between two vectors is
(a) $\cos^{-1}\left[-\frac{2(A^2 - B^2)}{(A^2 + B^2)}\right]$
(b) $\cos^{-1}\left[-\frac{A^2 - B^2}{A^2B^2}\right]$
(c) $\cos^{-1}\left[-\frac{(A^2 + B^2)}{2(A^2 - B^2)}\right]$
(d) $\cos^{-1}\left[-\frac{(A^2 - B^2)}{A^2 + B^2}\right]$.

43. If $\vec{a} + \vec{b} = \vec{c}$ and $a + b = c$, then the angle included between $\vec{a}$ and $\vec{b}$ is
(a) 90° (b) 180°
(c) 120° (d) zero.

44. The resultant of two forces $3P$ and $2P$ is R. If the first force is doubled, then the resultant is also doubled. The angle between the two forces is
(a) 120° (b) 60° (c) 180° (d) 90°.

45. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ acting at an angle of 150° is $\vec{R}$. If $\vec{R}$ is perpendicular to $\vec{Q}$, then magnitude of $\vec{P}$ is equal to
(a) R (b) $2R$
(c) Q (d) $2Q$.

46. The resultant of two forces $(A + B)$ and $(A - B)$ is a force $\sqrt{3A^2 + B^2}$. The angle between two given forces is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π.

47. Two vectors $\vec{A_1}$ and $\vec{A_2}$ each of magnitude A are inclined to each other such that their resultant is equal to $\sqrt{3}A$. Then the resultant of $\vec{A_1}$ and $-\vec{A_2}$ is
(a) $2A$ (b) $\sqrt{3}A$ (c) $\sqrt{2}A$ (d) A.

48. The resultant $\vec{R}$ of $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. Also $|\vec{P}| = |\vec{R}|$. The angle between $\vec{P}$ and $\vec{Q}$ is [tan 45° = 1]

(a) $\frac{5\pi}{4}$ (b) $\frac{7\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$.

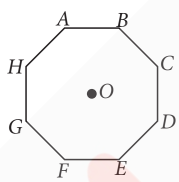
49. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is

(a) 45° (b) 180° (c) 0° (d) 90°.

50. The sum of magnitudes of two forces acting at a point is 16 and magnitude of their resultant is $8\sqrt{3}$. If the resultant is at 90° with the force of smaller magnitude, then their magnitudes are

(a) 3, 13 (b) 2, 14 (c) 5, 11 (d) 4, 12.

51. In an octagon *ABCDEFGH* of equal side, what is the sum of $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} + \overline{AG} + \overline{AH}$, if $\overline{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$?



(a) $16\hat{i} + 24\hat{j} + 32\hat{k}$

(b) $16\hat{i} + 24\hat{j} - 32\hat{k}$

(c) $-16\hat{i} - 24\hat{j} + 32\hat{k}$

(d) $16\hat{i} - 24\hat{j} + 32\hat{k}$

3.7 Motion in a Plane

52. The (*x*, *y*, *z*) coordinates of two points *A* and *B* are given respectively as (0, 4, –2) and (–2, 8, – 4). The displacement vector from *A* to *B* is

(a) $-2\hat{i} + 4\hat{j} - 2\hat{k}$ (b) $2\hat{i} - 4\hat{j} + 2\hat{k}$

(c) $2\hat{i} + 4\hat{j} - 2\hat{k}$ (d) $-2\hat{i} - 4\hat{j} - 2\hat{k}$

53. A person moves 30 m north, then 30 m east, then $30\sqrt{2}$ m south-west. His displacement from the original position is

(a) zero

(b) 28 m towards south

(c) 10 m towards west

(d) 15 m towards east.

54. A particle starts moving from point (2, 10, 1). Displacement for the particle is $8\hat{i} - 2\hat{j} + \hat{k}$. The final coordinates of the particle is

(a) (10, 8, 2) (b) (8, 10, 2)

(c) (2, 10, 8) (d) (8, 2, 10)

55. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, the displacement of the motorist at the third turn is

(a) 500 m (b) $500\sqrt{3}$ m

(c) 1000 m (d) $1000\sqrt{3}$ m

56. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is

(a) v (b) $\frac{\sqrt{3}}{2}v$ (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{2}$.

57. The velocity vector of the motion described by the position vector of a particle $\vec{r} = 2t\hat{i} + t^2\hat{j}$ is

(a) $\vec{v} = 2\hat{i} + 2t\hat{j}$ (b) $\vec{v} = 2t\hat{i} + 2t\hat{j}$

(c) $\vec{v} = t\hat{i} + t^2\hat{j}$ (d) $\vec{v} = 2\hat{i} + t^2\hat{j}$.

58. A particle has initial velocity $(2\hat{i} + 3\hat{j})$ and acceleration $(0.3\hat{i} + 0.2\hat{j})$. The magnitude of velocity after 10 seconds will be

(a) $9\sqrt{2}$ units (b) $5\sqrt{2}$ units

(c) 5 units (d) 9 units.

59. The initial velocity of a particle is $\vec{u} = 4\hat{i} + 3\hat{j}$. It is moving with uniform acceleration $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$ Its velocity after 10 seconds is

(a) 3 units (b) 4 units

(c) 5 units (d) 10 units.

60. The position of a particle is given by $\vec{r} = 3t\hat{i} + 2t^2\hat{j} + 5\hat{k}$, where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres. The direction of velocity of the particle at t = 1 s is

(a) 53° with *x*-axis (b) 37° with *x*-axis

(c) 30° with *y*-axis (d) 60° with *y*-axis.

61. The position vector of a particle is defined by $\vec{r} = 12t\hat{i} - 5t\hat{j} + 3\hat{k}$ metre.

The magnitude of velocity of the particle is

(a) 12 m s^{-1} (b) 13 m s^{-1}

(c) 5 m s^{-1} (d) 3 m s^{-1}.

62. A particle moving with a velocity equal to 0.4 m/s is subjected to an acceleration of 0.15 m/s^2 for 2 s in a direction at the right angle to its direction of motion. The resultant velocity is

(a) 0.7 m/s (b) 0.5 m/s

(c) 0.1 m/s

(d) Between 0.7 and 0.1 m/s

63. The velocity of a particle P due east is 4 m/s while that of Q is 3 m/s due south. The velocity of P with respect to Q will be

(a) $(4\hat{i}+3\hat{j})$ *E-N* (b) $(3\hat{i}+4\hat{j})$ *N-E*
(c) $(4\hat{i}-3\hat{j})$ *N-S* (d) $(3\hat{i}-4\hat{j})$ *S-N.*

64. For any arbitrary motion in space, which of the following relations is true?

(a) $\vec{v}_{\text{average}} = \frac{1}{2}[\vec{v}(t_1)+\vec{v}(t_2)]$
(b) $\vec{v}_{\text{average}} = \frac{\vec{r}(t_2)-\vec{r}(t_1)}{t_2-t_1}$
(c) $\vec{v}(t) = \vec{v}(0)+\vec{a}\,t$
(d) $\vec{r}(t) = \vec{r}(0)+\vec{v}(0)t+\frac{1}{2}\vec{a}t^2$

(The average stands for average of the quantity over the time interval t_1 to t_2)

65. Position of a particle in a horizontal plane is given by $x = 6t$ m and $y = (8t - 5t^2)$ m. The velocity with which the particle starts moving ?

(a) 8 m/s (b) 6 m/s (c) 10 m/s (d) 0 m/s

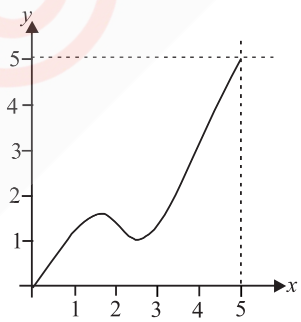
66. In the above problem, the direction of the initial velocity with the x-axis is

(a) $\tan^{-1}\frac{3}{4}$ (b) $\tan^{-1}\frac{4}{3}$
(c) $\sin^{-1}\frac{3}{4}$ (d) $\cos^{-1}\frac{3}{4}$.

67. A train is moving north with speed 20 m s^{-1}. If it turns west with same speed, then the change in velocity will be

(a) $20\sqrt{2}$ m s^{-1} SW (b) $20\sqrt{2}$ m s^{-1} NW
(c) 40 m s^{-1} NE (d) 20 m s^{-1} SE.

68. A particle moves on a path as shown. The particle takes 10 seconds in going from starting point to the final point. What is the average velocity vector of the particle?



(a) $0.5\hat{i}+\hat{j}$
(b) $0.5\hat{i}+2.5\hat{j}$
(c) $0.5\hat{i}+0.5\hat{j}$ (d) None of these

69. The coordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by

(a) $\sqrt{\alpha^2+\beta^2}$ (b) $3t\sqrt{\alpha^2+\beta^2}$
(c) $3t^2\sqrt{\alpha^2+\beta^2}$ (d) $t^2\sqrt{\alpha^2+\beta^2}$

70. A particle motion on a shape curve is governed by $x = 2\sin t$, $y = 3\cos t$ and $z = \sqrt{5}\sin t$. What is the magnitude of velocity of the particle at any time t ?

(a) $3\sqrt{2}\sin t$ (b) 3
(c) $3\sqrt{2}\cos t$ (d) $3\sqrt{2}$

71. A particle moves in the x-y plane with velocity $v_x = 8t - 2$ and $v_y = 2$. If it passes through the point $x = 14$ and $y = 4$ at $t = 2$ s, then the equation of the path is

(a) $x = y^3 - y^2 + 2$ (b) $x = y^2 - y + 2$
(c) $x = y^2 - 3y + 2$ (d) $x = y^3 - 2y^2 + 2$

72. A body moves in a plane so that the displacements along the x and y axes are given by $x = 3t^3$ and $y = 4t^3$. The velocity of the body is

(a) $9t$ (b) $15t$ (c) $15t^2$ (d) $25t^2$

73. A particle has initial velocity $(3\hat{i}+4\hat{j})$ and has acceleration $(0.4\hat{i}+0.3\hat{j})$. Its speed after 10 s is

(a) 7 units (b) $7\sqrt{2}$ units
(c) 8.5 units (d) 10 units.

74. A particle is moving with velocity 5 m/s towards east and its velocity changes to 5 m/s north in 10 s, find the acceleration.

(a) $\sqrt{2}$ North-West (b) $\frac{1}{\sqrt{2}}$ North-West
(c) $\frac{1}{\sqrt{2}}$ North-East (d) $\sqrt{2}$ North-East

75. The x and y co-ordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$ where x and y are in m and t in s. The acceleration of the particle at 5 s is

(a) zero (b) 8 m s^{-2}
(c) 20 m s^{-2} (d) 40 m s^{-2}.

76. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in metres and t in seconds. The acceleration of the particle at $t = 2$ s is

(a) 5 m s^{-2} (b) –4 m s^{-2}
(c) –8 m s^{-2} (d) 0

77. The position of a particle is given by $\vec{r} = 2t^2\hat{i} + 3t\hat{j} + 4\hat{k}$, where t is in second and the coefficients have proper units for $\vec{r}$ to be in metre. The $\vec{a}(t)$ of the particle at $t = 1$ s is

(a) 4 m s^{-2} along y-direction
(b) 3 m s^{-2} along x-direction
(c) 4 m s^{-2} along x-direction
(d) 2 m s^{-2} along z-direction.

78. If the magnitude of acceleration is A, then at all the times

(a) $A \propto y$ (b) $A \propto x$

(c) $A \propto \sqrt{x^2 + y^2}$ (d) $A = 0$

79. The acceleration of a particle is $a = 3t^2\hat{i} + 5t\hat{j} - (8t^3 + 400)\hat{k}$ ms^{-2}. The change in velocity from $t = 0$ to $t = 10$ s is $n_1\hat{i} + n_2\hat{j} - n_3\hat{k}$

(a) $n_1 = 1000$ m s^{-1} (b) $n_2 = 250$ m s^{-1}

(c) $n_3 = 24000$ m s^{-1} (d) all of these.

3.8 Motion in a Plane with Constant Acceleration

80. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i} + 4.0\hat{j})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is

(a) 50 (b) 40 (c) 32 (d) 60

81. A particle starts from the origin at $t = 0$ s with a velocity of $10\hat{j}$ m s^{-1} and moves in the $x - y$ plane with a constant acceleration of $(8\hat{i} + 2\hat{j})$ m s^{-2}. At an instant when the x-coordinate of the particle is 16 m, y-coordinate of the particle is

(a) 16 m (b) 28 m (c) 36 m (d) 24 m

3.9 Projectile Motion

82. In the entire path of a projectile, the quantity that remains unchanged is

(a) vertical component of velocity
(b) horizontal component of velocity
(c) kinetic energy
(d) potential energy.

83. If a body is projected with an angle θ to the horizontal, then

(a) its velocity is always perpendicular to its acceleration
(b) its velocity becomes zero at its maximum height
(c) its velocity makes zero angle with the horizontal at its maximum height
(d) the body just before hitting the ground, the direction of velocity coincides with the acceleration.

84. From the top of a tower a body A is projected vertically upwards, another body B is horizontally thrown and a third body C is thrown vertically downwards with the same velocity

(a) B strikes the ground with more velocity
(b) C strikes the ground with less velocity
(c) A, B, C strike the ground with same velocity
(d) A and C strike the ground with more velocity than B.

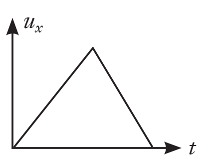
85. A ball of mass m is projected upward with a speed v_0. The speed at a height h is (Neglecting air resistance)

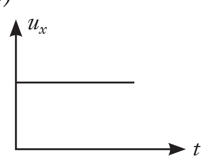
(a) independent of angle and direction of projection
(b) independent of mass, angle and the direction of projection
(c) dependent on the direction of projection
(d) dependent on the shape, size and mass of the ball and angle of projection

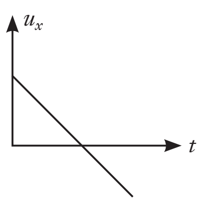
86. Two particles A and B having different masses are projected from a tower with same speed. A is projected vertically upward and B vertically downward. On reaching the ground

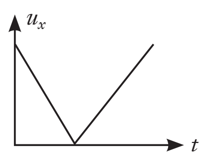
(a) velocity of A is greater than that of B.
(b) velocity of B is greater than that of A.
(c) both A and B attain the same velocity.
(d) the particle with the larger mass attains higher velocity.

87. In case of projectile motion, which one of the following figures represent variation of horizontal component of velocity (u_x) with time t ? (Assume that air resistance is negligible)

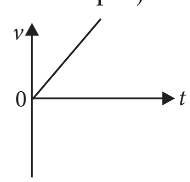
(a) 

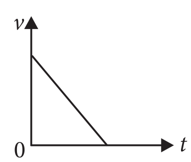
(b) 

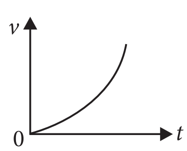
(c) 

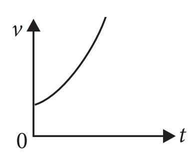
(d) 

88. Which of the following graphs represents the speed v of a projectile as a function of time t?

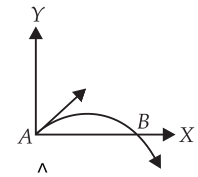
(a) 

(b) 

(c) 

(d) 

89. The velocity of a projectile at the initial point A is $(2\hat{i}+3\hat{j})$ m/s. Its velocity (in m/s) at point B is



(a) $2\hat{i}-3\hat{j}$ (b) $2\hat{i}+3\hat{j}$
(c) $-2\hat{i}-3\hat{j}$ (d) $-2\hat{i}+3\hat{j}$

90. A mountaineer standing on the edge of a cliff 441 m above the ground throws a stone horizontally with an initial speed of 20 m s^{-1}. What is the speed with which the stone reaches the ground?
(a) 90 m s^{-1} (b) 95.08 m s^{-1}
(c) 85 m s^{-1} (d) 92 m s^{-1}

91. The height y and the distance x along the horizontal plane of a projectile on a certain planet (with no surrounding atmosphere) are given by $y = 8t - 5t^2$ m and $x = 6t$ m, where t is in seconds. The velocity with which the projectile is projected is (Acceleration due to gravity = 9.8 m s^{-2})
(a) 6 m s^{-1} (b) 8 m s^{-1}
(c) 10 m s^{-1} (d) 14 m s^{-1}

92. A projectile is fired at an angle of 45° and reaches the highest point in its path after $2\sqrt{2}$ s. Find the velocity of projectile in m s^{-1}.
(a) 19.6 (b) 39.2 (c) 9.8 (d) 4.9

93. A body projected at an angle with the horizontal has a range 300 m. If the time of flight is 6 s, then the horizontal component of velocity is
(a) 30 m s^{-1} (b) 50 m s^{-1}
(c) 40 m s^{-1} (d) 45 m s^{-1}

94. An object is projected at an angle θ to the horizontal in a gravitational field and it follows a parabolic path $PQRST$. These points are the positions of the object after successive equal time intervals, T being the highest point reached. The displacements PQ, QR and ST
(a) are equal
(b) decrease at a constant rate
(c) have equal horizontal components
(d) increase at a constant rate.

95. A shell fired from a gun at sea level rises to a maximum height of 5 km when fired at a ship 20 km away. The muzzle velocity should be
(a) 107 m/s (b) 443 m/s
(c) 528 m/s (d) 356 m/s

96. The vertical component of velocity of a projectile at its maximum height (u - velocity of projection, θ - angle of projection) is
(a) $u\sin\theta$ (b) $u\cos\theta$ (c) $\dfrac{u}{\sin\theta}$ (d) 0

97. An aeroplane flying horizontally at a speed of 98 m s^{-1} releases an object which reaches the ground in 10 s. The angle made by the velocity of the object with the horizontal at the time of hitting the ground is
(a) 30° (b) 45° (c) 75° (d) 60°

98. A body is thrown up with a speed u, at an angle of projection θ. If the speed of the projectile becomes $\dfrac{u}{\sqrt{2}}$ on reaching the maximum height, the maximum vertical height attained by the projectile is
(a) $\dfrac{u^2}{4g}$ (b) $\dfrac{u^2}{3g}$ (c) $\dfrac{u^2}{2g}$ (d) $\dfrac{u^2}{g}$

99. A ball is projected from the ground with a speed 15 m s^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then 'tanθ' will be equal to
(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$ (c) 2 (d) 4

100. A cannon ball has a range R on a horizontal plane, such that the corresponding possible maximum heights reached are H_1 and H_2. Then, the correct expression for R is
(a) $\dfrac{H_1 + H_2}{2}$ (b) $(H_1H_2)^{1/2}$
(c) $2(H_1H_2)^{1/2}$ (d) $4(H_1H_2)^{1/2}$

101. The speed of a projectile at its maximum height is $\dfrac{\sqrt{3}}{2}$ times its initial speed. If the range of the projectile is P times the maximum height attained by it, then P is equal to
(a) $\dfrac{4}{3}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) $\dfrac{3}{4}$

102. The maximum range of a projectile fired with some initial velocity is found to be 1000 metre, in the absence of wind and air resistance. The maximum height h reached by this projectile is
(a) 250 m (b) 500 m
(c) 1000 m (d) 2000 m

103. The range of a particle when launched at an angle of 15° with the horizontal is 150 m. What is the range of projectile when launched at an angle of 45° to the horizontal?
(a) 150 m (b) 300 m (c) 450 m (d) 600 m

104. Three projectiles A, B and C are projected at an angle of 30°, 45°, 60° respectively. If R_A, R_B and R_C are ranges of A, B and C respectively then (velocity of projection is same for A, B and C)
(a) $R_A = R_B = R_C$ (b) $R_A = R_C > R_B$
(c) $R_A < R_B < R_C$ (d) $R_A = R_C < R_B$

105. The horizontal range of a projectile is maximum when the angle of projection is
(a) 0° (b) 30° (c) 45° (d) 60°

106. For which of the following pair of angles, horizontal range is same?
(a) 30°, 45° (b) 40°, 50°
(c) 30°, 50° (d) 30°, 40°

107. A projectile is thrown with an initial velocity of $\vec{v} = (p\hat{i} + q\hat{j})$ m/s. If the range of the projectile is double the maximum height reached by it, then
(a) $p = 2q$ (b) $q = 4p$
(c) $q = 2p$ (d) $q = p$

108. An artillery piece which consistently shoot its shells with the same muzzle speed has a maximum range R. To hit a target which is $R/2$ from the gun and on the same level, the elevation angle of the gun should be
(a) 15° (b) 45° (c) 30° (d) 60°

109. Two projectiles P and Q thrown with velocities v and $\frac{v}{2}$ respectively have the same range. If Q is thrown at an angle of 15° to the horizontal, P must be thrown at an angle of
(a) 30° (b) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$
(c) $\frac{1}{4}\sin^{-1}\left(\frac{1}{2}\right)$ (d) 60°

110. The equation of projectile is $y = ax - bx^2$ where a and b are the constants. Its horizontal range is
(a) ab (b) $\frac{a}{b}$ (c) b (d) a

111. If the angles of projection of a projectile with same initial velocity exceed or fall short of 45° by equal amounts α, then the ratio of horizontal ranges is
(a) 1 : 2 (b) 1 : 3 (c) 1 : 4 (d) 1 : 1

112. A particle is projected from origin in XY-plane at angle 37° with X-axis with a speed of 10 m s^{-1}. Its acceleration is $\vec{a} = \left(-5\hat{i} + 10\hat{j}\right)$ m s^{-2}. Find the speed of particle where its x-coordinate is zero.
(a) 10 ms^{-1} (b) 38 ms^{-1}
(c) 6 ms^{-1} (d) 38.8 ms^{-1}

3.10 Uniform Circular Motion

113. Uniform circular motion is an example of
(a) constant speed motion
(b) constant velocity motion
(c) non-accelerated motion
(d) zero accelerated motion.

114. A body moves in a circle covers equal distance in equal intervals of time. Which of the following remains constant?
(a) Velocity (b) Acceleration
(c) Speed (d) Displacement

115. Which one of the following statements is not correct in uniform circular motion?
(a) The speed of the particle remains constant.
(b) The acceleration always points towards the centre.
(c) The angular speed remains constant.
(d) The velocity remains constant.

116. For a particle performing uniform circular motion, choose the incorrect statement from the following.
(a) Magnitude of particle velocity (speed) remains constant.
(b) Particle velocity remains directed perpendicular to radius vector.
(c) Direction of acceleration keeps changing as particle moves.
(d) Magnitude of acceleration does not remain constant.

117. A particle is moving on a circular path of radius r with uniform speed v. What is the displacement of the particle after it has described an angle of 60°?
(a) $r\sqrt{2}$ (b) $r\sqrt{3}$ (c) r (d) $2r$

118. Which of the following statements is incorrect ?
(a) In one dimension motion, the velocity and the acceleration of an object are always along the same line.
(b) In two or three dimensions, the angle between velocity and acceleration vectors may have any value between 0°and 180°.
(c) The kinematic equations for uniform acceleration can be applied in case of a uniform circular motion.
(d) The resultant acceleration of an object in circular motion is towards the centre only if the speed is constant.

119. A particle is moving on a circular path of 10 m radius. At any instant of time, its speed is 5 m s^{-1} and the speed is increasing at a rate of 2 m s^{-2}. The magnitude of net acceleration at this instant is
(a) 5 m s^{-2} (b) 2 m s^{-2}
(c) 3.2 m s^{-2} (d) 4.3 m s^{-2}

120. a_r and a_t represent radial and tangential accelerations respectively. The motion of the particle is uniformly circular only if
(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ and $a_t \neq 0$
(c) $a_r \neq 0$ and $a_t = 0$ (d) $a_r \neq 0$ and $a_t \neq 0$

121. Centripetal acceleration is
(a) a constant vector
(b) a constant scalar
(c) a magnitude changing vector
(d) not a constant vector.

122. The earth moves round the sun in a near circular orbit of radius 1.5×10^{11} m. Its centripetal acceleration is
(a) 1.5×10^{-3} m/s^2 (b) 3×10^{-3} m/s^2
(c) 6×10^{-3} m/s^2 (d) 12×10^{-3} m/s^2

123. In uniform circular motion, the centripetal acceleration is
(a) towards the centre of the circular path and perpendicular to the instantaneous velocity
(b) a constant acceleration
(c) away from the centre of the circular path and perpendicular to the instantaneous velocity
(d) a variable acceleration making 45° with the instantaneous velocity.

124. The radii of circular paths of two particles of same mass are in ratio 6 : 8 then what will be velocities ratio if they have a constant centripetal force?
(a) $\sqrt{3}:4$ (b) $4:\sqrt{3}$ (c) $2:\sqrt{3}$ (d) $\sqrt{3}:2$

125. An object moves at a constant speed along a circular path in a horizontal X-Y plane, with the centre at the origin. When the object is at $x = -2$ m, its velocity is $-4\hat{j}$ m s^{-1}. What is the object's acceleration when it is $y = 2$ m?
(a) $-8\hat{j}$ m s^{-2} (b) $-8\hat{i}$ m s^{-2}
(c) $-4\hat{j}$ m s^{-2} (d) $4\hat{i}$ m s^{-2}

126. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. The linear speed of the insect is
(a) 4.3 cm s^{-1} (b) 5.3 cm s^{-1}
(c) 6.3 cm s^{-1} (d) 7.3 cm s^{-1}

127. What is the ratio of the angular speeds of the minute hand and second hand of a clock ?
(a) 1 : 12 (b) 12 : 1 (c) 1 : 60 (d) 60 : 1

128. Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be
(a) 1 : 1 (b) $r_A : r_B$ (c) $v_A : v_B$ (d) $r_B : r_A$

129. If the length of second's hand of a clock is 10 cm, the speed of its tip (in cm s^{-1}) is nearly
(a) 2 (b) 0.5 (c) 1.5 (d) 1

130. A particle is moving with a constant speed v in a circle. What is the magnitude of average velocity after half rotation?
(a) $2v$ (b) $\frac{2v}{\pi}$ (c) $\frac{v}{2}$ (d) $\frac{v}{2\pi}$

131. The angular velocity of earth due to its spin motion is
(a) 7.27×10^5 rad/s (b) 2.62×10^3 rad/s
(c) 2.62×10^{-1} rad/s (d) 7.27×10^{-5} rad/s

132. A particle moves with a uniform speed v and time period T in a circular path of radius r. If the speed of the particle is doubled, its new time period is
(a) T (b) $\frac{T}{2}$ (c) $2T$ (d) $\frac{T}{4}$

133. Two particles having mass M and m are moving in a circular path having radius R and r. If their time periods are same, then the ratio of angular velocity will be
(a) $\frac{r}{R}$ (b) $\frac{R}{r}$ (c) 1 (d) $\sqrt{\frac{R}{r}}$

134. The angular speed of a fly wheel making 120 rpm is
(a) 2π rad s^{-1} (b) π rad s^{-1}
(c) 4π rad s^{-1} (d) $4\pi^2$ rad s^{-1}

135. A stone tied to the end of a string 100 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 22 s, then the acceleration of the stone is
(a) 16 m s^{-2} (b) 4 m s^{-2}
(c) 12 m s^{-2} (d) 8 m s^{-2}

136. A particles is performing a uniform circular motion along circle of radius 'R'. In half the period of revolution, its displacement and distance covered are respectively
(a) $\sqrt{2}R, 2\pi R$ (b) $R, \pi R$
(c) $2R, 2\pi R$ (d) $2R, \pi R$

137. A stone tied to the end of a string 100 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the acceleration of the stone?

(a) $\left(\frac{88}{25}\right)^2$ m s^{-2} (b) $\left(\frac{25}{88}\right)^2$ m s^{-2}

(c) $\left(\frac{88}{25}\right)$ m s^{-2} (d) $\left(\frac{25}{88}\right)$ m s^{-2}

138. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 r.p.m., the acceleration of a point on the tip of the blade is

(a) 1600 m s^{-2} (b) 47.4 m s^{-2}

(c) 23.7 m s^{-2} (d) 50.55 m s^{-2}

139. An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1200 rpm, the acceleration of a point on the tip of the blade is about

(a) 1600 m s^{-2} (b) 4740 m s^{-2}

(c) 2370 m s^{-2} (d) 5055 m s^{-2}

140. A stone tied at the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 25 revolutions in 14 s, what is the magnitude of acceleration of the stone?

(a) 90 m/s^2 (b) 100 m/s^2

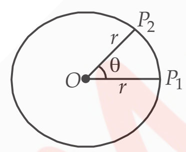
(c) 110 m/s^2 (d) 120 m/s^2

KCET Ready

1. A unit vector is represented as $(0.8\hat{i} + b\hat{j} + 0.4\hat{k})$. Hence, the value of '*b*' must be

(a) 0.4 (b) $\sqrt{0.6}$ (c) 0.2 (d) $\sqrt{0.2}$

2. A particle is moving on circular path as shown in the figure. Then displacement from P_1 to P_2 is



(a) $2r\cos\frac{\theta}{2}$ (b) $2r\tan\frac{\theta}{2}$

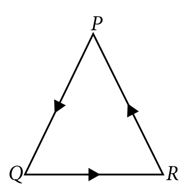
(c) $2\,r\sin\theta$ (d) $2r\sin\frac{\theta}{2}$

3. A vector $\vec{A}$ is rotated by a small angle $\Delta\theta$ radians ($\Delta\theta << 1$) to get a new vector $\vec{B}$. In that case $|\vec{B} - \vec{A}|$ is

(a) 0 (b) $|\vec{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$

(c) $|\vec{A}|\Delta\theta$ (d) $|\vec{B}|\Delta\theta - |\vec{A}|$

4. Three particles *P*, *Q* and *R* are at rest at the vertices of an equilateral triangle of side *s*. Each of the particles starts moving with constant speed *v* m s^{-1}. *P* is moving along *PQ*, *Q* along *QR* and *R* along *RP*. The particles will meet each other at time *t* given by



(a) $\frac{s}{v}$ (b) $\frac{3s}{v}$ (c) $\frac{3s}{2v}$ (d) $\frac{2s}{3v}$

5. Motion of a particle in *x*-*y* plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$ m. The path of the particle will be

(a) circular (b) helical

(c) parabolic (d) elliptical.

6. Four persons *K*, *L*, *M* and *N* are initially at the corners of a square of side of length *d*. If every person starts moving with the same speed *v* such that *K* is always headed towards *L*, *L* towards *M*, *M* is headed directly towards *N* and *N* towards *K*, then the four persons will meet after

(a) $\frac{d}{v}$ (b) $\frac{\sqrt{2}d}{v}$ (c) $\frac{d}{\sqrt{2}v}$ (d) $\frac{d}{2v}$

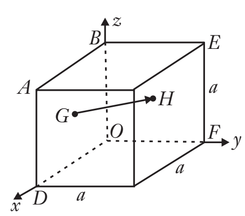
7. Two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ have a resultant $\vec{R}$ which makes an angle α with $\vec{A}$ and angle β with $\vec{B}$. Let the magnitudes of the vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ be represented by *A*, *B* and *R* respectively. Which of the following relations is not correct?

(a) $\frac{R}{\sin(\alpha+\beta)} = \frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$

(b) $R\sin\alpha = B\sin(\alpha + \beta)$

(c) $A\sin\alpha = B\sin\beta$ (d) $R\sin\beta = A\sin(\alpha + \beta)$

8. In the cube of side *a* shown in the figure, the vector from the central point of the face *ABOD* to the central point of the face *BEFO* will be



(a) $\frac{1}{2}a(\hat{j} - \hat{k})$

(b) $\frac{1}{2}a(\hat{j} - \hat{i})$

(c) $\frac{1}{2}a(\hat{k} - \hat{i})$ (d) $\frac{1}{2}a(\hat{i} - \hat{k})$

9. A ball falls from a table top with initial horizontal speed v_0. In the absence of air resistance, correct statement is
(a) The vertical component of the acceleration changes with time.
(b) The horizontal component of the velocity does not change with time.
(c) The horizontal component of the acceleration is non-zero and finite.
(d) The time taken by the ball to touch the ground depends on v_0.

10. A certain vector in the xy plane has an x-component of 12 m and a y-component of 8 m. It is then rotated in the xy plane so that its x-component is halved. Then its new y-component is approximately
(a) 14 m (b) 13.11 m
(c) 10 m (d) 2.0 m

11. A ball is projected with a speed v_0 at an angle α from a point on the playground. Then, its velocity is perpendicular to the initial velocity of projection at $t = v_0/g\sin\alpha$ for
(a) $0 < \alpha \le \pi/2$ (b) $0 < \alpha \le \pi/4$
(c) $\pi/4 \le \alpha \le \pi/2$ (d) all values of α.

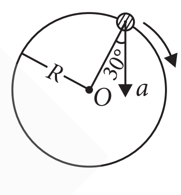
12. A particle is moving with a uniform speed v in a circular path of radius r with the centre at O. When the particle moves from a point P to Q on the circle such that $\angle POQ = \theta$, then the magnitude of the change in velocity is
(a) $2v\sin(2\theta)$ (b) zero
(c) $2v\sin\left(\frac{\theta}{2}\right)$ (d) $2v\cos\left(\frac{\theta}{2}\right)$.

13. The position vector of a particle $\vec{R}$ as a function of time is given by $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$. Where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x-and y-directions, respectively. Which one of the following statements is wrong for the motion of particle?
(a) Magnitude of the velocity of particle is 8π m/s.
(b) Path of the particle is a circle of radius 4π meter.
(c) Acceleration vector is along $-\vec{R}$.
(d) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle.

14. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction such that it is always directed towards the particle at the next consecutive corner. Calculate the time after which the particles will meet each other.
(a) $\frac{a}{2v}$ (b) $\frac{2a}{v}$ (c) $\frac{v}{2a}$ (d) $\frac{2v}{a}$

15. Two seconds after projection, a projectile is moving at 30° above the horizontal, after one more second it is moving horizontally. The initial speed of the projectile is (g = 10 m s^{-2})
(a) 10 m s^{-1} (b) $10\sqrt{3}$ m s^{-1}
(c) 20 m s^{-1} (d) $20\sqrt{3}$ m s^{-1}

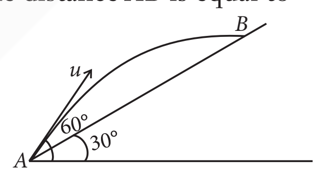
16. In the given figure, a = 15 m s^{-2} represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is



(a) 4.5 m s^{-1} (b) 5.0 m s^{-1}
(c) 5.7 m s^{-1} (d) 6.2 m s^{-1}.

17. If the sum of two unit vectors is also a unit vector, then magnitude of their difference is
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) 3

18. Time taken by the projectile to reach from A to B is t, then the distance AB is equal to



(a) $2ut$ (b) $\sqrt{3}\,ut$ (c) $\frac{\sqrt{3}}{2}ut$ (d) $\frac{ut}{\sqrt{3}}$.

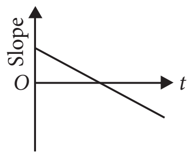
19. A person from a truck, moving with a constant speed of 60 km h^{-1}, throws a ball upwards with a speed of 60 km h^{-1}. Neglecting the effect of rotation of Earth, choose the correct answer from the given choices.
(a) The person cannot catch the ball when it comes down since the truck is moving
(b) The person can catch the ball when it comes down, if the truck is stopped immediately after throwing the ball
(c) The person can catch the ball when it comes down, if the truck moves with speed less than 60 km h^{-1} but does not stop
(d) The person can catch the ball when it comes down, if the truck continues to move with a constant speed of 60 km h^{-1}

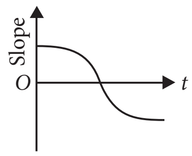
20. A projectile moves from the ground such that its horizontal displacement is $x = Kt$ and vertical displacement is $y = Kt(1 - \alpha t)$, where K and α are constants and t is time. Find out total time of flight (T) and maximum height attained (y_{max}).

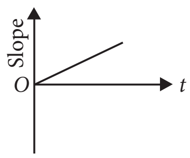
(a) $T = \alpha,\ y_{max} = \frac{K}{2\alpha}$ (b) $T = \frac{1}{\alpha},\ y_{max} = \frac{2K}{\alpha}$

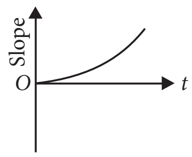
(c) $T = \frac{2}{\alpha},\ y_{max} = \frac{K}{6\alpha}$ (d) $T = \frac{1}{\alpha},\ y_{max} = \frac{K}{4\alpha}$

21. A heavy particle is projected with a velocity at an angle θ with the horizontal into a uniform gravitational field. The slope of the trajectory of the particle varies as

(a)


(b)


(c)


(d)


22. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be [Use $g = 10$ m s^{-2}]

(a) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(c) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (d) $\cot^{-1}\left(\frac{R}{20t^2}\right)$.

23. A fighter plane, flying horizontally with a speed 360 km h^{-1} at an altitude of 500 m drops a bomb for a target straight ahead of it on the ground. The bomb should be dropped at what approximate distance ahead of the target? Assume that acceleration due to gravity (g) is 10 m s^{-2}. Also neglect air drag.

(a) 1000 m (b) $50\sqrt{5}$ m

(c) $500\sqrt{5}$ m (d) 866 m

24. In U.C.M., when time interval $\delta t \to 0$, the angle between change in velocity $(\delta\vec{v})$ and linear velocity $(\vec{v})$ will be

(a) 0° (b) 90° (c) 180° (d) 45°

25. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10$ m s^{-2})

(a) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ m s^{-1}

(b) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ m s^{-1}

(c) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m s^{-1}

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ m s^{-1}.

26. A particle A is projected from the ground with an initial velocity of 10 m s^{-1} at an angle of 60° with horizontal. From what height should another particle B be projected horizontally with velocity 5 m s^{-1} so that both the particles collide in ground at point C if both are projected simultaneously. (Take $g = 10$ m s^{-2})

(a) 10 m (b) 15 m (c) 20 m (d) 30 m

27. A bomb is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is

(a) straight line vertically down the plane
(b) parabola in the direction opposite to the motion of plane
(c) parabola in the direction of motion of plane
(d) hyperbola.

28. The equation of motion of a projectile are given by $x = 36\,t$ metre and $2y = 96\,t - 9.8\,t^2$ metre. The angle of projection will be

(a) $\theta = \sin^{-1}\left(\frac{3}{5}\right)$ (b) $\theta = \sin^{-1}\left(\frac{4}{5}\right)$

(c) $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ (d) None of these

29. A stone is projected from ground level with speed u and at an angle θ with horizontal. Some how the acceleration due to gravity (g) becomes double (that is $2g$) immediately after the stone reaches the maximum height and remains same thereafter. Assuming direction of acceleration due to gravity to be vertically downwards, the horizontal range of the particle is $\frac{u^2 \sin 2\theta}{2g}\left(1 + \frac{1}{x}\right)$. Then the value of x is

(a) 1.41 (b) 1.73 (c) 2.24 (d) 2.45

30. A projectile is fired from the surface of the earth with a velocity of 5 m s^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 m s^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in m s^{-2}) is (Given $g_e = 9.8$ m s^{-2})

(a) 3.5 (b) 5.9

(c) 16.3 (d) 110.8

31. A particle moves in a circle with constant speed v. The angular separation of two points on the circumference of the circle is 60°. When the particle moves between the two points, the change in velocity is

(a) $0.5\,v$ (b) v
(c) $1.5\,v$ (d) $2.0\,v$

32. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, where a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is

(a) 1 (b) 1.5 (c) 2.0 (d) 2.5

33. A cricketer can throw a ball to a maximum horizontal distance of 100 m. With the same speed how much high above the ground can the cricketer throw the same ball?

(a) 50 m (b) 100 m (c) 150 m (d) 200 m

34. A motorboat is racing towards north at 25 km h^{-1} and the water current in that region is 10 km h^{-1} in the direction of 60° east of south. The resultant velocity of the boat is

(a) 11 km h^{-1} (b) 22 km h^{-1}
(c) 33 km h^{-1} (d) 44 km h^{-1}.

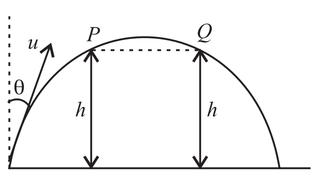
35. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$ where α is dimensionless constant. The displacement of the particle varies with time as

(a) t^3 (b) t^2 (c) t (d) $t^{1/2}$.

36. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with speed of 10 m s^{-1}, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration (in m/s^2) of the train is

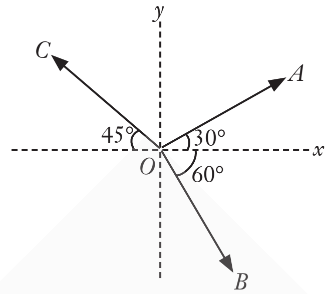
(a) 1 (b) 3 (c) 5 (d) 7.

37. A particle is thrown with velocity u making an angle θ with the vertical. It just crosses the top of two poles each of height h after 1 s and 3 s respectively. The maximum height of projectile is



(a) 9.8 m (b) 19.6 m
(c) 39.2 m (d) 4.9 m.

38. The magnitude of vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with x-axis will be



(a) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$ (b) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

(c) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$ (d) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$.

39. A particle is moving in a circle of radius R in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at $t = 0$ is v_0, the time taken to complete the first revolution is

(a) $\dfrac{R}{v_0}$ (b) $\dfrac{R}{v_0}(1-e^{-2\pi})$

(c) $\dfrac{R}{v_0}e^{-2\pi}$ (d) $\dfrac{2\pi R}{v_0}$.

40. The greatest and the least resultant of two force acting at a point are 29 N and 5 N respectively. If each force is increased by 3 N. What will be the resultant of two new forces acting at right angle to each other?

(a) 26°.42′ (b) 36°.52′
(c) 41°.32′ (d) 46°.36′

41. A bird moves with velocity 20 m s^{-1} in a direction making an angle of 60° with the eastern line and 60° with vertical upward. The velocity vector in rectangular form represent as

(a) $5\hat{i} + \dfrac{20}{\sqrt{2}}\hat{j} + 5\hat{k}$ (b) $5\hat{i} + \dfrac{10}{\sqrt{3}}\hat{j} + 5\hat{k}$

(c) $10\hat{i} + \dfrac{10}{\sqrt{3}}\hat{j} + 10\hat{k}$ (d) $10\hat{i} + \dfrac{20}{\sqrt{2}}\hat{j} + 10\hat{k}$

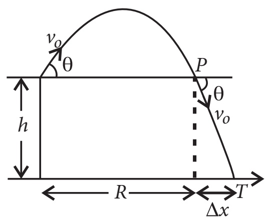
42. From the top of a tower of height 10 m, one fire is shot horizontally with a speed of $5\sqrt{3}$ m/s. Another fire is shot upwards at angle of 60° with the horizontal at some interval of time with the same speed of $5\sqrt{3}$ m/s. The shots collide in air at a certain point. The time interval between the two fires is

(a) 2 s (b) 1 s (c) 0.5 s (d) 0.25 s.

43. A projectile has a maximum range of 16 km. At the highest point of its motion, it explodes into two equal masses. One mass drops vertically downwards. The horizontal distance covered by the other mass from the time of explosion is

(a) 8 km (b) 16 km
(c) 24 km (d) 32 km.

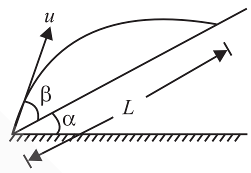
44. A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = \frac{v_0^2}{g}$. If a target farther away by distance Δx (beyond R) has to be hit with the same gun (see figure), then it could be achieved by raising the gun to a height at least



(a) $h = \Delta x\left[1+\frac{\Delta x}{R}\right]$ (b) $h = \Delta x\left[1-\frac{\Delta x}{R}\right]$

(c) $h = \Delta x\left[1+\frac{2\Delta x}{R}\right]$ (d) $h = \Delta x\left[1+\frac{\Delta x}{2R}\right]$.

45. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal. Then distance L is equal to



(a) $\frac{2u^2 \sin\alpha\cos(\alpha+\beta)}{g\cos^2\alpha}$ (b) $\frac{2u^2 \sin\beta\cos(\alpha+\beta)}{g\cos^2\beta}$

(c) $\frac{2u^2 \sin\beta\cos(\alpha+\beta)}{g\cos^2\alpha}$ (d) $\frac{2u^2 \sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$

KCET Exam Archive — 10 Years' PYQs (2014-2023)

1. Which of the following is not a vector quantity ?

(a) Momentum (b) Weight
(c) Potential energy (d) Nuclear spin *(2014)*

2. A stone is thrown vertically at a speed of 30 m s^{-1} making an angle of 45° with the horizontal. What is the maximum height reached by the stone ? Take g = 10 m s^{-2}.

(a) 15 m (b) 30 m
(c) 10 m (d) 22.5 m *(2014)*

3. A particle is projected with a velocity v so that its horizontal range twice the greatest height attained. The horizontal range is

(a) $\frac{2v^2}{3g}$ (b) $\frac{v^2}{2g}$ (c) $\frac{v^2}{g}$ (d) $\frac{4v^2}{5g}$ *(2015)*

4. The ratio of angular speed of a second-hand to the hour-hand of a watch is

(a) 60 : 1 (b) 72 : 1
(c) 720 : 1 (d) 3600 : 1 *(2015)*

5. Three projectiles A, B and C are projected at an angle of 30°, 45°, 60° respectively. If R_A, R_B and R_C are ranges of A, B and C respectively then (velocity of projection is same for A, B and C)

(a) $R_A = R_B = R_C$ (b) $R_A = R_C > R_B$
(c) $R_A < R_B < R_C$ (d) $R_A = R_C < R_B$ *(2016)*

6. The component of a vector $\vec{r}$ along x-axis will have a maximum value if

(a) $\vec{r}$ is along +ve x-axis
(b) $\vec{r}$ is along +ve y-axis
(c) $\vec{r}$ is along –ve y-axis
(d) $\vec{r}$ makes an angle of 45° with the x-axis *(2016)*

7. If $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to $\vec{B} = 4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of 'α' is

(a) 1 (b) $\frac{1}{2}$ (c) –1 (d) $-\frac{1}{2}$ *(2017)*

8. The angle between velocity and acceleration of a particle describing uniform circular motion is

(a) 180° (b) 45° (c) 90° (d) 60° *(2017)*

9. The trajectory of a projectile projected from origin is given by the equation $y = x - \frac{2x^2}{5}$. The initial velocity of the projectile is

(a) 25 m s^{-1} (b) $\frac{2}{5}$ m s^{-1}
(c) $\frac{5}{2}$ m s^{-1} (d) 5 m s^{-1} *(2019)*

10. A wheel starting from rest gains an angular velocity of 10 rad s^{-1} after uniformly accelerated for 5 sec. The total angle through which it has turned is

(a) 25π rad
(b) 50π rad about a vertical axis
(c) 25 rad
(d) 100 rad *(2020)*

11. The maximum range of a gun on horizontal plane is 16 km. If $g = 10\ \text{m s}^{-2}$, then muzzle velocity of a shell is
(a) $160\ \text{m s}^{-1}$ (b) $200\sqrt{2}\ \text{m s}^{-1}$
(c) $400\ \text{m s}^{-1}$ (d) $800\ \text{m s}^{-1}$ *(2021)*

12. The trajectory of a projectile is
(a) semicircle
(b) an ellipse
(c) a parabola always
(d) a parabola in the absence of air resistance. *(2021)*

13. For a projectile motion, the angle between the velocity and acceleration is minimum and acute at
(a) only one point (b) two points
(c) three points (d) four points. *(2021)*

14. Two objects are projected at an angle θ° and (90 – θ)°, to the horizontal with the same speed. The ratio of their maximum vertical heights is
(a) $1 : \tan\theta$ (b) $1 : 1$
(c) $\tan^2\theta : 1$ (d) $\tan\theta : 1$ *(2022)*

15. A particle is in uniform circular motion. Related to one complete revolution of the particle, which among the statements is incorrect?
(a) Average acceleration of the particle is zero.
(b) Displacement of the particle is zero.
(c) Average speed of the particle is zero.
(d) Average velocity of the particle is zero. *(2023)*

Hints & Explanations

Self Test - 1

1. (c) : Among the given quantities, potential energy is a scalar quantity whereas all others are vector quantities.

2. (c) : In Latin, the word vector means carrier.

3. (d) : Among the given physical quantities impulse is a vector quantity whereas all others are scalar quantities.

4. (a) : When a vector is displaced parallel to itself, neither its magnitude nor its direction changes.

5. (a) : A vector has a maximum value along its own direction only. Hence the component of a vector $\vec{r}$ along x-axis will have a maximum value if $\vec{r}$ is along positive x-axis.

Self Test - 2

1. (b) : $\vec{A} = 5$ units due east.

$\therefore \quad -5\vec{A} = -5$ (5 units due east)
$= -25$ units due east $= 25$ units due west

2. (d) : $\vec{A} = 4$ units due east;

$\therefore \quad -4\vec{A} = -16$

The magnitude of a vector $-4\vec{A}$ is 16 units and its direction is due west.

3. (c) **4. (a)** **5. (b)**

Self Test - 3

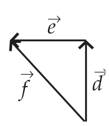
1. (c) : $0\vec{A} = \vec{0}$

2. (b) : Neither the magnitude of vectors nor the angle between the vectors is changed. So, magnitude of the resultant remains unchanged. However, the direction of the resultant will be changed.

3. (c) : As per the laws of vector addition,

$$\vec{d} + \vec{e} = \vec{f}$$

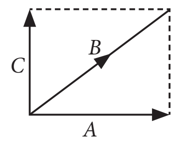
This is as shown in the figure.



4. (c) : $|\vec{A}| = \sqrt{9+4+1} = \sqrt{14}$

$|\vec{B}| = \sqrt{1+9+25} = \sqrt{35}$

$|\vec{C}| = \sqrt{4+1+16} = \sqrt{21}$



$\therefore$ Resultant of A and C is $B^2 = A^2 + C^2$

$\vec{A}\cdot\vec{C} = 0 \ \therefore\ \vec{A}$ and $\vec{C}$ are perpendicular to each other.

Also, $\vec{B} = \vec{A} + \vec{C}$

5. (c) : Join PR.

From the triangle law of addition, $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \vec{A} + \vec{B}$

Again from the triangle law of addition

$$\overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS} = \vec{A} + \vec{B} + \vec{C}$$

Self Test - 4

1. (a) : Given : $\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$;

$\vec{B} = 3\hat{i} + 6\hat{j} + 2\hat{k}$

As per question

$$\vec{C} = |\vec{B}|\,\hat{A} = \sqrt{(3)^2+(6)^2+(2)^2}\left(\frac{\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{(1)^2+(2)^2+(2)^2}}\right)$$

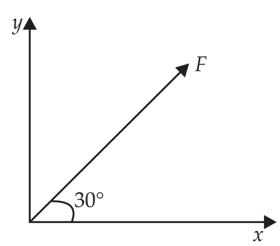
$$= \frac{7}{3}(\hat{i}+2\hat{j}+2\hat{k})$$

2. (c) : The x component of force F is

$$F_x = F\cos 30^\circ = F\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}F$$

The y component of force F is

$$F_y = F\sin 30^\circ = F\times\frac{1}{2} = \frac{F}{2}$$

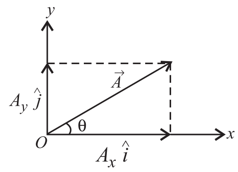


3. (c) : The angle subtended by vector $\vec{A}$ with x-axis is

$$\cos\alpha = \frac{A_x}{A} = \frac{4}{\sqrt{4^2+3^2+(12)^2}} = \frac{4}{13} \Rightarrow \alpha = \cos^{-1}\left(\frac{4}{13}\right)$$

4. **(a) :** $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$

5. **(b) :** $\vec{A} = A_x\hat{i} + A_y\hat{j}$



Also $A = \sqrt{A_x^2 + A_y^2}$

Here, $A = 5, A_x = 3 \therefore 5 = \sqrt{(3)^2 + A_y^2}$

Squaring both sides, we get

$25 = 9 + A_y^2$ or $A_y = 4$

Self Test - 5

1. **(a) :** Let $\vec{P} = 2\hat{i} + 3\hat{j}$ and $\vec{Q} = 2\hat{j} + 3\hat{k}$

Their resultant is

$$\vec{R} = \vec{P} + \vec{Q} = (2\hat{i} + 3\hat{j}) + (2\hat{j} + 3\hat{k}) = 2\hat{i} + 5\hat{j} + 3\hat{k}$$

Thus, the components of the resultant along x and y directions are $\therefore$ $R_x = 2$ and $R_y = 5$

2. **(d) :** Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\therefore \quad \vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$
$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

Given, $\vec{A} + \vec{B} = 0$

$\therefore \quad A_x + B_x = 0$ ...(i)

$A_y + B_y = 0$ ...(ii)

$A_z + B_z = 0$ ...(iii)

From equations (i), (ii) and (iii), we get

$A_x = -B_x, A_y = -B_y, A_z = -B_z$

3. **(c) :** $\vec{A} + 3\vec{B} - \vec{C} = 0$

$3\hat{i} + \hat{j} + 3\hat{k} + 3(3\hat{i} + 5\hat{j} - 2\hat{k}) - \vec{C} = 0$

$12\hat{i} + 16\hat{j} - 3\hat{k} - \vec{C} = 0$, $\vec{C} = 12\hat{i} + 16\hat{j} - 3\hat{k}$

4. **(a) :** The sum of three vectors, $\vec{r} = \vec{a} + \vec{b} + \vec{c}$

$$= (4\hat{i} - \hat{j}) + (-3\hat{i} + 2\hat{j}) + (-\hat{k}) = \hat{i} + \hat{j} - \hat{k}$$

Unit vector, $\hat{r} = \frac{\vec{r}}{r} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

5. **(b) :** $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= (2\hat{i} - 3\hat{j} - 2\hat{k}) + (5\hat{i} + 8\hat{j} + 6\hat{k}) + (-4\hat{i} - 5\hat{j} + 5\hat{k})$

$+ (-3\hat{i} + 4\hat{j} - 7\hat{k}) = 0\hat{i} + 4\hat{j} + 2\hat{k}.$

The X-component of force is missing, so the particle initially at rest will accelerate on the YZ plane.

Self Test - 6

1. **(a) :** $\theta = 60°$, $u_x = 10$ m s^{-1}, $u_y = ?$

As $\tan\theta = \frac{u_y}{u_x}$ $\therefore$ $u_y = u_x \tan\theta = 10 \times \sqrt{3} = 10\sqrt{3}$ m s^{-1}

2. **(a) :** Here, $x = 4t^2$, $y = 3t^2$

$\therefore \quad v_x = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$ and $v_y = \frac{dy}{dt} = \frac{d}{dt}(3t^2) = 6t$

The speed of the particle at any time t is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(8t)^2 + (6t)^2} = 10t$$

3. **(a) :** Here, $\vec{r} = 2t\hat{i} + t^2\hat{j}$, $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} + 2t\hat{j}$

4. **(d) :** Given : $\vec{v} = 4.0\hat{i} + 5.0t\hat{j}$ m s^{-1}

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(4.0\hat{i} + 5.0t\hat{j}) = 5.0\hat{j}$ m s^{-2}

$|\vec{a}| = \sqrt{(5.0)^2} = 5.0$ m s^{-2}

5. **(c) :** $x = 3t^2 - 6t$ $\therefore$ $v_x = \frac{dx}{dt} = 6t - 6$

$y = t^2 - 2t$ $\therefore$ $v_y = \frac{dy}{dt} = 2t - 2$

At time $t = 1$ s, $v_x = 6 \times 1 - 6 = 0$ and $v_y = 2 \times 1 - 2 = 0$

Self Test - 7

1. **(c) :** The position of the particle is given by

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

where, $\vec{r}_0$ is the position vector at $t = 0$ and $\vec{v}_0$ is the velocity at $t = 0$

Here, $\vec{r}_0 = 0$, $\vec{v}_0 = 5\hat{i}$ m s^{-1}, $\vec{a} = (3\hat{i} + 2\hat{j})$ m s^{-2}

$\therefore \quad \vec{r} = 5t\hat{i} + \frac{1}{2}(3\hat{i} + 2\hat{j})t^2 = (5t + 1.5t^2)\hat{i} + 1t^2\hat{j}$ ... (i)

Compare it with $\vec{r} = x\hat{i} + y\hat{j}$, we get

$x = 5t + 1.5t^2$, $y = 1t^2$; $\because$ $x = 84$ m; $\therefore$ $84 = 5t + 1.5t^2$

On solving, we get $t = 6$ s

At $t = 6$ s, $y = (1)(6)^2 = 36$ m

2. **(b) :** As, $\vec{r} = (5t + 1.5t^2)\hat{i} + 1t^2\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(5t + 1.5t^2)\hat{i} + 1t^2\hat{j} = (5 + 3t)\hat{i} + 2t\hat{j}$

At $t = 6$ s, $\vec{v} = 23\hat{i} + 12\hat{j}$

The speed of the particle is $|\vec{v}| = \sqrt{(23)^2 + (12)^2} \approx 26$ m s^{-1}

3. **(b) :** Given, $y = bx^2$

Differentiating w.r.t. t on both sides, we get

$\frac{dy}{dt} = 2bx\frac{dx}{dt}$ or $v_y = 2bxv_x$ ...(i)

Again, differentiating w.r.t. t on both sides, we get

$\frac{dv_y}{dt} = 2bv_x\frac{dx}{dt} + 2bx\frac{dv_x}{dt} = 2bv_x^2 + 0$ ...(ii)

[$\because$ $\frac{dv_x}{dt} = 0$, because the particle has constant acceleration along y-direction]

As per question, $\frac{dv_y}{dt} = a$

From (ii), $a = 2bv_x^2$ $\Rightarrow$ $v_x^2 = \frac{a}{2b}$ or $v_x = \sqrt{\frac{a}{2b}}$

4. **(d) :** Given, initial velocity, $u = 5\hat{j}$ m/s and acceleration, $a = (10\hat{i} + 4\hat{j})$ m/s^2

Also, at $t = t$, position coordinates $(x, y) = (20$ m, y_0 m)

Now, from equation of motion, $x = u_xt + \frac{1}{2}a_xt^2$

Here, $u_x = 0$, $x = 20$ m and $a_x = 10$ m/s^2

$\therefore \quad 20 = 0 + \frac{1}{2} \times 10 \times t^2$ or $t = 2$ s

Now, $y = u_y t + \frac{1}{2} a_y t^2$

where $u_y = 5$ m/s, $a_y = 4$ m/s^2 and $t = 2$ s

$\therefore \quad y = 5 \times 2 + \frac{1}{2} \times 4 \times 4 = 18$ m

Self Test - 8

1. **(b) :** Given, $\vec{u} = \hat{i} + 2\hat{j} = u_x\hat{i} + u_y\hat{j}$

Then $u_x = 1 = u\cos\theta$ and $u_y = 2 = u\sin\theta$

$\therefore \quad \tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1} = 2$

The equation of trajectory of a projectile motion is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta - \frac{gx^2}{2(u\cos\theta)^2}$$

$$\therefore \quad y = x \times 2 - \frac{10 \times x^2}{2(1)^2} = 2x - 5x^2$$

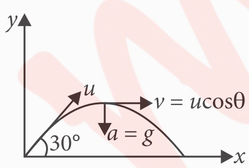
2. **(b) :** Here, $\theta = 45°$ i, Time of flight, $T = 1$s

Let u be the initial velocity of the particle

$\therefore \quad T = \frac{2u\sin\theta}{g}$ or $u = \frac{gT}{2\sin\theta}$

$\Rightarrow \quad u = \frac{9.8 \times 1}{2\sin 45°} = 4.9\sqrt{2}$ m/s = 6.93 m/s

3. **(c) :** The situation is shown in figure. At the highest point of its trajectory, the velocity and the acceleration are perpendicular, so the angle between them is 90°.



4. **(c) :** Given $R = 4H$

$\therefore \quad \frac{u^2\sin 2\theta}{g} = 4 \times \frac{u^2\sin^2\theta}{2g}$ or $\tan\theta = 1$ or $\theta = 45°$

5. **(a) :** Given, $5 = \frac{2u\sin\theta}{g}$ or $\frac{u\sin\theta}{g} = \frac{5}{2}$

$$\text{Maximum height } = \frac{u^2\sin^2\theta}{2g} = \frac{g}{2} \times \left(\frac{u^2\sin^2\theta}{g^2}\right)$$

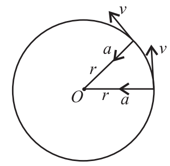
$$= \frac{g}{2} \times \left(\frac{5}{2}\right)^2 = \frac{10}{2} \times \frac{25}{4} = 31.25 \text{ m}$$

Self Test - 9

1. **(d) :** Centripetal acceleration, $a_c = \frac{v^2}{R}$

where v is the speed of an object and R is the radius of the circle. It is always directed towards the centre of the circle. Since v and R are constants for a given uniform circular motion, therefore the magnitude of centripetal acceleration is also constant. However, the direction of centripetal acceleration changes continuously. Therefore, a centripetal acceleration is not a constant vector.

2. **(b) :** In a uniform circular motion,the acceleration is directed towards the centre while velocity is acting tangentially. Therefore, velocity vector is perpendicular to acceleration vector.



3. **(a) :** Centripetal acceleration, $a = \frac{v^2}{r}$ ...(i)

If v is doubled, $a' = \frac{(2v)^2}{r} = \frac{4v^2}{r} = 4a$ (Using (i))

4. **(a) :** Here, $r = 100$ cm = 1 m

Frequency, $\upsilon = \frac{14}{22}$ rps ; $\therefore$ $\omega = 2\pi\upsilon = 2 \times \frac{22}{7} \times \frac{14}{22} = 4$ rad s^{-1}

The acceleration of the stone is $a_c = \omega^2 r = (4)^2\,(1) = 16$ m s^{-2}

5. **(d) :** In uniform circular motion, velocity and acceleration are perpendicular to each other.

KCET Connect

1. **(b) :** Movement of mercury level in a thermometer is an example of one dimensional motion.

2. **(d) :** Among the given quantities, electric potential is a scalar quantity whereas all others are vector quantities.

3. **(a) :** Magnetic moment is vector. Other quantities are scalars. Hence, magnetic moment is stranger in the group.

4. **(c) :** Among the given quantities, potential energy is a scalar quantity whereas all others are vector quantities.

5. **(c) :** A vector has a magnitude and a direction. The direction takes care of the sign. Magnitude can never be negative.

6. **(d) :** $\vec{A} = \hat{i} - 5\hat{j}$; $\vec{B} = 2\hat{i} - 10\hat{j} = 2(\hat{i} - 5\hat{j}) = 2\vec{A}$

Thus, $\vec{B} \,||\, \vec{A}$.

7. **(d) :** $+5\hat{k} = -a\hat{k}$ $\therefore$ $a = -5$

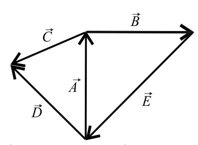
8. **(c) :** $\vec{A} = 8$ units due north.

$\therefore -8\vec{A} = -8$ (8 units due north)

= – 64 units due north = 64 due south

9. **(d) :** $\vec{A} + \vec{B} + \vec{E} = 0$ as they form a triangle when taken in order.

$\Rightarrow \quad \vec{E} = -(\vec{A} + \vec{B})$.



10. **(d) :** Let $\vec{P}$ and $\vec{Q}$ be two vectors. Then according to question

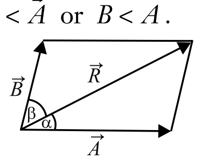
$|\vec{P} + \vec{Q}| = 17$ or $P + Q = 17$ ...(i)

$|\vec{P} - \vec{Q}| = 7$ or $P - Q = 7$ ...(ii)

On adding and subtracting the equations (i) and (ii) we get

$P = 12$; $Q = 5$

11. **(c) :** $\alpha < \beta$ if $\vec{B} < \vec{A}$ or $B < A$.



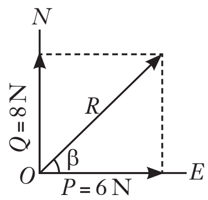
12. **(d) :** Resultant of these forces is

$$R = \sqrt{(6)^2 + (6)^2 + (\sqrt{72})^2} \text{ along } OE$$

$$= \sqrt{36 + 36 + 72} = 12 \text{ N along } OE$$

13. (c) : $\overline{OC}$ and $\overline{OA}$ are equal in magnitude and inclined to each other at an angle of 90°. So their resultant is $\sqrt{2}r$. It acts mid-way between $\overline{OC}$ and $\overline{OA}$, *i.e.*, along *OB*.
Now, both *r* and $\sqrt{2}r$ are along the same line and in the same direction.
Resultant $= r + \sqrt{2}r = r(1+\sqrt{2})$

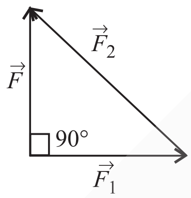
14. (a) : From the figure, If the resultant *R* makes angle β with the force of 6 N, then



$$\tan\beta = \frac{Q}{P} = \frac{8}{6} = 1.3333$$

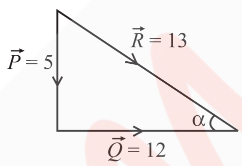
$\therefore \quad \beta = \tan^{-1}(1.3333) = 53°12'$.

15. (a) : As shown in the figure, *F* is the resultant of $\vec{F}_1$ and $\vec{F}_2$ and F_1 is the smaller force. Now



$F_2^2 = F_1^2 + F^2 = F_1^2 + (8)^2$ ...(i)
and $F_1 + F_2 = 16$ or $F_2 = 16 - F_1$ ...(ii)
Using, (ii) in (i), we have
$(16 - F_1)^2 = F_1^2 + 64,$
which gives $F_1 = 6$ N
Therefore, $F_2 = 16 - 6 = 10$ N

16. (c) : Here $P^2 + Q^2 = R^2$
$(5)^2 + (12)^2 = (13)^2$



$\therefore$ Angle between *P* and *Q* = 90°.
Let α be the angle between *Q* and *R*.

$$\therefore \cos\alpha = \left(\frac{12}{13}\right) \quad \text{or} \quad \alpha = \cos^{-1}\left(\frac{12}{13}\right)$$

17. (d) : The component of a vector may be equal to or less than or greater than the magnitude of vector itself depending upon the magnitudes of component vectors and their orientations with respect to each other.

18. (d) : $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{\vec{A}}{A}$

19. (b) : Given : $\vec{A} = 2\hat{i} + 3\hat{j}$, $\vec{B} = \hat{i} - \hat{j}$ (say)
Component of $\vec{A}$ along the direction of $\vec{B}$

$$= \frac{\vec{A}\cdot\vec{B}}{|B|} = \frac{(2\hat{i}+3\hat{j})\cdot(\hat{i}-\hat{j})}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

20. (c) : $\vec{r} = 3\hat{i} + 3\hat{j}$

Angle with *x*-axis, $\tan\theta = \dfrac{\hat{j}\text{ coefficient}}{\hat{i}\text{ coefficient}}$

$\tan\theta = \dfrac{3}{3} = 1$ or $\theta = 45°$

21. (d): $1 = \sqrt{a^2 + b^2}$; or $a^2 + b^2 = 1$ ...(i)
And $(a\hat{i} + b\hat{j})\cdot(\hat{i} + \hat{j}) = 0$ or $a + b = 0$; or $b = -a$
From (i), $a^2 + (-a)^2 = 1$
or $a = \dfrac{1}{\sqrt{2}}$ and $b = -\dfrac{1}{\sqrt{2}}$

22. (a) : $\hat{n} = \dfrac{\vec{A}}{|A|} = \dfrac{1/\sqrt{2}\cos\theta\,\hat{i} + 1/\sqrt{2}\sin\theta\,\hat{j}}{1/\sqrt{2}}$

$= \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

23. (b) : Given : $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 6\hat{i} + 9\hat{j} + 3\hat{k}$
$\therefore \quad \vec{B} = 3(2\hat{i} + 3\hat{j} + \hat{k})$; As $\vec{B} = 3\vec{A} \quad \therefore \quad \vec{B} \parallel \vec{A}$

24. (d) : Let $\vec{A} = \hat{i} + \hat{j} + \hat{k} \quad \therefore \quad A_x = 1, A_y = 1, A_z = 1$
and $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$
cos α, cos β and cos γ are the direction cosines of $\vec{A}$.

$$\therefore \quad \cos\alpha = \frac{A_x}{A} = \frac{1}{\sqrt{3}}, \cos\beta = \frac{A_y}{A} = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{A_z}{A} = \frac{1}{\sqrt{3}}$$

25. (b) : A vector perpendicular to both the given vectors is
$(2\hat{i} - 2\hat{j} + \hat{k}) \times (3\hat{i} + 4\hat{j} - 5\hat{k}) = 6\hat{i} + 13\hat{j} + 14\hat{k}$
$\therefore$ Unit vector perpendicular to the given vectors is

$$\frac{6\hat{i} + 13\hat{j} + 14\hat{k}}{\sqrt{6^2 + 13^2 + 14^2}} = \frac{1}{\sqrt{401}}\left(6\hat{i} + 13\hat{j} + 14\hat{k}\right)$$

26. (c) : $1 = 0.5\hat{i} + 0.8\hat{j} + c\hat{k}$
or $(1)^2 = (0.5)^2 + (0.8)^2 + c^2$ or $1 = 0.25 + 0.64 + c^2$
or $c^2 = 1 - 0.25 - 0.64 = 0.11 \therefore c^2 = 0.11$

27. (d)

28. (d) : $\cos\theta = \dfrac{(\hat{i} + \hat{j} + \hat{k})\cdot\hat{j}}{\sqrt{(1)^2 + (1)^2 + (1)^2}\sqrt{(1)^2}} = \dfrac{1}{\sqrt{3}}$

29. (b) : $F = \sqrt{(1)^2 + (1)^2 + (\sqrt{2})^2} = 2$

$\cos\gamma = \dfrac{F_z}{F} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}} = \cos 45°$; $\therefore \gamma = 45°$

30. (a) : Let $\vec{P} = 2\hat{i} + 3\hat{j}$ and $\vec{Q} = 2\hat{j} + 3\hat{k}$
Their resultant is

$$\vec{R} = \vec{P} + \vec{Q} = (2\hat{i} + 3\hat{j}) + (2\hat{j} + 3\hat{k}) = 2\hat{i} + 5\hat{j} + 3\hat{k}$$

Thus, the components of the resultant along *x* and *y* directions are $\therefore R_x = 2$ and $R_y = 5$

31. (c) : $\vec{A} + \vec{B} = 0$

i.e., $\vec{A} + \vec{B}$ and $\vec{A}$ are perpendicular.

32. (b) : A vector, its magnitude and the angle between two vectors do not depend on the choice of the orientation of the coordinate axes. So $\vec{A} + \vec{B}, |\vec{A} + \vec{B}|$, angle between $\vec{A}$ and $\vec{B}$ are independent of the orientation of the coordinate axes. But the quantity $A_x + B_y$ depends upon the magnitude of the components along *x* and *y*-axes, so it will change with change in coordinate axes.

33. (c) : $\left(|\vec{A}_1 + \vec{A}_2|\right)^2 = A_1^2 + A_2^2 + 2\vec{A}_1\cdot\vec{A}_2 = 25$

or $\vec{A}_1\cdot\vec{A}_2 = \dfrac{1}{2}(25 - 9 - 25) = -\dfrac{9}{2}$

So, $(2\vec{A}_1 + 3\vec{A}_2)\cdot(3\vec{A}_1 - 2\vec{A}_2) = 6A_1^2 - 4\vec{A}_1\cdot\vec{A}_2 + 9\vec{A}_2\cdot\vec{A}_1 - 6A_2^2$

$= 6(9) + 5\left(\dfrac{-9}{2}\right) - 6(25) = -118.5$

34. (c): Let the required vector be $\vec{C}$.
Resultant vector = $\vec{A} + \vec{B} + \vec{C}$
$\therefore \quad \hat{k} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + (-3\hat{i} - 2\hat{j} - 3\hat{k}) + \vec{C}$
or $\hat{k} = -\hat{i} - 5\hat{j} + \hat{k} + \vec{C}$ or $\vec{C} = (\hat{i} + 5\hat{j})$

35. (b) : $R^2 = P^2 + Q^2 + 2PQ\cos\theta$
R shall be maximum if $\cos\theta = +1$
$\therefore \quad \theta$ = zero

36 (d) : $|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$
since $A = B = |\vec{A}+\vec{B}|$
$A = \sqrt{A^2+A^2+2A^2\cos\theta}$ or $A^2 = 2A^2 + 2A^2\cos\theta$
$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$.

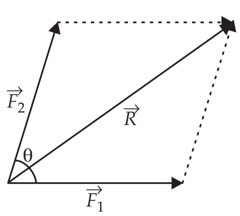
37. (b) : The angle between the given forces is 120° and not 60°.
Resultant force is
$|\vec{R}| = \sqrt{|\vec{A}|^2+|\vec{B}|^2+2\times|\vec{A}||\vec{B}|\cos\theta}$
$= \sqrt{(5)^2+(5)^2+2\times(5)\times(5)\times\left(-\frac{1}{2}\right)} = 5$ N

38. (a) : Given : Force F_1 = Force $F_2 = F$
or $\quad F_1 = F_2 = F$
Resultant force, $R = 40\sqrt{3}$ N
Angle between two forces F_1 and F_2 is $\theta = 60°$
$\therefore \quad R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$
or $\quad 40\sqrt{3} = \sqrt{F^2+F^2+2F^2\cos 60°} = \sqrt{3F^2}$
or $\quad F = 40$ N

39. (a) : Here, $\vec{A} = \vec{B}+\vec{C}$
Let the angle between $\vec{B}$ and $\vec{C}$ be θ. Then
$A^2 = B^2 + C^2 + 2BC\cos\theta$; $\therefore (5)^2 = 4^2 + 3^2 + 2(4)(3)\cos\theta$
or $\quad 0 = 24\cos\theta \quad$ or $\quad \theta = \frac{\pi}{2}$
In the right angled triangle, let the angle between $\vec{A}$ and $\vec{C}$ be α.
$\therefore \quad \cos\alpha = \frac{C}{A} = \frac{3}{5} \quad$ or $\quad \alpha = \cos^{-1}\left(\frac{3}{5}\right)$

40. (c) : $\tan 90° = \frac{B\sin\theta}{A+B\cos\theta}$ or $A + B\cos\theta = 0$
or $\quad \cos\theta = -\frac{A}{B}$ ...(i)
$R = \frac{B}{2} = [A^2+B^2+2AB\cos\theta]^{1/2}$
or $\quad \frac{B^2}{4} = A^2+B^2+2AB\left(-\frac{A}{B}\right) = B^2 - A^2$ (Using (i))
or $\quad A^2 = B^2 - \frac{B^2}{4} = \frac{3}{4}B^2$ or $\frac{A^2}{B^2} = \frac{3}{4}$ or $\frac{A}{B} = \frac{\sqrt{3}}{2}$
From (i), $\cos\theta = -\frac{\sqrt{3}}{2} = \cos 150°$ or $\theta = 150°$

41. (a) : Let $\vec{F}_1$ and $\vec{F}_2$ be the two forces acting on a particle simultaneously and θ be the angle between them.
The resultant of these two forces is
$R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$ ...(i)
According to question
$\frac{F_1}{F_2} = \frac{1}{2}$ or $F_2 = 2F_1$
and $\quad R = 3F_1$



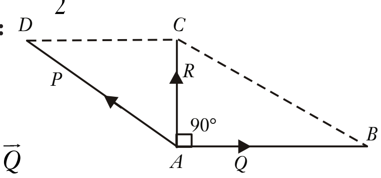
Substituting these values in equation (i), we get
$(3F_1)^2 = F_1^2 + (2F_1)^2 + 4F_1^2\cos\theta$
or $\quad 4\cos\theta = 4 \quad$ or $\quad \cos\theta = 1 \quad$ or $\quad \theta = \cos^{-1}(1) = 0°$

42. (c) : Given that,
$\sqrt{(|\vec{A}+\vec{B}|)^2+(|\vec{A}-\vec{B}_1)^2+2|\vec{A}+\vec{B}||\vec{A}-\vec{B}|\cos\theta}$
$= \sqrt{A^2+B^2}$
Squaring both sides, we get,
$(A^2 + B^2) + (A^2 + B^2) + 2(A^2 - B^2)\cos\theta = (A^2 + B^2)$
$(A^2 + B^2) = -2(A^2 - B^2)\cos\theta$
$\Rightarrow \quad \cos\theta = -\frac{(A^2+B^2)}{2(A^2-B^2)} \quad \Rightarrow \quad \theta = \cos^{-1}\left[-\frac{(A^2+B^2)}{2(A^2-B^2)}\right]$

43. (d) : Here, $\vec{a}+\vec{b} = \vec{c}$ and $c = a + b$
Let θ be angle between $\vec{a}$ and $\vec{b}$.
$\therefore \quad c = \sqrt{a^2+b^2+2ab\cos\theta}$; $a + b = \sqrt{a^2+b^2+2ab\cos\theta}$
Squaring both sides, we get
$(a + b)^2 = a^2 + b^2 + 2ab\cos\theta$
$a^2 + b^2 + 2ab = a^2 + b^2 + 2ab\cos\theta$
$2ab - 2ab\cos\theta = 0$ or $1 - \cos\theta = 0$
$\cos\theta = 1$ or $\theta = \cos^{-1}(1) = 0°$

44. (a) : $R^2 = (3P)^2 + (2P)^2 + 2(3P)(2P)\cos\theta$
$\Rightarrow R^2 = 13P^2 + 12P^2\cos\theta$ ...(i)
When the first force is doubled, the resultant is doubled,
so, $(2R)^2 = (6P)^2 + (2P)^2 + 2(6P)(2P)\cos\theta$
$\Rightarrow 4R^2 = 36P^2 + 4P^2 + 24P^2\cos\theta$
$\Rightarrow \quad R^2 = 10P^2 + 6P^2\cos\theta$ ...(ii)
Equating (i) & (ii), we get
$13P^2 + 12P^2\cos\theta = 10P^2 + 6P^2\cos\theta$
$3P^2 = -6P^2\cos\theta$
so, $\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$.

45. (b):



$\vec{R} = \vec{P}+\vec{Q}$
$R^2 = P^2 + Q^2 + 2PQ\cos 150°$ or $R^2 = P^2 + Q^2 - \frac{2PQ\times\sqrt{3}}{2}$
or $\quad R^2 = P^2 + Q^2 - \sqrt{3}\cdot PQ$ ...(i)
Also $P^2 = Q^2 + R^2$ by right angle ...(ii)
Eliminate P in (i) and (ii)
$\therefore \quad R^2 = (Q^2 + R^2) + Q^2 - \sqrt{3}\sqrt{(Q^2+R^2)}Q$
or $\quad 0 = 2Q^2 - \sqrt{3}\sqrt{(Q^2+R^2)}Q$
or $\quad \sqrt{3}\sqrt{Q^2+R^2} = 2Q \quad$ or $\quad 3(Q^2 + R^2) = 4Q^2$
or $\quad 3R^2 = Q^2 \quad$ or $\quad 3R^2 = P^2 - R^2$ from (ii)
or $\quad 4R^2 = P^2 \quad$ or $\quad P = 2R$

46. (b) : Here, $\vec{P} = (\vec{A}+\vec{B}), \vec{Q} = (\vec{A}-\vec{B}), \vec{R} = \sqrt{3A^2+B^2}$
Let θ be angle between the two given forces.
$\therefore \quad (3A^2 + B^2) = (A + B)^2 + (A - B)^2 + 2(A + B)(A - B)\cos\theta$
or $\quad 3A^2 + B^2 = A^2 + B^2 + 2AB + A^2 + B^2 - 2AB + 2(A^2 - B^2)\cos\theta$
or $\quad A^2 - B^2 = 2(A^2 - B^2)\cos\theta$
or $\quad \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$ or $\theta = \frac{\pi}{3}$

47. (d) : Let θ be the angle between $\vec{A}_1$ and $\vec{A}_2$.

$\therefore$ Resultant of $\vec{A}_1$ and $\vec{A}_2$ is

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$$

or $3A^2 = A^2 + A^2 + 2AA\cos\theta$

or $\cos\theta = \frac{1}{2} = \cos 60°$ or $\theta = 60°$

The angle between $\vec{A}_1$ and $-\vec{A}_2$ is $(180° - 60°) = 120°$

$\therefore$ Resultant of $\vec{A}_1$ and $-\vec{A}_2$ is

$$R' = [A_1^2 + A_2^2 + 2A_1A_2\cos(180° - 60°)]^{1/2}$$

$$= [A^2 + A^2 + 2\,AA\cos 120°]^{1/2} = A$$

48. (d) : Given, $\vec{R}$ is resultant of $\vec{P}$ and $\vec{Q}$; $\vec{R} \perp \vec{P}$ and $|\vec{R}| = |\vec{P}|$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

where α is the angle between $\vec{R}$ and $\vec{P}$

$\therefore \tan 90° = \frac{Q\sin\theta}{P + Q\cos\theta}$

$\Rightarrow \frac{1}{0} = \frac{Q\sin\theta}{P + Q\cos\theta} \Rightarrow P + Q\cos\theta = 0$

or $\cos\theta = -\frac{P}{Q}$ ...(i)

$\because |\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Squaring both sides we get,

$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow P^2 = P^2 + Q^2 + 2PQ\cos\theta$ $(\because |\vec{R}| = |\vec{P}|)$

$\Rightarrow Q^2 = -2PQ\cos\theta \Rightarrow Q = -2P\cos\theta$

$\Rightarrow Q = -2P\cdot\left(-\frac{P}{Q}\right) \Rightarrow Q = \sqrt{2}P$

$\Rightarrow \frac{P}{Q} = \frac{1}{\sqrt{2}} \Rightarrow -\cos\theta = +\frac{1}{\sqrt{2}}$ (Using (i))

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} = \cos\left(\frac{3\pi}{4}\right) \Rightarrow \theta = \frac{3\pi}{4}$

49. (d) : Let the two vectors be $\vec{A}$ and $\vec{B}$.

Then, magnitude of sum of $\vec{A}$ and $\vec{B}$,

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

and magnitude of difference of $\vec{A}$ and $\vec{B}$,

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}| \text{ (given)}$$

or $\sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$\Rightarrow 4AB\cos\theta = 0$

$\because 4AB \neq 0, \therefore \cos\theta = 0$ or $\theta = 90°$

50. (b) : $A + B = 16$; $8\sqrt{3} = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$

And $\tan 90° = \frac{B\sin\theta}{A + B\cos\theta}$ or $\infty = \frac{B\sin\theta}{A + B\cos\theta}$

or $A + B\cos\theta = 0$ or $B\cos\theta = -A$

$\therefore 8\sqrt{3} = [A^2 + B^2 + 2A(-A)]^{1/2}$

or $192 = B^2 - A^2 = (B - A)(B + A) = (B - A)\times 16$

or $B - A = \frac{192}{16} = 12$

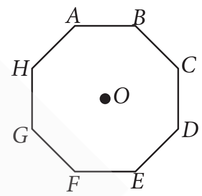
On solving, $A = 2$ and $B = 14$.

51. (b) : $\overrightarrow{OA} = \vec{a}$; $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k} = \vec{a}$

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$

$= (\vec{b} - \vec{a}) + (\vec{c} - \vec{a}) + (\vec{d} - \vec{a}) + (\vec{e} - \vec{a}) + (\vec{f} - \vec{a}) + (\vec{g} - \vec{a}) + (\vec{h} - \vec{a})$

$= \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} + \vec{g} + \vec{h} - 7\vec{a}$ ... (i)



Here, $\frac{\vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} + \vec{g} + \vec{h}}{8} = 0$

$\vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} + \vec{g} + \vec{h} = 0$

$\vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} + \vec{g} + \vec{h} = -\vec{a}$

On putting value of a in eqn (i), we get

$= -\vec{a} - 7\vec{a} = -8\vec{a}$

$= -8\,\overrightarrow{OA} = 8\overrightarrow{AO} = 8(2\hat{i} + 3\hat{j} - 4\hat{k}) = 16\hat{i} + 24\hat{j} - 32\hat{k}$

52. (a) : Here, $\vec{r}_A = 0\hat{i} + 4\hat{j} - 2\hat{k}$, $\vec{r}_B = -2\hat{i} + 8\hat{j} - 4\hat{k}$

Displacement vector from A to B is given by

$\vec{r} = \vec{r}_B - \vec{r}_A = (-2\hat{i} + 8\hat{j} - 4\hat{k}) - (0\hat{i} + 4\hat{j} - 2\hat{k}) = -2\hat{i} + 4\hat{j} - 2\hat{k}$

53. (a) : Resolving displacement $30\sqrt{2}$ m south-west into two rectangular components, we get

Displacement in south $= 30\sqrt{2}\cos 45° = 30\sqrt{2}\times\left(\frac{1}{\sqrt{2}}\right) = 30$ m

Displacement in west $= 30\sqrt{2}\sin 45° = 30\sqrt{2}\times\left(\frac{1}{\sqrt{2}}\right) = 30$ m

$\therefore$ Effective displacement due to 30 m north and 30 m south is zero.

Also effective displacement due to 30 m east and 30 m west is zero.

54. (a) : Here, Initial position vector of the particle, $\vec{r}_i = 2\hat{i} + 10\hat{j} + \hat{k}$

Let final position vector of the particle be

$$\vec{r}_f = x\hat{i} + y\hat{j} + z\hat{k}$$

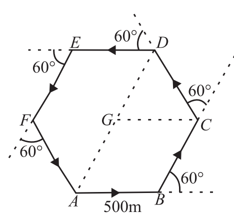
$\therefore$ Displacement, $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$$8\hat{i} - 2\hat{j} + \hat{k} = x\hat{i} + y\hat{j} + z\hat{k} - \left(2\hat{i} + 10\hat{j} + \hat{k}\right)$$

or $x\hat{i} + y\hat{j} + z\hat{k} = 10\hat{i} + 8\hat{j} + 2\hat{k}$

Hence, the final coordinates of the particle are (10, 8, 2).

55. (c) : The path followed by the motorist is a regular hexagon $ABCDEF$ of side length 500 m as shown in the figure. Let the motorist start from A and takes the third turn at D.

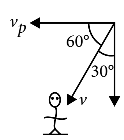


Therefore, the magnitude of this displacement is

$AD = AG + GD = 500$ m $+ 500$ m $= 1000$ m

56. (d) : $v_p = v\cos 60°$

$$= \frac{v}{2}$$



57. (a) : Here, $\vec{r} = 2t\hat{i} + t^2\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} + 2t\hat{j}$

58. (b) : Here, $\vec{u} = 2\hat{i} + 3\hat{j}$, $\vec{a} = 0.3\hat{i} + 0.2\hat{j}$, $t = 10$ s

As $\vec{v} = \vec{u} + \vec{a}t$ $\therefore$ $\vec{v} = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j})(10)$

$= 2\hat{i} + 3\hat{j} + 3\hat{i} + 2\hat{j} = 5\hat{i} + 5\hat{j}$

$|\vec{v}| = \sqrt{(5)^2 + (5)^2} = 5\sqrt{2}$ units

59. (d) : $v = u + at$

$v = (4\hat{i} + 3\hat{j}) + 10 \cdot (0.4\hat{i} + 0.3\hat{j}) = 8\hat{i} + 6\hat{j}$

$\therefore$ Magnitude of $V = \sqrt{8^2 + 6^2} = 10$ units

60. (a) : Given : $\vec{r} = 3t\hat{i} + 2t^2\hat{j} + 5$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t\hat{i} + 2t^2\hat{j} + 5\hat{k}) = 3\hat{i} + 4t\hat{j}$ m s^{-1}

Let θ be the angle which the direction of $\vec{v}$ makes with the x-axis. Then $\tan\theta = \frac{v_y}{v_x} = \frac{4t}{3} = \frac{4}{3}$ or $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

61. (b): $= \vec{r} = 12t\hat{i} - 5t\hat{j} + 3\hat{k}$

or $\frac{d\vec{r}}{dt} = 12\hat{i} - 5\hat{j}$ or velocity $\vec{v} = 12\hat{i} - 5\hat{j}$

$\therefore$ Magnitude of velocity $= \sqrt{(12)^2 + (5)^2}$

$= \sqrt{144 + 25} = \sqrt{169} = 13$ m/s.

62. (b) : $v(y) = u(y) + at$

$v(y) = 0 + (0.15)2$; $v(y) = 0.3$ m/s

The velocity along with x component is $v(x) = 0.4$ m/s

Therefore the resultant velocity is

$v(r) = \sqrt{(0.4)^2 + (0.3)^2} = 0.5$ m/s.

63. (a): The velocity of P w.r.t. $Q = v_{PQ}$

$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$

or $\vec{v}_P = 4$ m/s due east

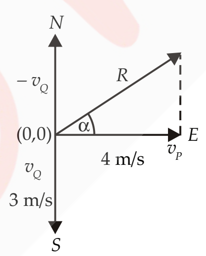
or $\vec{v}_P = 4\hat{i}$

$v_Q = 3$ m/s due south

or $v_Q = -3\hat{j}$

$\tan\alpha = 3/4$

R denotes resultant along α.



$\therefore$ $\vec{v}_{PQ} = 4\hat{i} - (-3\hat{j})$

or $\vec{v}_{PQ} = 4\hat{i} + 3\hat{j}$ along $(E - N)$ direction.

64. (b) : The relation (b) is true, others are false because relations (a), (c) and (d) hold only for uniformly accelerated motion.

65. (c) : $x = 6t$

$v = \frac{dx}{dt} = 6 = x$–component of initial velocity $= v_x$

$y = 8t - 5t^2$; $\frac{dy}{dt} = 8 - 10t$

At $t = 0$, $\frac{dy}{dt} = 8 = y$–component of initial velocity $= v_y$

Now, $|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(6)^2 + (8)^2} = 10$ m/s

66. (b) : $\tan\theta = \frac{v_x}{v_y} = \frac{8}{6}$; $\theta = \tan^{-1}(4/3)$.

67. (a) : Here, $\vec{v}_1 = 20$ m s^{-1} due north $= \overrightarrow{OA}$

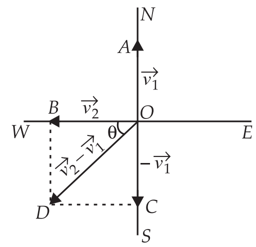
$\vec{v}_2 = 20$ m s^{-1} due west $= \overrightarrow{OB}$

Change in velocity

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$

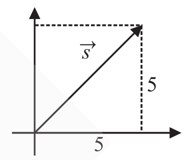
$|\Delta\vec{v}| = \sqrt{v_2^2 + v_1^2} = \sqrt{(20)^2 + (20)^2}$

$= 20\sqrt{2}$ m s^{-1}



$\tan\theta = \frac{v_2}{v_1} = \frac{20}{20} = 1 = \tan 45°$ $\Rightarrow$ $\theta = 45°$ SW

68. (c) : $\vec{s} = 5\hat{i} + 5\hat{j}$



Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

$= \frac{5\hat{i} + 5\hat{j}}{10} = 0 \cdot 5\hat{i} + 0 \cdot 5\hat{j}$

69. (c) : $x = \alpha t^3$, $y = \beta t^3$

$\frac{dx}{dt} = 3\alpha t^2$ $\therefore$ $v_x = 3\alpha t^2$

$\frac{dy}{dt} = 3\beta t^2$ $\therefore$ $v_y = 3\beta t^2$

Resultant velocity $v = \sqrt{v_x^2 + v_y^2} = 3t^2\sqrt{\alpha^2 + \beta^2}$

70. (b) : $x = 2\sin t$, $y = 3\cos t$, $z = \sqrt{5}\sin t$

$v_x = \frac{dx}{dt} = 2\cos t$, $v_y = \frac{dy}{dt} = -3\sin t$, $v_z = \frac{dz}{dt} = \sqrt{5}\cos t$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{4\cos^2 t + 9\sin^2 t + 5\cos^2 t}$

$= \sqrt{4\cos^2 t + 4\sin^2 t + 5\sin^2 t + 5\cos^2 t} = \sqrt{9} = 3$

71. (b): Velocity along x-axis, $v_x = \frac{dx}{dt} = 8t - 2$

or $dx = (8t - 2)\,dt$

Integrating it, we get $x = \frac{8t^2}{2} - 2t + C = 4t^2 - 2t + C$

where C is a constant of integration.

At $t = 2$, $x = 14$; so $14 = 4 \times 2^2 - 2 \times 2 + C$ or $C = 2$

$\therefore$ $x = 4t^2 - 2t + 2$ ...(i)

Also, $v_y = \frac{dy}{dt} = 2$ or $dy = 2\,dt$

Integrating it, we get $y = 2t + C'$.

where C' is a constant of integration.

At $t = 2$, $y = 4$; so $4 = 2 \times 2 + C'$ or $C' = 0$

$\therefore$ $y = 2t$ or $t = r/2$ ...(ii)

In order to find the equation of path of projectile we have to eliminate t from (i) and (ii).

From (ii), we get $x = 4\left(\frac{y}{2}\right)^2 - 2\left(\frac{y}{2}\right) + 2$ or $x = y^2 - y + 2$

72. (c) : Given : $x = 3t^3$; $y = 4t^3$

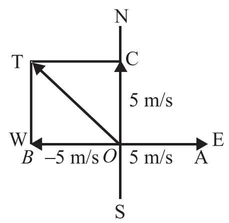
$\therefore$ $v_x = \frac{dx}{dt} = \frac{d}{dt}(3t^3) = 9t^2$; $v_y = \frac{dy}{dt} = \frac{d}{dt}(4t^3) = 12t^2$

The velocity of the body is

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9t^2)^2 + (12t^2)^2}$ $v = \sqrt{81t^4 + 144t^4} = 15t^2$

73. (b) : Here, Initial velocity, $\vec{u} = 3\hat{i} + 4\hat{j}$
Acceleration, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$; Time, $t = 10$ s
Let $\vec{v}$ be velocity of a particle after 10 s.
Using, $\vec{v} = \vec{u} + \vec{a}t \quad \therefore \quad \vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})(10)$
$= 3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$
Speed of the particle after 10 s $= |\vec{v}|$
$= \sqrt{(7)^2 + (7)^2} = 7\sqrt{2}$ units

74. (b) : In the given figure, change in velocity in 10 s is represented by diagonal OT of parallelogram $OBTC$ where $OT = \sqrt{5^2 + 5^2} = 5\sqrt{2}$



$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$
$= \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2 (North - West)

75. (b) : Given : $x = 7t + 4t^2$, $y = 5t$
$\therefore \quad v_x = \frac{dx}{dt} = 7 + 8t \text{ m s}^{-1}, \; v_y = \frac{dy}{dt} = 5 \text{ m s}^{-1}$
$a_x = \frac{dv_x}{dt} = 8 \text{ m s}^{-2}$ ...(i)
$a_y = \frac{dv_y}{dt} = 0$ ...(ii)
Equations (i) and (ii) show that acceleration does not depend on time *i.e.*,the particle moves with constant acceleration a.
$a = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{(8)^2 + (0)^2} = 8 \text{ m s}^{-2}$
Thus the acceleration of the particle at $t = 5$ s is 8 m s^{-2}.

76. (b) : $x = 5t - 2t^2$, $y = 10t$
$\frac{dx}{dt} = 5 - 4t, \quad \frac{dy}{dt} = 10 \quad \therefore \quad v_x = 5 - 4t, \quad v_y = 10$
$\frac{dv_x}{dt} = -4, \quad \frac{dv_y}{dt} = 0 \quad \therefore \quad a_x = -4, \quad a_y = 0$
Acceleration, $\vec{a} = a_x\hat{i} + a_y\hat{j} = -4\hat{i}$
$\therefore$ The acceleration of the particle at $t = 2$ s is -4 m s^{-2}.

77. (c) : $\vec{r} = 2t^2\hat{i} + 3t\hat{j} + 4\hat{k}$
$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(2t^2\hat{i} + 3t\hat{j} + 4\hat{k}) = 4t\hat{i} + 3\hat{j}$
$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(4t\hat{i} + 3\hat{j}) = 4\hat{i}$
Thus the acceleration of the particle at $t = 1$ s is 4 m s^{-2} along x-direction.

78. (a) : $x = at$, $y = b\sin ct$
$v_x = \frac{dx}{dt} = a, \; v_y = \frac{dy}{dt} = bc\cos ct$
$a_x = \frac{dv_x}{dt} = 0, \; a_y = \frac{dv_y}{dt} = -bc^2\sin ct = -c^2 y$
$A = \sqrt{a_x^2 + a_y^2} = c^2 y \Rightarrow A \propto y$

79. (d) : $a = \frac{dv}{dt}$; $\int_u^v dv = \int_0^{10} a\, dt$
$\Rightarrow \quad v - u = \int_0^{10} 3t^2 dt\,\hat{i} + \int_0^{10} 5t dt\,\hat{j} - \int_0^{10} (8t^3 - 400)dt\,\hat{k}$
$\Rightarrow \quad v - u = 1000\,\hat{i} + 250\,\hat{j} - 24000\,\hat{k}$
On comparing with $n_1\hat{i} + n_2\hat{j} - n_3\hat{k}$, we get
$n_1 = 1000$, $n_2 = 250$ and $n_3 = 24000$

80. (d) : For y-coordinate, $s = ut + \frac{1}{2}at^2$
or $32 = 0 + \frac{1}{2} \times 4 \times t^2$ or $t = 4$ s
For x-coordinate, $s = ut + \frac{1}{2}at^2$
or $D = 3(4) + \frac{1}{2}(6)(4)^2 = 60$ m

81. (d) : Given, initial velocity, $\vec{u} = 10\hat{j}$ m/s
Acceleration, $a = (8\hat{i} + 2\hat{j})$m/s^2
$S_x = 16$ *m*, $S_y = ?$
$S_x = u_x t + \frac{1}{2}a_x t^2$ or $16 = 0 + \frac{1}{2} \times 8 \times t^2 \quad \therefore \quad t = 2$ s
Also, $S_y = u_y t + \frac{1}{2}a_y t^2$
$S_y = 10 \times 2 + \frac{1}{2} \times 2 \times (2)^2 = 24$ m

82. (b) : The horizontal component of velocity remains unchanged during the entire path of a projectile.

83. (c) : In angular projection, the body at the highest point has velocity = $u\cos\theta$ in the horizontal direction which makes zero angle with the horizontal direction.

84. (c)

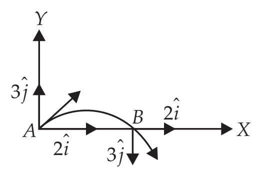
85. (b) : Upward deceleration is constant (g). It is independent of mass, angle and direction of projection.

86. (c) : Acceleration due to gravity is independent of mass.

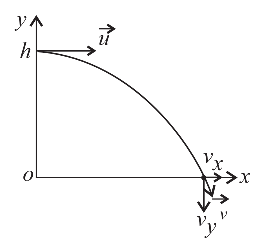
87. (b) : During projectile motion, horizontal acceleration is zero, so horizontal component of velocity always remains constant.

88. (b)

89. (a) : At point B, X component component of velocity remains unchanged while Y component reverses its direction.
$\therefore$ The velocity of the projectile at point B is $2\hat{i} - 3\hat{j}$ m/s.



90. (b) : Here,
Height of the cliff, $h = 441$ m
Initial speed with which the stone is thrown,
$u_x = 20$ m s^{-1} ; $u_y = 0$ m s^{-1}
Let v be the speed with which the stone reaches the ground.
Then, $v_x = 20$ m s^{-1}
Since, acceleration is acting downwards.



$v_y^2 - u_y^2 = 2gh$

$v_y = \sqrt{u_y^2 + 2gh} = \sqrt{2gh}, v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{(20)^2 + 2 \times 9.8 \times 441} = 95.09 \text{ m s}^{-1}$

91. (c) : Here, $x = 6t$ and $y = 8t - 5t^2$

$\therefore \quad v_x = \frac{dx}{dt} = \frac{d}{dt}(6t) = 6$ and $v_y = \frac{dy}{dt} = \frac{d}{dt}(8t - 5t^2) = 8 - 10t$

At $t = 0$, $v_x = u_x = 6$ and $v_y = u_y = 8$

Velocity of projection, $u = \sqrt{u_x^2 + u_y^2}$

$u = \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64} = 10 \text{ m s}^{-1}$

92. (b) : Given : Time taken to reach highest point $t = 2\sqrt{2}$ s

Angle of projection, $\theta = 45°$

Let u be the speed of projection

$\therefore \quad t = \frac{u\sin\theta}{g}$ or $u = \frac{gt}{\sin\theta}$ or $u = \frac{2\sqrt{2} \times 9.8}{\frac{1}{\sqrt{2}}} = 39.2 \text{ m s}^{-1}$

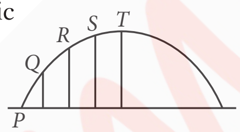
93. (b) : Horizontal range, $R = u\cos\theta \times T$

Here, $R = 300$ m, $T = 6$ s

$\therefore$ Horizontal component of velocity

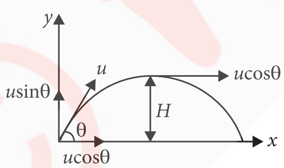
$= u\cos\theta = \frac{R}{T} = \frac{300}{6} = 50 \text{ m s}^{-1}$

94. (c) : As object follows a parabolic path *PQRST* as shown here. Since horizontal velocity is a constant, the horizontal component will be equal in equal time.



95. (b)

96. (d) : At maximum height (H) the vertical component of the projectile becomes zero whereas its horizontal component remains the same.
i.e., $u\cos\theta$.



97. (b) : For horizontal motion,

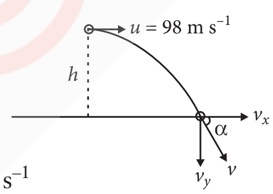
$v_x = u = 98 \text{ m s}^{-1}$

(As there is no acceleration in the horizontal direction)

For vertical motion, $u_y = 0$

$\therefore \quad v_y = 0 + g \times t = 9.8 \times 10 = 98 \text{ m s}^{-1}$

As $v_x = v_y = 98 \text{ m s}^{-1}$ $\therefore \alpha = 45°$



98. (a) : The speed of the projectile at the maximum height is $v = u\cos\theta$.

But $v = \frac{u}{\sqrt{2}}$ (given), $\therefore \frac{u}{\sqrt{2}} = u\cos\theta$ or $\frac{1}{\sqrt{2}} = \cos\theta$

or $\quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

The maximum height attained by the projectile is

$H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g}$

99. (d) : Range = Maximum height

$\frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g}; \quad \frac{2\sin\theta\cos\theta}{1} = \frac{\sin^2\theta}{2} \quad \therefore \tan\theta = 4$

100. (d) : The cannon ball will have same horizontal range for angle of projection θ and $(90° - \theta)$. So

$H_1 = \frac{u^2\sin^2\theta}{2g}$ and $H_2 = \frac{u^2\sin^2(90° - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$

$\therefore \quad H_1H_2 = \frac{1}{4}\left(\frac{u^2\sin\theta\cos\theta}{g}\right)^2 = \frac{1}{4} \times \frac{R^2}{4} \quad \left(\because R = \frac{u^2\sin 2\theta}{g}\right)$

or $\quad R = 4\sqrt{H_1H_2}$

101. (c) : Given, Speed at maximum height $u\cos\theta = \frac{\sqrt{3}u}{2}$

$\therefore \quad \cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = 30°$

Also, $PH_{max} = R \Rightarrow P = \frac{R}{H_{max}}$; But $H_{max} = \frac{R\tan\theta}{4}$

$\therefore \quad P = \frac{4}{\tan\theta} = \frac{4}{\tan 30°} = 4\sqrt{3}$

102. (a) : $R_{max} = \frac{u^2}{g} = 1000$ m

(R is maximum when $\theta = 45°$)

$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2}{2g} \times \sin^2 45° = \frac{1000}{4} = 250$ m

103. (b) : Range, $R_1 = \frac{u^2}{g}\sin 2 \times 15° \Rightarrow \frac{u^2}{g}\sin 30° = 150$

$\therefore \quad 150 \times 2 = \frac{u^2}{g}$ or $\frac{u^2}{y} = 300 \quad (\because \sin 30° = 1/2]$

Range, $R_2 = \frac{u^2}{g}\sin 2 \times 45° = 300 \times 1 = 300$ m

104. (d) : Range, $R = \frac{u^2\sin 2\theta}{g}$

Here g is constant and u is same for the projectiles A, B and C.

$\therefore \quad R \propto \sin 2\theta \Rightarrow R_A : R_B : R_C = \sin 60° : \sin 90° : \sin 120°$

$R_A : R_B : R_C = \frac{\sqrt{3}}{2} : 1 : \frac{\sqrt{3}}{2} = \sqrt{3} : 2 : \sqrt{3}$

Hence, $R_A = R_C < R_B$.

105. (c) : Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$

where u is the velocity of the projection and θ is the angle of the projection. Horizontal range is maximum when $\theta = 45°$.

106. (b) : Horizontal range is same for complementary angles *i.e.*, θ and $90° - \theta$. Hence, horizontal range is same for 40° and 50°, *i.e.* (90° – 40°).

107. (c)

108. (a) : Here, $R_{max} = R = \frac{u^2}{g}$ or $u^2 = Rg$

Also, Range $= \frac{u^2\sin 2\theta}{g} \Rightarrow \frac{R}{2} = \frac{Rg\sin 2\theta}{g}$

or $\quad \sin 2\theta = \frac{1}{2} = \sin 30°$ or $\theta = 15°$

109. (b) : Range $= \frac{u^2 \sin 2\theta}{g}$; $\frac{v^2 \sin 2\theta}{g} = \frac{v^2 \sin 30^\circ}{4g}$

or $\sin 2\theta = \frac{1}{4} \times \sin 30^\circ = \frac{1}{4} \times \frac{1}{2}$ or $\sin 2\theta = \frac{1}{8}$

or $2\theta = \sin^{-1}\left(\frac{1}{8}\right)$ $\therefore$ $\theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

110. (b) : The equation of the trajectory of a projectile motion is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta\left(1 - \frac{gx}{u^2 2\tan\theta\cos^2\theta}\right)$$

$$= x\tan\theta\left(1 - \frac{gx}{u^2\sin 2\theta}\right) = x\tan\theta\left(1 - \frac{x}{R}\right) \quad ...(i)$$

where R is the horizontal range of the projectile.
From the given equation of projectile

$$y = ax - bx^2 = ax\left(1 - \frac{bx}{a}\right) \quad ...(ii)$$

Comparing (i) and (ii), we get, $\frac{1}{R} = \frac{b}{a}$ or $R = \frac{a}{b}$

111. (d) : Let u be initial velocity of the projectile.
For angle of projection $(45^\circ + \alpha)$, horizontal range is

$$R_1 = \frac{u^2\sin 2(45^\circ + \alpha)}{g} = \frac{u^2\sin(90^\circ + 2\alpha)}{g} = \frac{u^2\cos 2\alpha}{g}$$

For angle of projection $(45^\circ - \alpha)$, horizontal range is

$$R_2 = \frac{u^2\sin 2(45^\circ - \alpha)}{g} = \frac{u^2\sin(90^\circ - 2\alpha)}{g} = \frac{u^2\cos 2\alpha}{g}$$

$$\therefore \quad \frac{R_1}{R_2} = 1$$

112. (b) : $s_x = u_x t + \frac{1}{2}a_x t^2$

$$s_x = u\cos 37^\circ\, t - \frac{1}{2}\times 5\times t^2 \qquad \left[\because\ a = -5\ \text{m/s}^2\right]$$

$$0 = u\cos 37^\circ t - \frac{1}{2}\times 5\times t^2\ ;\ u\cos 37^\circ t = \frac{1}{2}\times 5\times t^2$$

$$t = u\times\frac{2}{5}\times\cos 37^\circ\ ;\ t = u\times\frac{2}{5}\times\frac{4}{5} = \frac{8u}{25}\text{ sec}$$

at this time x = coordinate will become zero again
$u = 10$ m/s ; $v_x = u_x + a_x t$

$$v_x = u\cos 37^\circ - 5\times\frac{8u}{25}\ ;\ u_x = 10\times\frac{4}{5} - \frac{8\times 10}{5}$$

$v_x = 8 - 16 = -8$ m/sec and y coordinate
$v_y = u\sin 37^\circ + a_y t$

$$v_y = 10\times\frac{3}{5} + \frac{10\times 8\times 10}{25}\ ;\ v_y = 6 + 32 = 38\text{ m/s.}$$

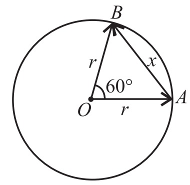
113. (a) : In uniform circular motion, the speed of a particle remains constant.

114. (c)

115. (d) : Velocity does not remain constant as it continuously changes direction.

116. (d) : For a particle performing uniform circular motion, magnitude of the acceleration remains constant.

117. (c) : According to cosine formula



$$\cos 60^\circ = \frac{r^2 + r^2 - x^2}{2r^2}$$

$2r^2\cos 60^\circ = 2r^2 - x^2$
$x^2 = 2r^2 - 2r^2\cos 60^\circ = 2r^2[2\sin^2 30^\circ] = r^2$
$\therefore \quad x = r$
Displacement $AB = x = r$

118. (c) : The kinematic equations for uniform acceleration do not apply in case of uniform circular motion because in this case the magnitude of acceleration is constant but its direction is changing.

119. (c) : Here, $r = 10$ m, $v = 5$ m s^{-1}, $a_t = 2$ m s^{-2},

$$a_r = \frac{v^2}{r} = \frac{5\times 5}{10} = 2.5\text{ m s}^{-2}$$

The net acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(2.5)^2 + 2^2} = \sqrt{10.25} = 3.2\text{ m s}^{-2}$$

120. (c) : If $a_r = 0$, there is no radial acceleration and circular motion is not possible.
So, $a_r \neq 0$
If $a_t \neq 0$, the motion is not uniform as angular velocity will change
So, $a_r \neq 0$ and $a_t = 0$ for uniform circular motion.

121. (d) : Centripetal acceleration, $a_c = \frac{v^2}{R}$

where v is the speed of an object and R is the radius of the circle.
It is always directed towards the centre of the circle. Since v and R are constants for a given uniform circular motion, therefore the magnitude of centripetal acceleration is also constant. However, the direction of centripetal acceleration changes continuously. Therefore, a centripetal acceleration is not a constant vector.

122. (c) : Centripetal acceleration, $a_c = \omega^2 R$

$$= \frac{(2\pi)^2\times 1.5\times 10^{11}}{(365\times 86400)^2} \approx 6\times 10^{-3}\text{ m/s}^2$$

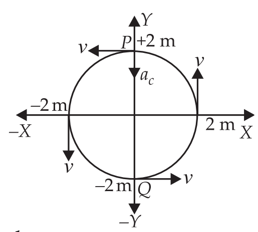
123. (a)

124. (d) : Here, $\frac{r_1}{r_2} = \frac{6}{8}, \frac{m_1}{m_2} = 1$

$$\frac{F_1}{F_2} = 1 \text{ or, } F_1 = F_2 \text{ or } \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$$\text{or } \frac{v_1^2}{v_2^2} = \frac{r_1}{r_2}\times\frac{m_2}{m_1} = \frac{6}{8}\times 1 = \frac{3}{4} \qquad \therefore \frac{v_1}{v_2} = \frac{\sqrt{3}}{2}$$

125. (a) : Here, radius of circular path, $r = 2$ m
Speed of the object, $v = 4$ m s^{-1}
The magnitude of centripetal acceleration



$$a_c = \frac{v^2}{r} = \frac{4^2}{2} = 8\text{ m s}^{-2}$$

This acceleration is directed towards the centre.
Therefore, when the object is at $y = 2$ m, its position is P (see figure). Its acceleration is $-8\hat{j}$ m s^{-2}.

126. (b) : Here, r = 12 cm, frequency, $\upsilon = \frac{7}{100}$ rps
The angular speed of the insect is

$\omega = 2\pi\upsilon = 2\pi \times \frac{7}{100} = 0.44$ rad s^{-1}

The linear speed of the insect is $v = \omega r = 0.44 \times 12 = 5.3$ cm s^{-1}

127. (c) : The time period of minute hand of a clock is 1 hour and of second hand of a clock is one minute.

$\therefore \quad \omega_m = \frac{2\pi}{60}$ rad/min and $\omega_s = \frac{2\pi}{1}$ rad/min

$\therefore \quad \frac{\omega_m}{\omega_s} = \frac{1}{60}$

128. (a) : Time period, $T = \frac{2\pi}{\omega}$

As $T_A = T_B$

So, $\frac{2\pi}{\omega_A} = \frac{2\pi}{\omega_B}$ or $\omega_A : \omega_B = 1 : 1$

129. (d) : Here, R = 10 cm
The second hand of a clock completes one rotation in 60 s.

Its angular speed is $\omega = \frac{2\pi \text{ rad}}{60 \text{ s}} = \frac{\pi}{30}$ rad s^{-1}

The speed of its tip is $v = \omega R = \frac{\pi}{30} \times 10$ cm s$^{-1} \approx 1$ cm s^{-1}

130. (b)

131. (d) : Here, T = 24 hr = 24 × 60 × 60 s = 86400 s

$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{86400} \quad \therefore \ \omega = 7.27 \times 10^{-5}$ rad/s

132. (b) : The time period T is; $T = \frac{2\pi r}{v}$ ...(i)

When the speed of the particle is doubled, its new time period becomes

$T' = \frac{2\pi r}{2v} = \frac{1}{2}\left(\frac{2\pi r}{v}\right) = \frac{T}{2}$ (using (i))

133. (c) : As $\omega = \frac{2\pi}{T}$ and T is same, therefore, ω must be same *i.e.*, $\frac{\omega_1}{\omega_2} = 1$.

134. (c) : $\upsilon = 120 \text{ rpm} = \frac{120}{60} = 2$ rps

$\omega = 2\pi\upsilon = 2\pi(2) = 4\pi$ rad s^{-1}

135. (a) : Here, r = 100 cm = 1 m

Frequency, $\upsilon = \frac{14}{22}$ rps; $\therefore \ \omega = 2\pi\upsilon = 2 \times \frac{22}{7} \times \frac{14}{22} = 4$ rad s^{-1}

The acceleration of the stone is $a_c = \omega^2 r = (4)^2 (1) = 16$ m s^{-2}

136. (d) : In half the period, particle is diametrically opposite to its inital position. Hence, its displacement is $2R$. It has covered a semicircle, hence distance covered bny particle is πR.

137. (a) : Here, r = 100 cm = 1 m, $\upsilon = \frac{14}{25}$ s^{-1}

$\therefore \quad \omega = 2\pi\upsilon = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25}$ rad s^{-2}

Centripetal acceleration,

$a_c = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 1.0 = \left(\frac{88}{25}\right)^2$ m s^{-2}

138. (b) : Frequency of rotation, n = 120 r.p.m. = 2 r.p.s.
Length of each blade, r = 30 cm = 0.3 m
Centripetal acceleration, $a = \omega^2 r = (2\pi n)^2 r$
$= 4\pi^2 n^2 r = 4\pi^2 (2)^2 (0.3) = 47.4$ m s^{-2}

139. (b) : Acceleration of a point at the tip of the blade = centripetal acceleration = $\omega^2 R = (2\pi\upsilon)^2 R$

$= \left(2 \times \frac{22}{7} \times \frac{1200}{60}\right)^2 \times \frac{30}{100} = 4740$ m s^{-2}

140. (b) : Length of the string, l = 80 cm = 0.8 m
Number of revolutions = 25, Time taken = 14 s

$\therefore \quad \upsilon = \frac{25}{14} = 1.78$ s^{-1}

$\therefore \quad \omega = 2\pi\upsilon = 2\pi \times 1.78 = 11.18$ rad/s
$a = \omega^2 l = (11.18)^2 \times (0.8) = 99.99$ m/s$^2 \approx 100$ m/s^2

KCET Ready

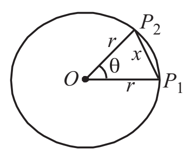
1. (d) : For unit vector, $|0.8\hat{i} + b\hat{j} + 0.4\hat{k}| = 1$

$\sqrt{(0.8)^2 + (b)^2 + (0.4)^2} = 1$

$\sqrt{0.64 + b^2 + 0.16} = 1$; $\sqrt{0.80 + b^2} = 1$

$0.8 + b^2 = 1 \Rightarrow b^2 = 0.2 \Rightarrow b = \sqrt{0.2}$

2. (d) : According to cosine formula,



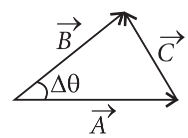
$\cos\theta = \frac{r^2 + r^2 - x^2}{2r^2}$

or $2r^2 \cos\theta = r^2 + r^2 - x^2$

or $x^2 = 2r^2 - 2r^2\cos\theta = 2r^2 [1 - \cos\theta] = 2r^2 \left[2\sin^2\frac{\theta}{2}\right]$

Displacement from P_1 to P_2 is $x = 2r\sin\frac{\theta}{2}$

3. (c) : By triangle rule ,



$\vec{A} + \vec{C} = \vec{B}$; $\vec{B} - \vec{A} = \vec{C}$

$|\vec{B} - \vec{A}| = |\vec{C}| = |\vec{B}| \sin\Delta\theta$ ($\because \Delta\theta << 1$)

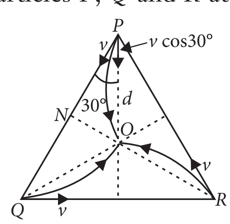
$|\vec{B} - \vec{A}| = |\vec{B}| \Delta\theta$ ($\because \sin\Delta\theta \simeq \Delta\theta$)

Again $|\vec{B}| \cos\Delta\theta = |\vec{A}|$

$|\vec{B}| = |\vec{A}|$ ($\because \cos\Delta\theta \simeq 1$)

So, $|\vec{B} - \vec{A}| = |\vec{B}| \Delta\theta = |\vec{A}| \Delta\theta$

4. (d) : By symmetry, all the three particles P, Q and R at the vertices of an equilateral triangle will meet at O, the centroid of the triangle while following the curved paths shown in figure.
The initial distance of each particle from the centroid O is
$d = PO = QO = RO$



From ΔPNO, $\cos 30° = \frac{PN}{PO}$

or $PO = \frac{PN}{\cos 30°} = \frac{s/2}{(\sqrt{3}/2)} = \frac{s}{\sqrt{3}}$ $\left(\because PN = \frac{1}{2}PQ = \frac{s}{2}\right)$

$\therefore \quad d = \frac{s}{\sqrt{3}}$

The component of velocity towards O is

$$v' = v\cos 30° = v\left(\frac{\sqrt{3}}{2}\right)$$

$\therefore$ The time of meeting is $t = \dfrac{d}{v'} = \dfrac{(s/\sqrt{3})}{v(\sqrt{3}/2)} = \dfrac{2s}{3v}$

5. (a) : Here, $x = 4\sin\left(\dfrac{\pi}{2} - \omega t\right)\text{m} = 4\cos\omega t \text{ m}$ ...(i)

and $y = 4\sin\omega t$ m ....(ii)

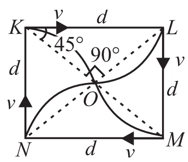
Squaring and adding equations (i) and (ii),

we get, $x^2 + y^2 = 16\cos^2\omega t + 16\sin^2\omega t$

or $x^2 + y^2 = 16(\cos^2\omega t + \sin^2\omega t) = 16$ or $x^2 + y^2 = 4^2$

This is the equation of a circle. Hence particle follows a circular path.

6. (a) : The four persons K, L, M and N will meet at O *i.e.*, the centre of the diagonal of the square $KLMN$. The person K will travel a distance KO, with velocity along $KO = v\cos 45° = \dfrac{v}{\sqrt{2}}$

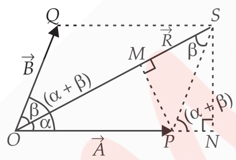


Here, $KO = d\cos 45° = \dfrac{d}{\sqrt{2}}$

$\therefore$ Time of meeting, $t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{d/\sqrt{2}}{v/\sqrt{2}} = \dfrac{d}{v}$

7. (a) : Let $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ represent two vectors $\vec{A}$ and $\vec{B}$ making an angle $(\alpha + \beta)$.

Using the parallelogram method of vector addition,



Resultant vector, $\vec{R} = \vec{A} + \vec{B}$

SN is normal to OP and PM is normal to OS.

From the geometry of the figure,

$OS^2 = ON^2 + SN^2 = (OP + PN)^2 + SN^2$

$= (A + B\cos(\alpha + \beta))^2 + (B\sin(\alpha + \beta))^2$

$R^2 = A^2 + B^2 + 2AB\cos(\alpha + \beta)$

In ΔOSN, $SN = OS\sin\alpha = R\sin\alpha$

and in ΔPSN, $SN = PS\sin(\alpha + \beta) = B\sin(\alpha + \beta)$

$\therefore$ $R\sin\alpha = B\sin(\alpha + \beta)$ or $\dfrac{R}{\sin(\alpha+\beta)} = \dfrac{B}{\sin\alpha}$ ... (i)

Similarly, $PM = A\sin\alpha = B\sin\beta$

$\dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha}$ ... (ii)

Combining (i) and (ii), we get

$\dfrac{R}{\sin(\alpha+\beta)} = \dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha}$ ... (iii)

From eqn. (iii), $R\sin\beta = A\sin(\alpha + \beta)$

8. (b) : The position vectors for points G and H are $\left(\dfrac{a}{2}\hat{k} + \dfrac{a}{2}\hat{i}\right)$ and $\left(\dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}\right)$ respectively.

So, the displacement vector from G to H is

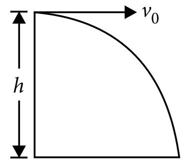
$$\overrightarrow{OH} - \overrightarrow{OG} = \frac{a}{2}(\hat{k} + \hat{j} - \hat{k} - \hat{i}) = \frac{a}{2}(\hat{j} - \hat{i})$$

9. (b) : Let h be the height of table.

So, acceleration, $\vec{a} = -g\hat{j}$

$\therefore$ The vertical components of velocity only changes.

Using second equation of motion in vertical direction,



$h = \dfrac{1}{2}gT^2$ or $T = \sqrt{\dfrac{2h}{g}}$

$\therefore$ Time of flight is independent of initial horizontal speed.

10. (b) : Let $\vec{A}$ be vector in xy plane. Its x and y components are $A_x = 12$ m, $A_y = 8$ m

The magnitude of vector $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(12)^2 + (8)^2} = \sqrt{208}\text{ m}$$

When this vector is rotated in xy plane such that its x component becomes halved and its new y component be A'_y. Then

$$A = \sqrt{\left(\frac{A_x}{2}\right)^2 + A_y'^2}\ ;\ \sqrt{208} = \sqrt{(6)^2 + A_y'^2}$$

Squaring both sides, we get

$208 = (6)^2 + A_y'^2$ or $A_y'^2 = 208 - 36 = 172$

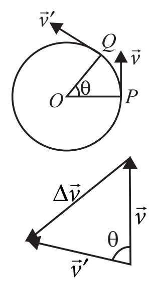
$A'_y = \sqrt{172} = 13.11$ m

11. (c) : $\vec{v}\cdot\vec{u} = 0 \Rightarrow t = \dfrac{v_0}{g\sin\alpha}$ $\therefore$ $\sin\alpha = \dfrac{v_0}{gt} > 0$ or $\alpha < \dfrac{\pi}{2}$

Maximum time is the time of flight, $T = \dfrac{2v_0\sin\alpha}{g}$

$T \ge t \Rightarrow \dfrac{2v_0\sin\alpha}{g} \ge \dfrac{v_0}{g\sin\alpha}$ or $\sin\alpha \ge \dfrac{1}{\sqrt{2}}$ $\therefore$ $\dfrac{\pi}{4} \le \alpha \le \dfrac{\pi}{2}$

12. (c) : $|\vec{v}'| = |\vec{v}| = v$



$\Delta\vec{v} = \vec{v}' - \vec{v}$

$\therefore |\Delta\vec{v}| = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$

$= \sqrt{2v^2(1 - \cos\theta)} = 2v\sin\dfrac{\theta}{2}$

13. (a) : Here, $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$

The velocity of the particle is

$\vec{v} = \dfrac{d\vec{R}}{dt} = \dfrac{d}{dt}[4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}]$

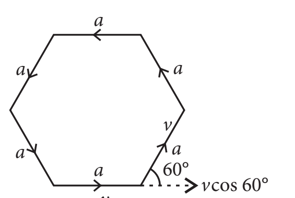
$= 8\pi\cos(2\pi t)\hat{i} - 8\pi\sin(2\pi t)\hat{j}$

Its magnitude is

$|\vec{v}| = \sqrt{(8\pi\cos(2\pi t))^2 + (-8\pi\sin(2\pi t))^2}$

$= \sqrt{64\pi^2\cos^2(2\pi t) + 64\pi^2\sin^2(2\pi t)} = \sqrt{64\pi^2} = 8\pi$ m/s

14. (b) : Due to symmetry of the problems, we can say that the six particles will meet at the centre of the hexagon.



The separation between any two consecutive particles decreases at the rate of approach velocity.

Velocity of approach

$v_{app} = v - v\cos 60° = v - v/2 = v/2$

The initial separation between two consecutive particles is a and it decreases to zero with a constant velocity of approach equal to $v/2$

$$t = \frac{a}{v_{app}} = \frac{a}{v/2} = \frac{2a}{v}$$

15. (d) : Let, θ_0 = angle at $t = 0$

$$\tan\theta = \frac{dy}{dx} = \frac{v_y}{v_x} = \frac{u\sin\theta_0 - gt}{u\cos\theta_0};\ \tan\theta = \tan\theta_0 - \frac{gt}{u}\sec\theta_0$$

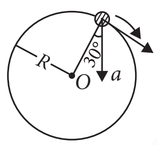
$$\tan 30° = \tan\theta_0 - \left(\frac{2g}{u}\sec\theta_0\right) \quad ...(i)$$

$$\tan 0° = \tan\theta_0 - \left(\frac{3g}{u}\sec\theta_0\right) \quad ...(ii)$$

$\tan\theta_0 = \sqrt{3}$ or $\theta_0 = 60°$

$\theta_0 = 60°$ putting in eq (i), $u = 20\sqrt{3}$ m s^{-1}

16. (c) : Here, $a = 15$ m s^{-2}, $R = 2.5$ m

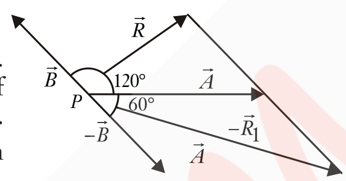


From figure, $a_c = a\cos 30° = 15 \times \frac{\sqrt{3}}{2}$ m s^{-2}

As we know, $a_c = \frac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$

$$\therefore\quad v = \sqrt{15 \times \frac{\sqrt{3}}{2} \times 2.5} = 5.69 \simeq 5.7 \text{ m s}^{-1}$$

17. (b) : Let $\vec{A} + \vec{B} = \vec{R}$.



Then, $R^2 = A^2 + B^2 + 2AB\cos\theta$. According to law of parallelogram of vectors. Here, $\hat{A}, \hat{B}$ and $\hat{R}$ are given to be unit vectors.

$\therefore (1)^2 = (1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos\theta$

$1 = 2 + 2\cos\theta$ or $2(1 + \cos\theta)$

or $1 + \cos\theta = \frac{1}{2} \Rightarrow \cos\theta = -\frac{1}{2} = \cos 120°$

or $\theta = 120°$ ...(i)

$$\vec{A} - \vec{B} = |\vec{A} + (-\vec{B}) \Rightarrow |(\hat{A} - \hat{B})| = |\hat{A} + (-\hat{B})|$$

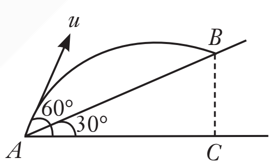
The angle between $\hat{A}$ and $(-\hat{B}) = 180° - 120° = 60°$

$\therefore$ Resultant of $|\hat{A} + (-\hat{B})| = |R_1|$

$\therefore$ $R_1^2 = (1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos 60°$

or $R_1^2 = 1 + 1 + \left(\frac{2\times1}{2}\right) = 3$ or $R_1 = \sqrt{3}$.

18. (d) : From figure, horizontal component of velocity at A is



$$u_H = u\cos 60° = \frac{u}{2}$$

$$\therefore\ AC = u_H \times t = \frac{ut}{2}$$

$$AB = AC\sec 30° = \frac{ut}{2} \times \frac{2}{\sqrt{3}} = \frac{ut}{\sqrt{3}}$$

19. (d) : When the person is sitting in truck and throws a ball upward, then ball gains velocity in two directions but the only acceleration present in the upward motion.

$\therefore$ The ball will trace a parabola and the range of ball and distance travelled by the person in truck is same.

Hence, the person can catch the ball when it comes down, if the truck continues to move with a constant speed of 60 km h^{-1}.

20. (d) : If T is the time of flight, then,

$y = 0 = Kt(1 - \alpha t) = 0 \quad \therefore \quad T = \frac{1}{\alpha}$.

Maximum height will occur at $T/2$,

$$y_{max} = \frac{dy}{dt} = 0 \Rightarrow k(1 - 2\alpha t)$$

$$\therefore\ y_{max} = K\left(\frac{1}{2\alpha}\right)\left(1 - \alpha\frac{1}{2\alpha}\right) = \frac{K}{4\alpha}.$$

21. (a) : Equation of trajectory is of the form

$y = ax - bx^2$; Slope, $\frac{dy}{dx} = a - 2bx$; where, $x = (u\cos\theta)t$

$$\frac{dy}{dx} = a - (2bu\cos\theta)t$$

Slope versus t is a straight line with negative slope.

22. (d) : Here, $u = 25$ m/s, angle of projection = θ

If the inclination is zero, it means it is at maximum height. So,

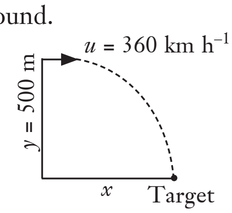
$$t = \frac{T}{2};\ t = \frac{u\sin\theta}{g};\ \text{Range, } R = u\cos\theta \times 2t \Rightarrow \frac{t}{R} = \frac{\tan\theta}{g(2t)}$$

$$\tan\theta = \frac{2gt^2}{R} \Rightarrow \theta = \tan^{-1}\left(\frac{2gt^2}{R}\right) \text{ or } \theta = \cot^{-1}\left(\frac{R}{20t^2}\right)$$

$(\because g = 10 \text{ m s}^{-2})$

23. (a) : $u_x = 360$ km h^{-1} $= 360 \times \frac{5}{18}$ m s^{-1} $= 100$ m s^{-1}

Let the bomb takes time t to hit the ground.



Using second equation of motion for vertical direction, $s_y = u_y t + \frac{1}{2}a_y t^2$

$$-500 = 0 - \frac{1}{2} \times 10 \times t^2$$

$\Rightarrow \quad t = 10$ s

Horizontal distance, $x = u_x \times t = 100 \times 10 = 1000$ m

$\therefore$ The plane has to drop the bomb 1000 m ahead of the target.

24. (b) : In case of uniform circular motion, for time interval $\delta t \to 0$, angle between change in velocity $(\delta\vec{v})$ and linear velocity $(\vec{v})$ will be 90°.

25. (a) : Given trajectory of particle, $y = 2x - 9x^2$

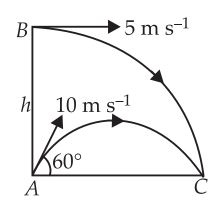
Comparing it with equation of projectile

$$y = x\tan\theta_0 - \frac{g}{2u^2\cos^2\theta_0}x^2$$

$$\tan\theta_0 = 2 \Rightarrow \cos\theta_0 = \frac{1}{\sqrt{5}} \Rightarrow \theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$\text{and}\quad \frac{g}{2u^2\cos^2\theta_0} = 9 \Rightarrow u = v_0 = \frac{5}{3}\text{ m s}^{-1}$$

26. (b) : Both the particles will meet at C, if their time of flight is same. The time of flight of A is

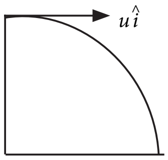


$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \sin 60°}{10} = \sqrt{3}\text{ s}$$

For vertical downward motion of particle B from B to C, we have

$$h = \frac{1}{2}gT^2 = \frac{1}{2} \times 10 \times (\sqrt{3})^2 = 15 \text{ m}$$

27. (a) : When a bomb moves with horizontal speed u velocity of bomb after time t



$v_B = u\hat{i} - gt\hat{j}$

velocity of bomb with respect to man

$v_{B/m} = \vec{v}_B - \vec{v}_m = u\hat{i} - gt\hat{j} - u\hat{i} = -gt\hat{j}$

So, motion is vertically down in a straight line.

28. (b) : Given; $x = 36t$ and $2y = 96t - 9.8t^2$

or $y = 48t - 4.9t^2$

Let the initial velocity of projectile be u and angle of projection is θ.

Initial horizontal component of velocity,

$$u_x = u\cos\theta = \left(\frac{dx}{dt}\right)_{t=0} = 36 \quad i.e., \ u\cos\theta = 36$$

Initial vertical component of velocity,

$$u_y = u\sin\theta = \left(\frac{dy}{dt}\right)_{t=0} = 48 \quad i.e., \ u\sin\theta = 48 \quad \text{...(ii)}$$

From (i) and (ii), we get

$$\tan\theta = \frac{48}{36} = \frac{4}{3} \quad \therefore \sin\theta = \frac{4}{5} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{4}{5}\right)$$

29. (a) : The time taken to reach maximum height and maximum height are $t = \frac{u\sin\theta}{g}$ and $H = \frac{u^2\sin^2\theta}{2g}$

For remaining half, the time of flight is

$$t' = \sqrt{\frac{2H}{(2g)}} = \sqrt{\frac{u^2\sin^2\theta}{2g^2}} = \frac{t}{\sqrt{2}}$$

∴ Total time of flight is $t + t' = T$

$$T = t\left(1 + \frac{1}{\sqrt{2}}\right) \Rightarrow T = \frac{u\sin\theta}{g}\left(1 + \frac{1}{\sqrt{2}}\right)$$

So horizontal range $R = u\cos\theta \times T = \frac{u^2\sin 2\theta}{2g}\left(1 + \frac{1}{\sqrt{2}}\right)$

30. (a) : The equation of trajectory is

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

where θ is the angle of projection and u is the velocity with which projectile is projected.

For equal trajectories for same angles of projection,

$$\frac{g}{u^2} = \text{constant}$$

As per question, $\frac{9.8}{5^2} = \frac{g'}{3^2}$

where g' is acceleration due to gravity on the planet.

$$g' = \frac{9.8 \times 9}{25} = 3.5 \text{ m s}^{-2}$$

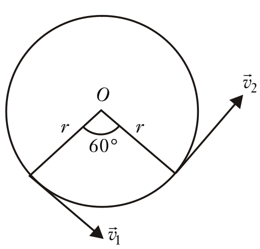
31. (b) : We know that:

$(\Delta v)^2 = v_1^2 + v_2^2 - 2v_1v_2\cos\theta$

$= (v)^2 + (v)^2 - 2(v)(v)\cos 60°$

$= 2v^2 - \frac{2v^2 \times 1}{2} = v^2$

or $\Delta v = v$



32. (c) : $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$

$$|\vec{A}+\vec{B}| = |(a + a\cos\omega t)\hat{i} + a\sin\omega t\,\hat{j}| = 2a\cos\frac{\omega t}{2} \quad \text{...(i)}$$

$$|\vec{A}-\vec{B}| = |(a - a\cos\omega t)\hat{i} - \sin\omega t\,\hat{j}| = 2a\sin\frac{\omega t}{2} \quad \text{...(ii)}$$

Using equations (i) and (ii) in $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$

$$2a\cos\frac{\omega t}{2} = \sqrt{3}\left(2a\sin\frac{\omega t}{2}\right) \Rightarrow \tan\frac{\omega t}{2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\omega t}{2} = \frac{\pi}{6} \text{ or } \frac{\pi}{6}\cdot t = \frac{\pi}{3} \Rightarrow t = 2.00 \text{ s} \quad \left(\because \omega = \frac{\pi}{6}\right)$$

33. (a) : Let u be the velocity of projection of the ball. The ball will cover maximum horizontal distance when angle of projection with horizontal, θ = 45°. Then $R_{max} = \frac{u^2}{g}$

Here, $R_{max} = 100$ m ∴ $\frac{u^2}{g} = 100$ m ... (i)

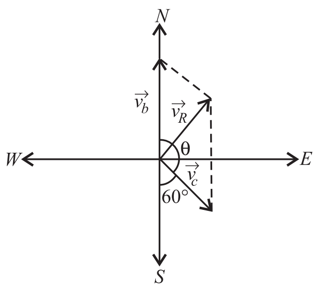
As $v^2 - u^2 = 2as$

Here, $v = 0$ (At highest point velocity is zero)

$a = -g$, $s = H$ ↑+ve

$$\therefore H = \frac{u^2}{2g} = \frac{100}{2} = 50 \text{ m} \quad \text{(Using (i))}$$

34. (b) : The velocity of the motorboat and the velocity of the water current are represented by vectors $\vec{v}_b$ and $\vec{v}_c$ as shown in the figure.



Here, θ = 180° – 60° = 120°

$v_b = 25$ km h^{-1}, $v_c = 10$ km h^{-1}

∴ According to parallelogram law of vector addition, the magnitude of the resultant velocity of the boat is

$$v_R = \sqrt{v_b^2 + v_c^2 + 2v_bv_c\cos 120°} \approx 22 \text{ km h}^{-1}$$

35. (b) : As $v = \alpha\sqrt{x}$ (given)

But $v = \frac{dx}{dt}$ ∴ $\frac{dx}{dt} = \alpha\sqrt{x}$

$$\text{or} \quad \frac{dx}{\sqrt{x}} = \alpha\, dt \quad \text{or} \quad x^{-1/2}dx = \alpha\, dt \quad \text{...(i)}$$

Integrating eqn. (i) within the given limits, we get

$$\int_0^x x^{-1/2}dx = \int_0^t \alpha\, dt \quad \text{or} \quad \left[\frac{x^{1/2}}{1/2}\right]_0^x = \alpha[t]_0^t$$

$$2\sqrt{x} = \alpha t \quad \text{or} \quad \sqrt{x} = \frac{\alpha t}{2}$$

Squaring both sides, we get

$$x = \frac{\alpha^2 t^2}{4} \quad \text{or} \quad x \propto t^2$$

36. (c) : Here, $u = 10$ m s^{-1}, θ = 60°

∴ $u_x = u\cos\theta = 10 \times \cos 60° = 5$ m s^{-1}

and $u_y = u\sin\theta = 10 \times \sin 60° = 5\sqrt{3}$ m s^{-1}

$$\therefore t = \frac{2u_y}{g} = \frac{2 \times 5\sqrt{3}}{10} = \sqrt{3} \text{ s}$$

and $1.15 = u_x t - \frac{1}{2}at^2$, where a is the acceleration of train

$1.15 = 5\times\sqrt{3} - \frac{1}{2}\times a\times(\sqrt{3})^2$

$\frac{3a}{2} = 5\sqrt{3} - 1.15 = 8.66 - 1.15 = 7.51$

$a = 7.5\times\frac{2}{3} = 5 \text{ m s}^{-2} \Rightarrow a = 5 \text{ m/s}^2$

37. (b): Using the relation, $s = ut + \frac{1}{2}at^2$, we have

$h = u\cos\theta t_1 - \frac{1}{2}gt_1^2 = u\cos\theta\, t_2 - \frac{1}{2}gt_2^2$

or $u\cos\theta\times 1 - \frac{1}{2}\times 9.8\times 1^2 = u\cos\theta\times 3 - \frac{1}{2}\times 9.8\times 3^2$

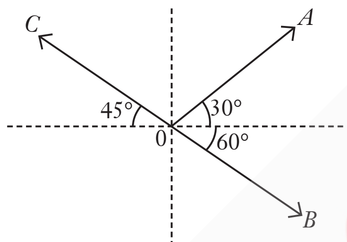
or $u\cos\theta\,(3-1) = 4.9\times(9-1)$

or $u\cos\theta = \frac{4.9\times 8}{2} = 19.6 \text{ m/s}$

Now, $H_{max} = \frac{u^2\cos^2\theta}{2g} = \frac{(19.6)^2}{2\times 9.8} = 19.6 \text{ m}$

38. (b): Let the magnitude of each vector is K.

$\overline{OA} = K\left[\cos 30\hat{i} + \sin 30\hat{j}\right]$; $\overline{OA} = K\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$



$\overline{OB} = K\left[\cos 60\,\hat{i} - \sin 60\hat{j}\right] = K\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$

$\overline{OC} = K\left[-\cos 45\,\hat{i} + \sin 45\hat{j}\right] = K\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$

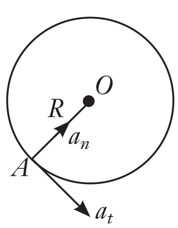
$\overline{OA} + \overline{OB} - \overline{OC} = \left[\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}\right]\hat{i}\,K + \left[\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right]\hat{j}\,K$

The direction is given by

$$\tan\theta = \frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}} = \frac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}$$

$$\theta = \tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$$

39. (b): Normal acceleration, $a_n = \frac{v^2}{R}$



Tangential acceleration, $a_t = \frac{dv}{dt}$

According to problem, $a_t = a_n$

$\frac{dv}{dt} = \frac{v^2}{R}$ or $\frac{dt}{R} = \frac{dv}{v^2}$

Integrating the above equation, we get

$\int_0^t \frac{dt}{R} = \int_{v_0}^{v}\frac{dv}{v^2}$, $\frac{t}{R} = -\left[\frac{1}{v}\right]_{v_0}^{v}$; $v = \frac{v_0 R}{(R - v_0 t)}$

As $v = \frac{dr}{dt} = \frac{v_0 R}{(R-v_0 t)}$; $\frac{dr}{R} = \frac{v_0 dt}{(R-v_0 t)}$

Integrate the above equation, we get, $\int_0^{2\pi R}\frac{dr}{R} = \int_0^T \frac{v_0\,dt}{(R-v_0 t)}$

$T = \frac{R}{v_0}(1-e^{-2\pi})$

40. (b): Let P and Q be the two forces. Then

Greatest resultant = $P + Q = 29$ N ...(i)

Least resultant = $P - Q = 5$ N ...(ii)

On solving equations (i) and (ii), we get $P = 17$ N, $Q = 12$ N

When each force is increased by 3 N, new force are

$p = P + 3 = 17 + 3 = 20$ N

$\Rightarrow q = Q + 3 = 12 + 3 = 15$ N

As the new forces act at right angle to each other, their resultant is

$R = \sqrt{p^2+q^2} = \sqrt{20^2+15^2} = \sqrt{625} = 25$ m

If the resultant R makes angle β with the force p, then

$\tan\beta = \frac{q}{p} = \frac{15}{20} = 0.75$ or $\beta = \tan^{-1}(0.75) = 36°.52'$

41. (d): Let eastern line be taken as X-axis, northern as Y-axis and vertical upward as Z-axis. Let the velocity vector $\vec{v}$ make angles α, β and γ with X-,Y-and Z-axis respectively. Then $\alpha = 60°$, $\gamma = 60°$.

As $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \cos^2 60° + \cos^2\beta + \cos^2 60° = 1$

or $\left(\frac{1}{2}\right)^2 + \cos^2\beta + \left(\frac{1}{2}\right)^2 = 1$

or $\cos^2\beta = 1 - \frac{1}{2} = \frac{1}{2}$ or $\cos\beta = \frac{1}{\sqrt{2}}$

$\therefore \vec{v} = v\cos\alpha\hat{i} + v\cos\beta\hat{j} + v\cos\gamma\hat{k}$

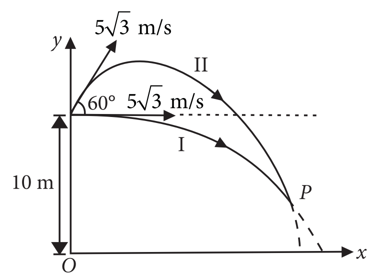
$= 20\times\frac{1}{2}\hat{i} + 20\times\frac{1}{\sqrt{2}}\hat{j} + 20\times\frac{1}{2}\hat{k} = 10\hat{i} + 20\frac{1}{\sqrt{2}}j + 10\hat{k}$

42. (b): The situation is as shown in figure.

Let the two shots collide at P.

Taking the foot of the tower as origin, with upward direction of motion positive and using the equation of motion,

$S = S_0 + ut + \frac{1}{2}at^2$



For the first shot, horizontal and vertical distances travelled by the shot in time t_1 are

$x_1 = 5\sqrt{3}\,t_1$

and $y_1 = 10 - \frac{1}{2}gt_1^2$

Similarly, for the second shot,

$x_2 = 5\sqrt{3}\cos 60° t_2 = 5\sqrt{3}\times\frac{1}{2}t_2 = \frac{5\sqrt{3}}{2}t_2$

and $y_2 = 10 + 5\sqrt{3}\sin 60° t_2 - \frac{1}{2}gt_2^2$

$= 10 + 5\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)t_2 - \frac{1}{2}gt_2^2 = 10 + \frac{15}{2}t_2 - \frac{1}{2}gt_2^2$

For collision, $x_1 = x_2$ and $y_1 = y_2$

$$\therefore \quad 5\sqrt{3}t_1 = \frac{5\sqrt{3}}{2}t_2 \text{ or } t_2 = 2t_1 \quad \text{...(i)}$$

and $10 - \frac{1}{2}gt_1^2 = 10 + \frac{15}{2}t_2 - \frac{1}{2}gt_2^2$

$$g\left(t_2^2 - t_1^2\right) = 15t_2 \text{ or } 10\left(t_2^2 - t_1^2\right) = 15t_2$$

$$t_2^2 - t_1^2 = \frac{3}{2}t_2 \quad \text{...(ii)}$$

On solving eqns. (i) and (ii), we get

$t_1 = 1$ s and $t_2 = 2$ s

So, time interval between the two fires,

$t_2 - t_1 = 2\text{ s} - 1\text{ s} = 1\text{ s}$

43. (b) : Maximum range,

$$R_{max} = \frac{u^2 \sin 2\theta}{g} = \frac{u^2}{g} = 16\,\text{km}$$

Range is maximum when $\theta = 45°$

Initial momentum at the highest point

$$= mu\cos 45° = \frac{mu}{\sqrt{2}}$$

After explosion, the projectile breaks into two equal masses. As one mass drops vertically downwards, hence its velocity and momentum is zero in the x-direction.

Hence, if v be the velocity of second mass, then according to law of conservation of momentum, $mu\cos 45° = \frac{m}{2}v$

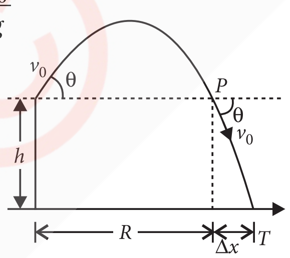
$\therefore \quad v = 2u\cos 45°$

Horizontal distance covered from the time of explosion

$$= v \times \frac{T}{2} = v \times \frac{1}{2} \times \frac{2u\sin 45°}{g}$$

$$= 2u\cos 45° \times \frac{u\sin 45°}{g} = \frac{u^2}{g} \times \sin 90° = \frac{u^2}{g} = 16 \text{ km}$$

44. (a) : Maximum range, $R = \frac{v_0^2}{g}$



Here, $\sin 2\theta = 1$

$\Rightarrow 2\theta = 90°$

$\Rightarrow \theta = 45°$

Consider the motion of a shell after point P.

For horizontal direction,

$$\Delta x = (v_0 \cos\theta)t \quad \Rightarrow \quad t = \frac{\Delta x}{v_0 \cos\theta}$$

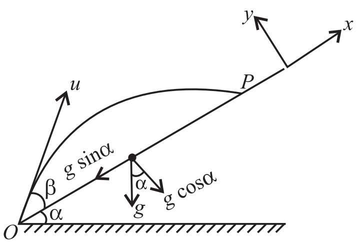
For vertical direction, $h = (v_0 \sin\theta)t + \frac{1}{2}gt^2$

$$= (v_0 \sin\theta)\left(\frac{\Delta x}{v_0\cos\theta}\right) + \frac{1}{2}g\left(\frac{\Delta x}{v_0\cos\theta}\right)^2$$

$$\Rightarrow h = \Delta x\left[\tan\theta + \frac{\Delta x}{2\left(\frac{v_0^2}{g}\right)\cos^2\theta}\right]$$

$$\Rightarrow h = \Delta x\left[\tan 45° + \frac{\Delta x}{2R\cos^2 45°}\right] \Rightarrow h = \Delta x\left[1 + \frac{\Delta x}{R}\right]$$

45. (c) :



Take the x-axis along the incline and y-axis perpendicular to the plane.

$\therefore \quad u_x = u\cos\beta$

$u_y = u\sin\beta$

$a_x = -g\sin\alpha$

$a_y = -g\cos\alpha$

When the particle lands at P its y coordinate becomes zero.

$$\therefore \quad 0 = u_y t + \frac{1}{2}a_y t^2$$

$$0 = u\sin\beta t - \frac{1}{2}g\cos\alpha\, t^2$$

$$\text{or} \quad t = \frac{2u\sin\beta}{g\cos\alpha} \quad \text{...(i)}$$

For motion along inclined plane

$$x = u_x t + \frac{1}{2}a_x t^2 \quad \therefore \quad L = u\cos\beta\, t - \frac{1}{2}g\sin\alpha\, t^2$$

Substituting the value of t from Eq. (i), we get

$$L = u\cos\beta\left(\frac{2u\sin\beta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\beta}{g\cos\alpha}\right)^2$$

$$= \frac{2u^2\sin\beta\cos\beta}{g\cos\alpha} - \frac{2u^2\sin\alpha\sin^2\beta}{g\cos^2\alpha}$$

$$= \frac{2u^2}{g\cos^2\alpha}[\sin\beta\cos\beta\cos\alpha - \sin\alpha\sin^2\beta]$$

$$= \frac{2u^2\sin\beta}{g\cos^2\alpha}[\cos\beta\cos\alpha - \sin\alpha\sin\beta] = \frac{2u^2\sin\beta\cos(\alpha+\beta)}{g\cos^2\alpha}$$

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1. (c) : Among the given quantities potential energy is a scalar quantity whereas all others are vector quantities.

2. (d) : Maximum height reached by the stone is

$$H_{max} = \frac{u^2\sin^2\theta}{2g}$$

Here, $u = 30$ m s^{-1}, $\theta = 45°$, $g = 10$ m s^{-2}

$$\therefore \quad H_{max} = \frac{(30)^2\sin^2 45°}{2\times 10} = \frac{30\times 30\times\left(\frac{1}{\sqrt{2}}\right)^2}{2\times 10} = 22.5 \text{ m}$$

$$= \frac{45}{2} = 22.5 \text{ m}$$

3. (c)

4. (c) : The second-hand completes one rotation in 1 min and the hour-hand completes in 12 h.

$\therefore$ The angular speed of the second-hand is

$$\omega_s = \frac{2\pi \text{ rad}}{1 \text{ min}} = \frac{2\pi \text{ rad}}{60 \text{ s}}$$

and that of the hour-hand is

$$\omega_h = \frac{2\pi \text{ rad}}{12 \text{ h}} = \frac{2\pi \text{ rad}}{12\times60\times60 \text{ s}}$$

Their corresponding ratio is

$$\frac{\omega_s}{\omega_h} = \frac{\dfrac{2\pi \text{ rad}}{60 \text{ s}}}{\dfrac{2\pi \text{ rad}}{12\times60\times60 \text{ s}}} = \frac{12\times60\times60}{60} = \frac{720}{1}$$

5. (d) : Range, $R = \dfrac{u^2 \sin 2\theta}{g}$

Here g is constant and u is same for the projectiles A, B and C. $\therefore R \propto \sin 2\theta$

$\Rightarrow R_A : R_B : R_C = \sin 60° : \sin 90° : \sin 120°$

$$R_A : R_B : R_C = \frac{\sqrt{3}}{2} : 1 : \frac{\sqrt{3}}{2} = \sqrt{3} : 2 : \sqrt{3}$$

Hence, $R_A = R_C < R_B$.

6. (a) : A vector has a maximum value along its own direction only. Hence the component of a vector $\vec{r}$ along x-axis will have a maximum value if $\vec{r}$ is along positive x-axis.

7. (d) : $\vec{A}$ is perpendicular to $\vec{B}$. $\therefore \vec{A}\cdot\vec{B} = 0$

or $(2\hat{i}+3\hat{j}+8\hat{k})\cdot(-4\hat{i}+4\hat{j}+\alpha\hat{k}) = 0$

$-8 + 12 + 8\alpha = 0$ or $\alpha = -1/2$

8. (c)

9. (d) : $y = x\tan\theta - \dfrac{1}{2} g \dfrac{x^2}{u^2\cos^2\theta}$

Comparing with given equation, $\tan\theta = 1 \Rightarrow \theta = 45°$.

$$\frac{1}{2} g \frac{1}{u^2\cos^2\theta} = \frac{2}{5} \Rightarrow u^2\cos^2\theta = \frac{25}{2}$$

$\Rightarrow u = 5 \text{ m s}^{-1}$

10. (c) : Given $\omega_0 = 0$, $\omega = 10$ rad/s, $t = 5$ s

From kinematic equation of rotational motion,

$\omega = \omega_0 + \alpha t$

or $\omega = \alpha t \Rightarrow \alpha = \dfrac{\omega}{t} = \dfrac{10}{5} = 2 \text{ rad s}^{-2}$

Now, using second equation of rotational motion,

Angular displacement, $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

$\therefore \theta = \dfrac{1}{2}\times 2\times(5)^2 = 25$ rad

11. (c) : Given that, maximum range $R = 16$ km, $g = 10$ m/s^2

We know that in projection of a particle for maximum range, $\theta = 45°$

Let the muzzle speed is u.

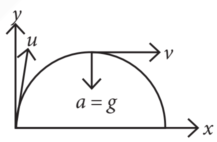
The maximum range is given by

$$R_{max} = \frac{u^2\sin 2\theta}{g} \Rightarrow R = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g}$$

$\therefore 16 \times 1000 \times 10 = u^2$ or $v = 400$ m/s

12. (d) : The path attained by a projectile in absence of air resistance is parabolic in nature.

13. (a) : The acceleration acting on projectile is acceleration due to gravity which acts perpendicularly downwards. At the peak point, the angle between velocity and acceleration is 90°. Thus, angle will be acute for whole second half of the projectile motion but will be minimum and acute only at one point where the projectile just strikes the ground.



14. (c) : $H_1 = \dfrac{u^2\sin^2\theta}{2g}$ ...(i)

$H_2 = \dfrac{u^2\sin^2(90°-\theta)}{2g} = \dfrac{u^2\cos^2\theta}{2g}$ ...(ii)

Divide (i) by (ii), we get $\dfrac{H_1}{H_2} = \dfrac{\tan^2\theta}{1}$

15. (c) : In one complete revolution of uniform circular motion, displacement is zero, therefore average velocity is zero and hence average acceleration is zero. Distance covered will be equal to perimeter thus average speed will be non zero.

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