SAMPLE CHAPTER

2024 EDITION

OBJECTIVE

KARNATAKA CET

Karnataka Common Entrance Test

MCQs Extracted from Each Line of Latest I & II PUC Textbooks

6000+ MCQs

3 MOCK TEST PAPERS AS PER LATEST PATTERN

KEY FEATURES...

- Topicwise 10 Years' Trend Analysis
- Comprehensive Theory with Illustrations, Self Tests and Concept Map
- Three Types of Exercises with Detailed Solutions
  - Topicwise KCET Connect
  - KCET Ready
  - KCET Exam Archive : Previous 10 Years' (2014-2023) Questions

MATHEMATICS

4 Complex Numbers and Quadratic Equations

10 Years' KCET Topicwise Trend at a Glance

NCERT Topic	No. of Questions									
	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
4.2 Complex Numbers	–	–	–	–	–	–	–	–	1	–
4.3 Algebra of Complex Numbers	–	–	1	1	1	–	–	1	–	–
4.4 The Modulus and the Conjugate of a Complex Number	–	1	–	–	–	–	–	–	–	1
4.5 Argand Plane and Polar Representation	–	1	–	–	–	–	–	–	–	–

4.1 Introduction

As we studied that the square root of negative numbers do not exist in real number system, we need to extend the system to a larger system, so that we can find the solution of the equation of the type $x^2 = -1$, which is not possible in the system of real numbers. In fact, the main objective is to solve the equation $ax^2 + bx + c = 0$, where $D = b^2 - 4ac < 0$, which is not possible in the system of real. Euler was the first to introduce the symbol i (read iota) and indicated $\sqrt{-1}$, and defined $i^2 = -1$. Now the equation $x^2 + 1 = 0$ has two solutions $x = \pm i$ and the equation $x^2 + 3 = 0$ also has two solutions $x = \pm i\sqrt{3}$.

4.2 Complex Numbers

Definition

Any number which can be expressed in the form $a + ib$ is known as complex number. We usually denote the complex number by z, *i.e.*, $z = a + ib$ ($a, b \in \mathbf{R}$) where 'a' is called (known as) the real part of z, denoted by $\text{Re}(z)$ and 'b' is called the imaginary part of z, denoted by $\text{Im}(z)$.

Thus if $z = 3 + 7i$, then $\text{Re}(z) = 3$ and $\text{Im}(z) = 7$. Also, if $z = -i$, then we can write $z = 0 + (-1)i$ and so $\text{Re}(z) = 0$ and $\text{Im}(z) = -1$.

The system of number $\{z : z = a + ib;\ a, b \in \mathbf{R}\}$ is called the set of complex numbers and denoted by C.

Purely Real Complex Numbers

A complex number $z = a + ib$ ($a_1, b \in \mathbf{R}$) is called purely real if its imaginary part is zero. *i.e.*, $b = 0$.

For example; $z = 2 + 0i$, $z = 7$ etc.

Purely Imaginary Complex Numbers

A complex number $z = a + ib$ ($a, b, \in \mathbf{R}$) is called purely imaginary if its real part is zero *i.e.*, $a = 0$

For example; $z = 0 + 3i$, $z = 5i$, etc.

Equality of two Complex Numbers

Two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are said to be equal if and only if $a = c$ and $b = d$. However, there is no order relation in the set of complex numbers *i.e.*, expressions of the form $a + ib < c + id$ and $a + ib > c + id$ are meaningless unless, $b = d = 0$.

Info Shots

- If $z = a + ib$, $a, b \in \mathbf{R}$, we also denote $z = a + ib$ by an ordered pair (a, b).
- The complex number $0 = 0 + i0$ is both purely real and purely imaginary.

Illustration : If $3 + yi - 2i = x - i$, then find y.

Soln.: We have, $3 + yi - 2i = x - i \Rightarrow 3 + (y - 2)i = x - i$

Equating the real and the imaginary parts, we get

$$x = 3,\ y - 2 = -1 \Rightarrow y = 1$$

Illustration : Find the real values of x and y for which the following equality hold.

$(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$.

Soln.: The given equality can be rewritten as

$$(x^4 - 3x^2) + i(2x - y) = 4 + i(2y - 5)$$

$\Rightarrow\quad x^4 - 3x^2 = 4,\ 2x - y = 2y - 5$

$\Rightarrow\quad (x^2 - 4)(x^2 + 1) = 0 \Rightarrow x = \pm 2 \qquad (\because x^2 \neq -1)$

$\therefore\quad$ At $x = 2$, $y = 3$ and at $x = -2$, $y = 1/3$

Illustration : If $(x + iy)^3 = u + iv$, then show that $\dfrac{u}{x} + \dfrac{v}{y} = 4(x^2 - y^2)$.

Soln.: We have, $(x + iy)^3 = u + iv$

Consider, $(x + iy)^3 = x^3 + (iy)^3 + 3x^2(iy) + 3x(iy)^2$

$$= x^3 - y^3 i + 3x^2 yi - 3xy^2 = (x^3 - 3xy^2) + i(3x^2 y - y^3)$$

$\Rightarrow\quad (x^3 - 3xy^2) + i(3x^2 y - y^3) = u + iv$

Comparing the real and imaginary parts we get

$$u = x^3 - 3xy^2,\ v = 3x^2 y - y^3$$

$\Rightarrow\quad u = x(x^2 - 3y^2),\ v = y(3x^2 - y^2)$

$\Rightarrow\quad \dfrac{u}{x} = (x^2 - 3y^2),\ \dfrac{v}{y} = 3x^2 - y^2$

$\therefore\quad \dfrac{u}{x} + \dfrac{v}{y} = 4x^2 - 4y^2 = 4(x^2 - y^2)$.

Self Test - 1

1. Find the real and imaginary parts of the complex number $\dfrac{a^2+ib^2}{a^2-ib^2}$.
   (a) $\dfrac{a^4-b^4}{a^4+b^4}, \dfrac{2a^2b^2}{a^4+b^4}$ (b) $\dfrac{a^4+b^4}{a^4-b^4}, \dfrac{2a^2b^2}{a^4+b^4}$
   (c) $\dfrac{a^4-b^4}{a^4+b^4}, \dfrac{2a^2b^2}{a^4-b^4}$ (d) $\dfrac{a^4+b^4}{a^4-b^4}, \dfrac{2a^2b^2}{a^4-b^4}$

2. The real values of x and y for which the following equality holds, are respectively
   $(x^4 + 2xi) - (8x^2 + iy) = (8 - 5i) + (1 + 2iy)$
   (a) $3, \dfrac{11}{3}$ or $-3, -\dfrac{1}{3}$ (b) 1, 3 or –1, 1/3
   (c) 2, 1/3 or –2, 3 (d) 2, 1/3 or –2, –1/3

3. Find the real values of x and y, if
   $(3x - 7) + 2iy = -5y + (5 + x)i$.
   (a) –1, 2 (b) 1, 2
   (c) 1, –2 (d) –1, –2

4. Let $z_1 = (1 + i)y^2 + (6 + i)$ and $z_2 = (2 + i)x$. Find the values of x and y so that $z_1 = z_2$.
   (a) (5, 2) (b) (5, – 2)
   (c) Both (a) and (b) (d) Neither (a) nor (b)

5. If $z_1 = 3 + iy$ and $z_2 = x + 4i$ are equal, then find the sum of x and y.
   (a) 3 (b) 4
   (c) 1 (d) 7

4.3 Algebra of Complex Numbers

Addition of two Complex Numbers : Let $z_1 = x + iy$ and $z_2 = u + iv$ be two complex numbers. Then, the sum $z_1 + z_2$ is defined as $z_1 + z_2 = (x + u) + i(y + v)$.

Difference of two Complex Numbers : Let $z_1 = x + iy$ and $z_2 = u + iv$ be two complex numbers. Then, the difference $z_1 - z_2$ is defined as $z_1 - z_2 = z_1 + (-z_2) = (x - u) + i(y - v)$.

Multiplication of two Complex Numbers : Let $z_1 = x + iy$ and $z_2 = u + iv$ be any two complex numbers. Then the product z_1z_2 is defined as $z_1z_2 = (xu - yv) + i(xv + yu)$

Division of two Complex Numbers : Let $z_1 = x + iy$ and $z_2 = u + iv$ be two complex numbers, where $z_2 \neq 0$, then the quotient $\dfrac{z_1}{z_2}$ is defined by $\dfrac{z_1}{z_2} = z_1 \cdot \left(\dfrac{1}{z_2}\right) = \dfrac{x}{u+iv} + i\dfrac{y}{u+iv}$.

Properties of Addition and Multiplication

- Let z_1, z_2 and z_3 be three complex numbers.

Property	Operation	Description
Closure	Addition	$z_1 + z_2$ is a complex number
	Multiplication	$z_1 \cdot z_2$ is a complex number
Commutative	Addition	$z_1 + z_2 = z_2 + z_1$
	Multiplication	$z_1z_2 = z_2z_1$
Associative	Addition	$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$
	Multiplication	$z_1(z_2z_3) = (z_1z_2)z_3$
Existence of identity	Addition	For every complex number z, the complex number $0 = 0 + i0$ is the identity element for addition *i.e.* $z + 0 = z = 0 + z$
	Multiplication	For every complex number z, the complex number $1 = 1 + i0$ is the identity element for multiplication *i.e.* $z \cdot 1 = z = 1 \cdot z$
Existence of inverse	Addition	For every complex number $z = x + iy$, we have $-x + i(-y)$ (denoted as $-z$) is called additive inverse or negative of z.
	Multiplication	For every non-zero complex number $z = x + iy$ or $x + yi$ ($x \neq 0$, $y \neq 0$), we have the complex number $\dfrac{x}{x^2+y^2} + i\dfrac{-y}{x^2+y^2}$ denoted by $\dfrac{1}{z}$ or z^{-1}, called the multiplicative inverse of z such that $z \cdot \dfrac{1}{z} = 1$ (the multiplicative identity)
Distributive	$z_1(z_2 + z_3) = z_1z_2 + z_1z_3$ $(z_1 + z_2)z_3 = z_1z_3 + z_2z_3$	

Integral Powers of *i*

Positive integral powers of *i* : We have, $i = \sqrt{-1}$

$\therefore \quad i^2 = -1,\ i^3 = i^2 \times i = -i,\ i^4 = (i^2)^2 = (-1)^2 = 1$

- The value of i^n for $n > 4$ is i^r, where r is the remainder when n is divided by 4.

Negative integral powers of *i* : By the law of indices, we have

$$i^{-1} = \frac{1}{i} = \frac{i^3}{i^4} = i^3 = -i,\quad i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$$

$$i^{-3} = \frac{1}{i^3} = \frac{i}{i^4} = i,\quad i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$$

- If $n > 4$, then $i^{-n} = \frac{1}{i^r}$, where r is the remainder when n is divided by 4.

Info Shots: i^0 is defined as 1.

Identities Related to Complex Numbers

For any complex number z_1, z_2, we have

(i) $(z_1 + z_2)^2 = z_1^2 + 2z_1z_2 + z_2^2$

(ii) $(z_1 - z_2)^2 = z_1^2 - 2z_1z_2 + z_2^2$

(iii) $(z_1 + z_2)^3 = z_1^3 + 3z_1^2z_2 + 3z_1z_2^2 + z_2^3$

(iv) $(z_1 - z_2)^3 = z_1^3 - 3z_1^2z_2 + 3z_1z_2^2 - z_2^3$

(v) $z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$

***Illustration* :** Find the sum and product of the complex numbers $-\sqrt{3} + \sqrt{-2}$ and $2\sqrt{3} - i$.

Soln.: Let $z_1 = -\sqrt{3} + \sqrt{-2} = -\sqrt{3} + i\sqrt{2}$

and $z_2 = 2\sqrt{3} - i$

Then $z_1 + z_2 = (-\sqrt{3} + i\sqrt{2}) + (2\sqrt{3} - i)$

$= (-\sqrt{3} + 2\sqrt{3}) + i(\sqrt{2} - 1) = \sqrt{3} + i(\sqrt{2} - 1)$

and $z_1z_2 = (-\sqrt{3} + i\sqrt{2})(2\sqrt{3} - i)$

$= (-6 + \sqrt{2}) + i(\sqrt{3} + 2\sqrt{6})$.

***Illustration* :** Express $i^9 + i^{10} + i^{11} + i^{12}$ in the form of $a + ib$.

Soln.: We have, $i^9 + i^{10} + i^{11} + i^{12} = i^8[i + i^2 + i^3 + i^4]$

$= (i^4)^2\,[i - 1 - i + 1] \qquad (\because i^2 = -1, i^3 = -i, i^4 = 1)$

$= 1(0) = 0 + 0i = a + ib$, where $a = 0$, $b = 0$.

***Illustration* :** If $\left(\frac{1+i}{1-i}\right)^n = 1$, then find the least positive integral value of n.

Soln.: We have, $\left(\frac{1+i}{1-i}\right)^n = 1$

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n = 1$

$\Rightarrow \left(\frac{1-1+2i}{1+1}\right)^n = 1 \Rightarrow \left(\frac{2i}{2}\right)^n = 1 \Rightarrow i^n = 1$

$\Rightarrow$ n is multiple of 4.

$\therefore$ The least positive integral value of n is 4.

***Illustration* :** If $\frac{(a^2+1)^2}{2a-i} = x + iy$, then find the value of $x^2 + y^2$.

Soln.: We have, $x + iy = \frac{(a^2+1)^2}{2a-i}$ ...(i)

Changing i to $-i$, we get

$x - iy = \frac{(a^2+1)^2}{2a+i}$ ...(ii)

Multiplying (i) and (ii), we get

$(x+iy)(x-iy) = \frac{(a^2+1)^2}{2a-i} \times \frac{(a^2+1)^2}{2a+i}$

$\Rightarrow \quad x^2 + y^2 = \frac{(a^2+1)^4}{4a^2+1}$

Self Test - 2

1. Find x and y if $(3x - 2iy)(2 + i)^2 = 10(1 + i)$.

(a) $\frac{-14}{15}, \frac{1}{5}$ (b) $\frac{14}{15}, \frac{-1}{5}$

(c) $\frac{14}{15}, \frac{1}{5}$ (d) $\frac{-14}{15}, \frac{-1}{5}$

2. If $(1 + 2i)(2 + 3i)(3 + 4i) = x + iy$, then $x^2 + y^2 =$

(a) 1600 (b) 1627

(c) 1625 (d) 1725

3. If $z = x - iy$ and $z^{1/3} = p + iq$ then $\left(\frac{x}{p} + \frac{y}{q}\right) \div (p^2 + q^2) =$

(a) 1 (b) -2

(c) 2 (d) -1

4. If $(1 + i)^3/(1 - i)^3 - (1 - i)^3/(1 + i)^3 = x + iy$, then

(a) $x = 0, y = -2$ (b) $x = -2, y = 0$

(c) $x = 1, y = 1$ (d) $x = -1, y = 1$

5. If $\dfrac{(1+i)^2}{2-i} = x + iy$, then $x + y =$

(a) $-2/5$ (b) $6/5$
(c) $2/5$ (d) $-6/5$

6. Let $z = \dfrac{11-3i}{1+i}$. If α is a real number such that $z - i\alpha$ is real, then the value of α is

(a) 4 (b) – 4 (c) 7 (d) –7

7. The imaginary part of $\dfrac{(1+i)^2}{i(2i-1)}$ is

(a) $\dfrac{4}{5}$ (b) 0 (c) $\dfrac{2}{5}$ (d) $-\dfrac{4}{5}$

8. The value of $\dfrac{\sin 60^\circ + i\cos 60^\circ}{\cos 15^\circ - i\sin 15^\circ}$ is

(a) $\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{2}} - i\dfrac{1}{\sqrt{2}}$

(c) $-\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$

9. If $x = \dfrac{-1+i\sqrt{3}}{2}$, then $x^2 + x + 1 =$

(a) 2 (b) $\dfrac{1}{2}$

(c) 0 (d) 1

10. Express $(1 - i) + (-1 - 6i)$ in the standard form $a + ib$.

(a) $2 - 7i$ (b) $-3i$
(c) $0 + (-7)i$ (d) $7 - 2i$

4.4 The Modulus and the Conjugate of a Complex Number

Modulus of a Complex Number

The modulus of a complex number $z = a + ib$ is denoted by $|z|$ and is defined as

$$|z| = \sqrt{a^2 + b^2} = \sqrt{\{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2}$$

Clearly, $|z| \geq 0$ for all $z \in C$.

Conjugate of a Complex Number

Let $z = a + ib$ be a complex number. Then the conjugate of z is denoted by $\bar{z}$ and is equal to $a - ib$.

Thus, $z = a + ib \Rightarrow \bar{z} = a - ib$

It follows from this definition that the conjugate of a complex number is obtained by replacing i by $-i$.

Note :

- A complex number z is purely real if and only if $\bar{z} = z$.
- A complex number z is purely imaginary if and only if $\bar{z} = -z$.

Multiplicative Inverse

Let $z = x + iy$ be a non-zero complex number. Then, z^{-1} is called multiplicative inverse of z and it is given by

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2} = \frac{\bar{z}}{|z|^2} \Rightarrow z^{-1} = \frac{\bar{z}}{|z|^2}.$$

Properties of Modulus

If $z, z_1, z_2 \in C$, then

(i) $|z| = 0 \Leftrightarrow z = 0$, *i.e.*, $\text{Re}(z) = \text{Im}(z) = 0$

(ii) $|z| = |\bar{z}| = |-z|$

(iii) $-|z| < \text{Re}(z) < |z|$; $-|z| < \text{Im}(z) < |z|$

(iv) $z\bar{z} = |z|^2$

(v) $|z_1 z_2| = |z_1||z_2|$ (vi) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}; z_2 \neq 0$

Info Shots

- $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\text{Re}(z_1\bar{z}_2)$
- $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\text{Re}(z_1\bar{z}_2)$
- $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$
- $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)\left(|z_1|^2 + |z_2|^2\right)$, where $a, b \in R$

Properties of Conjugate

If z, z_1, z_2 are complex numbers, then

(i) $\overline{(\bar{z})} = z$ (ii) $z + \bar{z} = 2\,\text{Re}(z)$

(iii) $z - \bar{z} = 2i\,\text{Im}(z)$ (iv) $z = \bar{z} \Leftrightarrow z$ is purely real

(v) $z + \bar{z} = 0 \Rightarrow z$ is purely imaginary

(vi) $z\bar{z} = \{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2$

(vii) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$ (viii) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}, z_2 \neq 0$

(ix) $\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$

(x) $\alpha = f(z) \Rightarrow \bar{\alpha} = f(\bar{z})$, where $\alpha \in C$

Info Shots

Triangle Inequality

For any two complex numbers z_1 and z_2, we have (a) $|z_1 + z_2| < |z_1| + |z_2|$
(b) $|z_1 - z_2| > ||z_1| - |z_2||$

Illustration : Find the multiplicative inverse of $\sqrt{5}+3i$.

Soln.: Let $z=\sqrt{5}+3i$

$\bar{z}=\sqrt{5}-3i$ and $|z|^2=(\sqrt{5})^2+(3)^2=14$

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3}{14}i$$

Illustration : Find the modulus of $\frac{1+i}{1-i}$.

Soln.: $\left|\frac{1+i}{1-i}\right|=\frac{|1+i|}{|1-i|}=\frac{\sqrt{1+1}}{\sqrt{1+1}}=\frac{\sqrt{2}}{\sqrt{2}}=1$

Illustration : If $z=2+\sqrt{3}i$, then find the value of $z\bar{z}$.

Soln.: $z=2+\sqrt{3}i$

$\therefore \quad \bar{z}=2-\sqrt{3}i$

$z\bar{z}=(2+\sqrt{3}i)(2-\sqrt{3}i)=4-3i^2=4+3=7$

Illustration : Find the conjugate and modulus of the complex number $(1-i)^{-2}+(1+i)^{-2}$.

Soln.: We have, $(1-i)^{-2}+(1+i)^{-2}$

$$=\frac{1}{(1-i)^2}+\frac{1}{(1+i)^2}=\frac{(1+i)^2+(1-i)^2}{(1-i)^2(1+i)^2}$$

$$=\frac{0}{(1-i)^2(1+i)^2}=0+0i$$

Hence conjugate is $0-0i$ and modulus is 0.

Self Test - 3

1. On multiplying $3-2i$ by its conjugate, we get
   (a) 10 (b) 11 (c) 12 (d) 13

2. Find the multiplicative inverse of $2-3i$.
   (a) $\frac{2}{13}-\frac{3}{13}i$ (b) $\frac{2}{13}+\frac{3}{13}i$
   (c) $-\frac{2}{13}-\frac{3}{13}i$ (d) $-\frac{2}{13}+\frac{3}{13}i$

3. The conjugate of a complex number is $\frac{1}{i-1}$. Then, the complex number is
   (a) $\frac{1}{i-1}$ (b) $\frac{-1}{i-1}$ (c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$

4. The modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$ is
   (a) $\sqrt{5}$ units (b) $\frac{\sqrt{11}}{5}$ units
   (c) $\frac{\sqrt{5}}{5}$ units (d) $\frac{\sqrt{12}}{5}$ units

5. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is
   (a) $\left(\frac{-8}{25},\frac{31}{25}\right)$ (b) $\left(\frac{-8}{25},\frac{-31}{25}\right)$
   (c) $\left(\frac{8}{25},\frac{-31}{25}\right)$ (d) $\left(\frac{8}{25},\frac{31}{25}\right)$

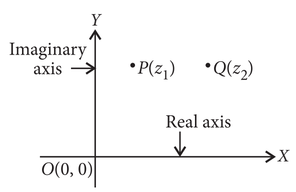
6. Let x_1 and y_1 be real numbers. If z_1 and z_2 are complex numbers such that $|z_1|=|z_2|=4$, then $|x_1z_1-y_1z_2|^2+|y_1z_1+x_1z_2|^2=$
   (a) $32(x_1^2+y_1^2)$ (b) $16(x_1^2+y_1^2)$
   (c) $4(x_1^2+y_1^2)$ (d) 32

7. What should be the positive value of p so that the magnitude of $(3+pi)$, where $i=\sqrt{(-1)}$ is twice that of $\left(\frac{3}{4}\right)+pi$?
   (a) 1 (b) 0 (c) $\frac{3}{2}$ (d) 3

8. If $\frac{z-1}{z+1}$ is purely imaginary, then
   (a) $|z|=\frac{1}{2}$ (b) $|z|=1$
   (c) $|z|=2$ (d) $|z|=3$

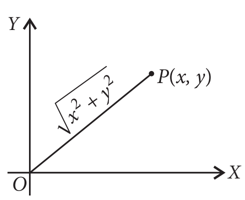
4.5 Argand Plane and Polar Representation

The plane having a complex number assigned to each of its point is called the complex plane or the argand plane.



If two complex numbers z_1 and z_2 represented by the points P and Q in the complex plane, then $|z_1-z_2|=PQ$.

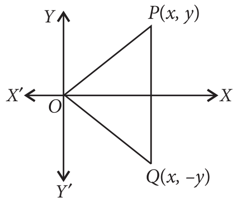
Geometrically, in the argand plane, the modulus of the complex number $zx+iy=\sqrt{x^2+y^2}$ is the distance between the points $P(x, y)$ and the origin $O(0, 0)$.



i.e. $|z|=\sqrt{x^2+y^2}$

A purely real number a, *i.e.* $a + 0i$ is represented by the point $(a, 0)$ on x-axis. Therefore, x-axis is called real axis. A purely imaginary number ib, *i.e.*, $0 + ib$ is represented by the point $(0, b)$ on y-axis. Therefore, y-axis is called imaginary axis.

Geometrically, the mirror image of the complex number $z = x + iy$, represented by the ordered pair (x, y), about the x-axis is called conjugate of z, which is represented by the ordered pair $(x, -y)$.



Info Shots

- **Cube Roots of Unity**

The roots of $z^3 = 1$ are $1, \omega, \omega^2$, where

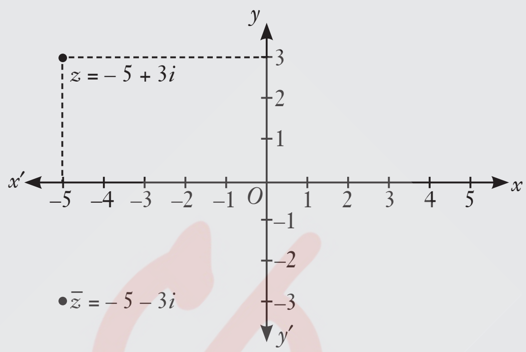
$$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2} \text{ and } \omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

Note : (i) $1 + \omega + \omega^2 = 0$ (ii) $\omega^3 = 1$

Illustration **:** Represent the conjugate of the complex number $z = -5 + 3i$ in the argand plane.

Soln.: We have, $z = -5 + 3i$

Now, $\bar{z} = -\overline{5 + 3i} = -5 - 3i$



Self Test - 4

1. It $z = -1 + i$, then z lies in
   (a) I quadrant (b) II quadrant
   (c) III quadrant (d) IV quadrant

2. If a complex number lies in the III quadrant, then find the quadrant in which its conjugate lies.
   (a) I quadrant (b) II quadrant
   (c) III quadrant (d) IV quadrant

3. If $z = x + iy$ lies in the third quadrant, then $\frac{\bar{z}}{z}$ also lies in the third quadrant, if
   (a) $x > y > 0$ (b) $x < y < 0$
   (c) $y < x < 0$ (d) $y > x > 0$

4. On a argand plane, a cube root of unity is represented by
   (a) a vertex of an equilateral triangle
   (b) a vertex of an isosceles triangle
   (c) a vertex of a right angled triangle
   (d) None of these

5. If $z = -3 - 2i$, then the mirror image of z with respect to x-axis is
   (a) $3 + 2i$
   (b) $3 - 2i$
   (c) $-3 + 2i$
   (d) $-3 - 2i$

CONCEPT MAP

COMPLEX NUMBERS

All complex numbers have the form $a + ib$, where a and b are real numbers. The number a is called the real part and b is called the imaginary part of the complex number.

Complex Numbers

Algebra of Complex Numbers

Let $z_1 = x + iy$ and $z_2 = u + iv$ be two complex numbers. Then

- $z_1 + z_2 = (x + u) + i(y + v)$.
- $z_1 - z_2 = z_1 + (-z_2)$ $= (x - u) + i(y - v)$.
- $z_1 z_2 = (xu - yv) + i(xv + yu)$
- $\dfrac{z_1}{z_2} = z_1 \cdot \left(\dfrac{1}{z_2}\right) = \dfrac{x}{u + iv} + i\dfrac{y}{u + iv}$

Modulus of a Complex Number

The modulus of a complex number $z = a + ib$ is denoted by $|z|$ and is defined as

$$|z| = \sqrt{a^2 + b^2} = \sqrt{\{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2}$$

Conjugate of a Complex Number

Let $z = a + ib$ be a complex number. Then the conjugate of z is denoted by $\bar{z}$ and is equal to $a - ib$.

Properties of Modulus

If $z, z_1, z_2 \in C$, then

(i) $|z| = 0 \Leftrightarrow z = 0$, *i.e.*, $\text{Re}(z) = \text{Im}(z) = 0$

(ii) $|z| = |\bar{z}| = |-z|$

(iii) $-|z| < \text{Re}(z) < |z|$; $-|z| < \text{Im}(z) < |z|$

(iv) $z\bar{z} = |z|^2$

(v) $|z_1 z_2| = |z_1| \cdot |z_2|$

(vi) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}; z_2 \neq 0$

(vii) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$

(viii) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1\bar{z}_2)$

(ix) $|z_1 + z_2|^2 + |z_1 - z_2|^2$ $= 2\left(|z_1|^2 + |z_2|^2\right)$

(x) $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$ $= (a^2 + b^2)\left(|z_1|^2 + |z_2|^2\right)$, where $a, b \in R$.

Properties of Conjugate

If z, z_1, z_2 are complex numbers, then

(i) $\overline{(\bar{z})} = z$

(ii) $z + \bar{z} = 2\text{Re}(z)$

(iii) $z - \bar{z} = 2i\,\text{Im}(z)$

(iv) $z = \bar{z} \Leftrightarrow z$ is purely real

(v) $z + \bar{z} = 0 \Rightarrow z$ is purely imaginary

(vi) $z\bar{z} = \{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2$

(vii) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

(viii) $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\bar{z}_1}{\bar{z}_2}, z_2 \neq 0$

(ix) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

(x) $\alpha = f(z) \Rightarrow \bar{\alpha} = f(\bar{z})$, where $\alpha \in C$

Cube Roots of Unity

The roots of $z^3 = 1$ are $1, \omega, \omega^2$,

where $\omega = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$ and $\omega^2 = -\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}$

Argand Plane

The plane having a complex number assigned to each of its points is called the complex plane or the argand plane.

EXERCISE

KCET Connect

4.2 Complex Numbers

1. Find the values of x and y which satisfy the following equations ($x, y, \in R$).
$(x + 2y) + (2x - 3y)i + 4i = 5$
(a) $x = 1, y = 2$ (b) $x = 2, y = 1$
(c) $x = -1, y = 2$ (d) $x = -2, y = -1$

2. If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then find the values of x and y respectively.
(a) 1/4, 3/4 (b) 3/4, 33/4
(c) 33/4, –3/4 (d) –3/4, –33/4

3. Let $x, y \in R$, then $x + iy$ is a non real complex number if
(a) $x = 0$ (b) $y = 0$
(c) $x \neq 0$ (d) $y \neq 0$

4. Find a and b such that $2a + i4b$ and $2i$ represent the same complex number.
(a) $a = 0, b = \frac{1}{2}$ (b) $a = 1, b = \frac{1}{2}$
(c) $a = \frac{1}{2}, b = 0$ (d) $a = \frac{1}{2}, b = 1$

5. If $z_1 = 2 - iy$ and $z_2 = x + 3i$ are equal, then find x and y.
(a) $x = 2, y = 3$ (b) $x = 2, y = -3$
(c) $x = 3, y = -2$ (d) $x = -3, y = -2$

6. If $3 + yi - 2i = x - i$, then find y.
(a) 3 (b) 0 (c) 2 (d) 1

7. If $(a + b) - i(3a + 2b) = 5 + 2i$, then find $a + b$.
(a) –5 (b) 0 (c) 5 (d) 12

4.3 Algebra of Complex Numbers

8. If $x + iy = \frac{a+i}{a-i}$, then the value of $ay - 1$ is equal to
(a) x (b) y (c) 0 (d) 1

9. If $(x + iy)^{1/3} = 2 + 3i$, then $3x + 2y$ is equal to
(a) – 20 (b) – 60 (c) – 120 (d) 60

10. If the imaginary part of $\frac{2+i}{ai-1}$ is zero, where a is a real number, then the value of a is equal to
(a) $\frac{1}{2}$ (b) 2 (c) $-\frac{1}{2}$ (d) –2

11. The least value of $n \in N$ for which $\left[\frac{1-i}{1+i}\right]^n$ is real, is
(a) 8 (b) 4 (c) 2 (d) 1

12. The value of $(1 + i)^3 + (1 - i)^3 =$
(a) 1 (b) –2 (c) 0 (d) – 4

13. If $z_1 = 2\sqrt{2}(1+i)$ and $z_2 = 1 + i\sqrt{3}$, then $z_1^2 z_2^3$ is equal to
(a) $128i$ (b) $64i$ (c) $-64i$ (d) $-128i$

14. If $z = \frac{2-i}{i}$, then $\text{Re}(z^2) + \text{Im}(z^2)$ is equal to
(a) 1 (b) –1 (c) 2 (d) –2

15. The expression $\frac{(1+i)^n}{(1-i)^{n-2}}$ equals
(a) $-i^{n+1}$ (b) i^{n+1}
(c) $-2i^{n+1}$ (d) 1

16. If $z \neq 0$, Re $z = 0$, then
(a) Re $z^2 = 0$ (b) Im $z^2 = 0$
(c) Re z^2 = Im z^2 (d) none of these

17. Let $x, y \in R$. If $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$, then $x + y =$
(a) 1 (b) 2 (c) –1 (d) –3

18. The value of $x^3 + 7x^2 - x + 16$ when $x = 1 + 2i$, is
(a) $17 - 24i$ (b) $-17 + 24i$
(c) $-17 - 24i$ (d) $17 + 24i$

19. If $z_1 = 2 + 3i$ and $z_2 = 1 + 2i$, then $\frac{z_1}{z_2}$ is equal to
(a) $\frac{8}{5} - \frac{1}{5}i$ (b) $\frac{8}{5} + \frac{1}{5}i$
(c) $-\frac{8}{5} - \frac{1}{5}i$ (d) $-\frac{8}{5} + \frac{1}{5}i$

20. The real part of $\frac{(1+i)^2}{(3-i)}$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $-\frac{1}{3}$ (d) $-\frac{1}{5}$

21. The least positive integer n, for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is positive is
(a) 4 (b) 3 (c) 2 (d) 1

22. If $z_1 = (3 + 2i)$ and $z_2 = (1 - i)$ then $z_1 \cdot z_2$ is
(a) $2 - 2i$ (b) $5 - i$
(c) $5 + i$ (d) $2 - 3i$

23. If $z_1 = 2 - i$, $z_2 = 3 + i$ and $z_3 = \dfrac{1+i}{2-i}$, then $z_1 - z_2 + z_3$ is equal to
(a) $4 + 7i$ (b) $\dfrac{4+7i}{5}$
(c) $\dfrac{-4-7i}{5}$ (d) $\dfrac{7i}{5}$

24. If $z_1 = \dfrac{-3-2i}{2+3i}$ and $z_2 = \dfrac{1+2i}{2+3i}$, then $(z_1 + z_2)\cdot(z_1 - z_2)$ is
(a) $\dfrac{56-136i}{144}$ (b) $\dfrac{56-136i}{169}$
(c) $\dfrac{136-56i}{144}$ (d) $\dfrac{136-56i}{169}$

25. If $z_1 = \dfrac{1}{3+i}$, then the value of $z_1 + \dfrac{1}{z_1}$ is
(a) $\dfrac{3}{5} + \dfrac{2}{5}i$ (b) $\dfrac{3}{5} - \dfrac{i}{5}$
(c) $\dfrac{1}{5} + \dfrac{3i}{5}$ (d) $\dfrac{1}{5} - \dfrac{3i}{5}$

26. If $z_1 = 3x - iy$ and $z_2 = 5x + 3iy$, the value of $\dfrac{z_1}{z_2} = \dfrac{Ax^2 - By^2}{25x^2 + 9y^2} - i\dfrac{Cxy}{25x^2 + 9y^2}$, then find $A - B + C$.
(a) 15 (b) 34 (c) 26 (d) 24

27. If $z = 3 + i$, then find the value of $(z-2)^2 + \dfrac{1}{(z-2)^2}$.
(a) $\dfrac{3}{2}i$ (b) $2i$ (c) $3i$ (d) $\dfrac{5}{2}i$

28. If $z = (1 + i)^2 - (2 + i)$, then find $\dfrac{z+1}{z-1}$.
(a) $\dfrac{i-1}{i+3}$ (b) $\dfrac{i-1}{i-3}$ (c) $\dfrac{i-3}{i-1}$ (d) $\dfrac{i+3}{i-1}$

29. $\left(\dfrac{1+i}{1-i}\right)^n$ is real
(a) for every real number n
(b) for every odd integer n
(c) for every rational n
(d) for every even positive integer n

30. The value of $\left(\dfrac{1+i}{1-i}\right)^{1000} + \left(\dfrac{1-i}{1+i}\right)^{2000}$ is equal to
(a) $1 + i$ (b) 2 (c) $-i$ (d) 1

31. The value of $\left(\dfrac{1+i}{1-i}\right)^{100}$ is equal to
(a) 1 (b) -1 (c) i (d) $-i$

32. The value of $\dfrac{(1+i)^{2016}}{(1-i)^{2014}}$ is
(a) $-2i$ (b) $2i$ (c) 2 (d) -2

33. If $\left(\dfrac{1+i}{1-i}\right)^3 - \left(\dfrac{1-i}{1+i}\right)^3 = x + iy$, then find (x, y).
(a) $(0, -2)$ (b) $(-2, 0)$ (c) $(0, 2)$ (d) $(2, 0)$

34. The smallest positive integer n, for which $(1 + i)^{2n} = (1 - i)^{2n}$ is
(a) 1 (b) 2 (c) 4 (d) 0

35. Find the value of $i^2 - i^4 - i^6 + i^8 - 4i^3$.
(a) $2 - 4i$ (b) $4i$ (c) $-4i$ (d) $1 + 4i$

36. Complex number i^{-99} in the form of $a + ib$ is
(a) $-i$ (b) $1 - i^2$
(c) $0 + 1\cdot i$ (d) $0 + 1\cdot i^4$

37. If $z = \dfrac{i^2 + i^3}{i^4 + i^5}$, then the value of z^2 is
(a) 1 (b) -1 (c) i (d) $-i$

38. Find the value of $\dfrac{i^4 - i^2}{\sqrt{-5}}$.
(a) 1 (b) 0 (c) $\dfrac{2i}{\sqrt{5}}$ (d) $\dfrac{-2i}{\sqrt{5}}$

39. Find the value of $\dfrac{i^{4n+1} - i^{4n-1}}{2}$.
(a) -1 (b) $-i$ (c) 1 (d) i

40. Evaluate: $i^5 + i^6 + i^7 + i^8$
(a) -1 (b) -2 (c) 0 (d) 1

41. If $\dfrac{i^4 + i^9 + i^{16}}{2 - i^8 + i^{10} + i^3} = a + ib$, then (a, b) is
(a) $(1, 2)$ (b) $(-1, 2)$
(c) $(2, 1)$ (d) $(-2, -1)$

42. Let $i^2 = -1$. Then
$$\left(i^{10} - \frac{1}{i^{11}}\right) + \left(i^{11} - \frac{1}{i^{12}}\right) + \left(i^{12} - \frac{1}{i^{13}}\right) + \left(i^{13} - \frac{1}{i^{14}}\right) + \left(i^{14} + \frac{1}{i^{15}}\right) =$$
(a) $-1 + i$ (b) $-1 - i$ (c) $1 + i$ (d) $-i$

43. The value of $\sum_{n=1}^{13} (i^n + i^{n+1})$ where $i = \sqrt{-1}$ equals
(a) 0 (b) i (c) $-i$ (d) $i - 1$

44. If $z = i^9 + i^{19}$, then z is equal to

(a) $0 + 0i$ (b) $1 + 0i$
(c) $0 + i$ (d) $1 + 2i$

45. The value of $(-\sqrt{-1})^{4n+3}$, $n \in N$ is

(a) 1 (b) −1 (c) i (d) $-i$

46. $\sqrt{-3}\sqrt{-6}$ is equal to

(a) $3\sqrt{2}$ (b) $-3\sqrt{2}$
(c) $3\sqrt{2}\,i$ (d) $-3\sqrt{2}\,i$

4.4 The Modulus and the Conjugate of a Complex Number

47. If $\bar{z}_1 = 3 + i$ and $\bar{z}_2 = -5 - i$ then the value of $|z_1 + z_2|$ is

(a) −2 (b) 2 (c) 1 (d) 8

48. Find the multiplicative inverse of $\dfrac{1}{\sqrt{a} - i\sqrt{b}}$.

(a) $\sqrt{b} + i\sqrt{a}$ (b) $\sqrt{a} + \sqrt{b}i$
(c) $\sqrt{a} - i\sqrt{b}$ (d) $\sqrt{b} - i\sqrt{a}$

49. Find the modulus of $\dfrac{\sqrt{a} - \sqrt{b}i}{\sqrt{x} + \sqrt{y}i}$.

(a) $\dfrac{a+b}{x+y}$ (b) $\left(\dfrac{a+b}{x+y}\right)^{1/2}$

(c) $\dfrac{a-b}{x-y}$ (d) $\left(\dfrac{a-b}{x-y}\right)^{1/2}$

50. Find the modulus of $\dfrac{2+i}{4i + (1+i)^2}$.

(a) $\dfrac{5}{6}$ (b) $\dfrac{\sqrt{5}}{6}$ (c) $\dfrac{\sqrt{6}}{5}$ (d) $\dfrac{5}{\sqrt{6}}$

51. Find the complex conjugate of $\dfrac{a - b^2 i}{a^2 + bi}$.

(a) $\dfrac{a^3 - b^3 - i(a^2b^2 + ab)}{a^4 + b^2}$

(b) $\dfrac{a^3 - b^3 + i(a^2b^2 + ab)}{a^4 + b^2}$

(c) $\dfrac{a^3 + b^3 - i(a^2b^2 + ab)}{a^4 + b^2}$

(d) $\dfrac{a^3 + b^3 + i(a^2b^2 + ab)}{a^4 + b^2}$

52. Find the value of $|\sqrt{3} - \sqrt{-5}|$.

(a) $\sqrt{8}$ (b) $\sqrt{5}$ (c) $\sqrt{2}$ (d) $\sqrt{34}$

53. If $2^{101} z = (\sqrt{3} + i)^{104}$, then modulus of z is

(a) 4 (b) 8 (c) 16 (d) 32

54. Reciprocal of $2 + \sqrt{7}i$ is

(a) $\dfrac{-2 + \sqrt{7}i}{\sqrt{11}}$ (b) $\dfrac{-2 - \sqrt{7}i}{11}$

(c) $\dfrac{2 - \sqrt{7}i}{11}$ (d) $\dfrac{\sqrt{7} + \sqrt{2}i}{\sqrt{11}}$

55. If $z_1 = \dfrac{\sqrt{3} + \sqrt{2}i}{\sqrt{2} - \sqrt{3}i}$, $z_2 = \dfrac{\sqrt{5} + \sqrt{7}i}{\sqrt{7} - \sqrt{5}i}$, then find the value of $|\overline{z_1 + z_2}|$.

(a) 1 (b) 2 (c) −2 (d) 5

56. Find the modulus of $\dfrac{x^2 + \sqrt{y}i}{y^2 - \sqrt{x}i}$.

(a) $\sqrt{\dfrac{x^2 + y}{y^2 - x}}$ (b) $\sqrt{\dfrac{x^4 + y}{x + y^4}}$

(c) $\sqrt{\dfrac{x + y^4}{y + x^4}}$ (d) $\sqrt{\dfrac{x^4 + y}{y - x^4}}$

57. Find the multiplicative inverse of $\dfrac{4 + 5i}{3 - 4i}$.

(a) $\dfrac{-8}{25} - \dfrac{31i}{25}$ (b) $\dfrac{-8}{41} - \dfrac{31i}{41}$

(c) $\dfrac{-8}{25} + \dfrac{31i}{25}$ (d) $\dfrac{8}{41} + \dfrac{31i}{41}$

58. If $\dfrac{z - \sqrt{2}}{z + \sqrt{2}}$ is purely imaginary, then

(a) $|z| = 1$ (b) $|z| = \sqrt{2}$

(c) $|z| = 3$ (d) $|z| = \dfrac{1}{\sqrt{2}}$

59. If $z = \dfrac{8 + i}{\sqrt{2} - 1}$, then $z\bar{z}$ is

(a) $\dfrac{60}{3}$ (b) $\dfrac{61}{3}$ (c) $\dfrac{64}{3}$ (d) $\dfrac{65}{3}$

60. If $z + \dfrac{1}{z} = 1 + i$, then find the value of $\left|z^3 + \dfrac{1}{z^3}\right|$.

(a) $\sqrt{20}$ (b) $\sqrt{24}$ (c) $\sqrt{26}$ (d) $\sqrt{22}$

61. If $z = \dfrac{5 - 4i}{3 + 2i}$. If β is a real number such that $|z - i\beta|$ is $3\sqrt{5}$, then the value $|3\beta^2 + 44\beta|$ is

(a) 510 (b) −584 (c) 544 (d) − 544

62. If $z_1 = 1 + i$, $z_2 = 2 - 2i$ and $z_3 = -3 + i$, then the value of $(z_1^3 + z_2^3 + z_3^3)$ is

(a) $-12 + 36i$ (b) $36 + 12i$
(c) $-36 + 12i$ (d) $-36 - 12i$

63. Let z_1 and z_2 be two non-zero complex numbers such that $|z_1| = |z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right| = 6$, then the value of $|z_1 + z_2|$ is
(a) 206 (b) 216 (c) 306 (d) 236

64. What is the value of $\sqrt{i} + 2\sqrt{-i}$?
(a) $\frac{3+i}{\sqrt{2}}$ (b) $\frac{3-i}{\sqrt{2}}$
(c) $\frac{-3+i}{\sqrt{2}}$ (d) $\frac{1+3i}{\sqrt{2}}$

65. What is the value of $(-\sqrt{-1})^{4n+4} + (i^{49} + i^{-257})^{10}$?
(a) 0 (b) 1 (c) −1 (d) 2

66. If $z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$, then what is the value of $z^2 + z\bar{z}$?
(a) 1 (b) 0 (c) 2 (d) 4

67. If $A + iB = = \frac{1+\sqrt{2}i}{1-\sqrt{2}i}$, where, $i = \sqrt{-1}$, then the value of AB is
(a) $\frac{2\sqrt{2}}{9}$ (b) $\frac{-2\sqrt{2}}{9}$ (c) 0 (d) $\frac{2\sqrt{2}}{6}$

68. A and B are two complex numbers, then modulus of the quotient of A and B is
(a) Greater than the quotient of their modulus
(b) Less than the quotient of their modulus
(c) Less than or equal to the quotient of their modulus
(d) Equal to the quotient of their modulus

69. If z_1 and z_2 are complex number with $|z_1| = |z_2|$, then which of the following is correct?
(a) $z_1 = z_2$
(b) Real part of z_1 = Real part of z_2
(c) Imaginary part of z_1 = imaginary pant of z_2
(d) None of these

70. If $3x = 6 + 15i$, then the value of $|x^3 + x^2 - 8x + 179|$ is
(a) 80 (b) 107 (c) 85 (d) 192

71. If $(1+i)z = (1-i)\bar{z}$, then z is equal to
(a) $-i\bar{z}$ (b) $-iz$ (c) $i\bar{z}$ (d) iz

72. If $\frac{z-1}{z+1}$ is a purely imaginary number ($z \neq -1$), then find the value of $|z|$.
(a) 2 (b) 1 (c) 0 (d) 4

73. The value of $(z+3)(\bar{z}+3)$ is equivalent to
(a) $|z+3|^2$ (b) $|z-3|$
(c) z^2+3 (d) None of these

74. A real value of x satisfies the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta \ (\alpha, \beta \in \text{R})$, if $\alpha^2 + \beta^2 =$
(a) 1 (b) −1 (c) 2 (d) −2

75. If $a + ib = c + id$, then
(a) $a^2 + c^2 = 0$ (b) $b^2 + c^2 = 0$
(c) $b^2 + d^2 = 0$ (d) $a^2 + b^2 = c^2 + d^2$

76. If z is a complex number, then
(a) $|z^2| > |z|^2$ (b) $|z^2| = |z|^2$
(c) $|z^2| < |z|^2$ (d) $|z^2| \geq |z|^2$

77. If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then $|f(z)|$ is
(a) $\frac{|z|}{2}$ (b) $|z|$
(c) $2|z|$ (d) None of these

78. Find the multiplicative inverse of $(4 - 3i)$.
(a) $\frac{4}{25} + \frac{3}{25}i$ (b) $\frac{4}{25} + \frac{3}{25}i$
(c) $\frac{3}{25} + \frac{4}{25}i$ (d) $\frac{3}{25} - \frac{4}{25}i$

79. If $\frac{z-\alpha}{z+\alpha}$ $(\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is
(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) 2

80. The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is
(a) $\frac{1}{i-1}$ (b) $\frac{-1}{i-1}$ (c) $\frac{1}{i+1}$ (d) $\frac{-1}{i+1}$

81. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to
(a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{4}$ (c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{3}$

82. The conjugate of the complex number $\frac{(1+i)^2}{1-i}$ is
(a) $1-i$ (b) $1+i$
(c) $-1+i$ (d) $-1-i$

83. If $\frac{5z_2}{11z_1}$ is purely imaginary, then the value of $\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right|$ is
(a) $\frac{37}{33}$ (b) 2 (c) 1 (d) 3

84. If z is a complex number such that $z = -\bar{z}$, then
(a) z is any complex number
(b) Real part of z is same as its imaginary part
(c) z is purely real
(d) z is purely imaginary

85. If $(\sqrt{5} + \sqrt{3}i)^{33} = 2^{49}z$, then modulus of the complex number z is equal to
(a) 1 (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) 4

86. If $\frac{2z_1}{3z_2}$ is a purely imaginary number, then $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$ is equal to
(a) 3/2 (b) 1 (c) 2/3 (d) 4/9

87. If $(3+i)(z+\bar{z})-(2+i)(z-\bar{z})+14i=0$, then $z\bar{z}=$
(a) 5 (b) 8 (c) 10 (d) 40

88. If $x + iy = \sqrt{\left[\frac{(a+ib)}{(c+id)}\right]}$, then $x^2 + y^2 =$
(a) $(a^2 + b^2)/(c^2 + d^2)$
(b) $\sqrt{(a^2 - b^2)/(c^2 - d^2)}$
(c) $(a^2 - b^2)(c^2 - d^2)$
(d) $\sqrt{(a^2 + b^2)/(c^2 + d^2)}$

89. If Z is a complex number with $|Z| = 1$ and $Z + \frac{1}{Z} = x + iy$, then $xy =$
(a) 0 (b) 1
(c) 2 (d) None of these

90. If $iz^3 + z^2 - z + i = 0$, then $|z|$ is equal to
(a) 0 (b) 1
(c) 2 (d) None of these

91. If $z = \frac{4}{1-i}$, then $\bar{z}$ is (where $\bar{z}$ is complex conjugate of z)
(a) $2(1 + i)$ (b) $(1 + i)$
(c) $\frac{2}{1-i}$ (d) $\frac{4}{1+i}$

92. If the conjugate of $(x + iy)(1 - 2i)$ is $1 + i$, then
(a) $x = -\frac{1}{5}$ (b) $x - iy = \frac{1+i}{1-2i}$
(c) $x + iy = \frac{1-i}{1-2i}$ (d) $x = \frac{1}{5}$

93. If $\text{Re}(1 + iy)^3 = -26$, where y is a real number, then the value of $|y|$ is
(a) 2 (b) 3 (c) 4 (d) 6

94. If $z = x + iy$ is a complex number such that $|z| = \text{Re}(iz) + 1$, then the locus of z is
(a) $x^2 + y^2 = 1$ (b) $x^2 = 1 - 2y$
(c) $y^2 = 2x - 1$ (d) $y^2 = 1 - 2x$

95. If $f(z) = \frac{1 - z^3}{1 - z}$, where $z = x + iy$ with $z \neq 1$, then $\text{Re}\{f(z)\} = 0$ reduces to
(a) $x^2 + y^2 + x + 1 = 0$ (b) $x^2 - y^2 + x - 1 = 0$
(c) $x^2 - y^2 - x + 1 = 0$ (d) $x^2 - y^2 + x + 1 = 0$

96. If $|z| = 5$ and $w = \frac{z-5}{z+5}$, then $\text{Re}(w)$ is equal to
(a) 0 (b) $\frac{1}{25}$ (c) 25 (d) 1

97. If z_1, z_2, z_3 are any three complex numbers, then $z_1\ (\text{Im}(\bar{z}_2 z_3)) + z_2(\text{Im}(\bar{z}_3 z_1)) + z_3(\text{Im}(\bar{z}_1 z_2))$ is equal to
(a) 0 (b) $z_1 + z_2 + z_3$
(c) $z_1 z_2 z_3$ (d) $\left(\frac{z_1 + z_2 + z_3}{z_1 z_2 z_3}\right)$

98. The modulus of $\frac{1+2i}{1-(1-i)^2}$ is
(a) $\sqrt{2}$ (b) 2 (c) 1 (d) $\sqrt{3}$

4.5 Argand Plane and Polar Representation

99. In Argand's plane, the point corresponding to $\frac{(1 - i\sqrt{3})(1 + i)}{(\sqrt{3} + i)}$ lies in quadrant
(a) I (b) II (c) III (d) IV

100. The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by
(a) $1 + i$ (b) $2 + 2i$
(c) $-2 - 2i$ (d) $-1 - i$

KCET Ready

1. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2$ is
(a) 18 (b) 54 (c) 6 (d) 12

2. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$, are
(a) $-1, -1, -1$ (b) $-1, -1 + 2\omega, -1 - 2\omega^2$
(c) $-1, 1 + 2\omega, 1 + 2\omega^2$ (d) $-1, 1 - 2\omega, 1 - 2\omega^2$

3. The value of $\left(i^{18} + \left(\frac{1}{i}\right)^{25}\right)^3$ is equal to
(a) $\frac{1+i}{2}$ (b) $2 + 2i$
(c) $2 - 2i$ (d) $\sqrt{2} - \sqrt{2}i$

4. If $Z_1 = a + ib$ and $Z_2 = c + id$ are two complex numbers where a, b, c, d are all real numbers, then
(a) $Z_1 + Z_2$ is always a complex number
(b) $Z_1 Z_2$ is always an imaginary number
(c) $Z_1 - Z_2$ is always an imaginary number
(d) $Z_1 Z_2$ is always a real number

5. Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}(i=\sqrt{-1})$, where x and y are real numbers, then $y - x$ equals
(a) –91 (b) –85 (c) 85 (d) 91

6. Let $z \in C$ with Im(z) = 10 and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n. Then
(a) $n = 20$ and Re(z) = 10
(b) $n = 20$ and Re(z) = –10
(c) $n = 40$ and Re(z) = –10
(d) $n = 40$ and Re(z) = 10

7. Let $z_1 = \frac{2\sqrt{3}+i6\sqrt{7}}{6\sqrt{7}+i2\sqrt{3}}$ and $z_2 = \frac{\sqrt{11}+i3\sqrt{13}}{3\sqrt{13}-i\sqrt{11}}$.
Then $\left|\frac{1}{z_1}+\frac{1}{z_2}\right|$ is equal to
(a) 47 (b) 264
(c) $|z_1 - z_2|$ (d) $|z_1 z_2|$

8. If $z_1, z_2, ..., z_n$ are complex numbers such that $|z_1| = |z_2| = ... = |z_n| = 1$, then $|z_1 + z_2 + ... + z_n|$ is equal to
(a) $|z_1 z_2 z_3 ... z_n|$ (b) $|z_1| + |z_2| + ... + |z_n|$
(c) $\left|\frac{1}{z_1}+\frac{1}{z_2}+...+\frac{1}{z_n}\right|$ (d) n

9. Let $z \neq 1$ be a complex number and let $\omega = x + iy$, $y \neq 0$. If $\frac{\omega - \bar{\omega}z}{1-z}$ is purely real, then $|z|$ is equal to
(a) $|\omega|$ (b) $|\omega|^2$ (c) $\frac{1}{|\omega|^2}$ (d) 1

10. If $\frac{(1+i)(2+3i)(3-4i)}{(2-3i)(1-i)(3+4i)} = a + ib$, then $a^2 + b^2 =$
(a) 132 (b) 25 (c) 144 (d) 1

11. If $\left|z - \frac{3}{z}\right| = 2$, then the greatest value of $|z|$ is
(a) 1 (b) 2 (c) 3 (d) 4

12. If $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ and z_1, z_2, z_3 are imaginary numbers, then $|z_1 + z_2 + z_3|$ is
(a) equal to 1 (b) less than 1
(c) greater than 1 (d) equal to 3

13. If a, b, c are real numbers and z is a complex number such that $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a)z$, then the value of $\frac{1+iz}{1-iz}$ is
(a) $\frac{a-ib}{1-c}$ (b) $\frac{a+ib}{1+c}$
(c) $\frac{a-ib}{1+c}$ (d) $\frac{-a+ib}{1+c}$

14. The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} + 2$ is
(a) 1 (b) –1 (c) 2 (d) 0

15. If $z = (3\sqrt{7}+4i)^2(3\sqrt{7}-4i)^3$, then Re($z$) =
(a) $79 \times 3\sqrt{7}$ (b) $(79)^2(3\sqrt{7})$
(c) $-4(79)^2$ (d) None of these

16. If $\left|z - \frac{8}{z}\right| = 2$, then the greatest value of $|z|$ is
(a) 3 (b) 2 (c) 10 (d) 4

17. If ω is a complex cube root of unity, then $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)(1-\omega^8+\omega^{16})$ is equal to
(a) 12 (b) 14
(c) 16 (d) none of these

18. If ω is an imaginary cube root of unity then $(1+\omega-\omega^2)^7$ equals
(a) 128ω (b) -128ω
(c) $128\omega^2$ (d) $-128\omega^2$

19. If z_1 and z_2 are $1 - i$ and $-2 + 4i$ respectively, then $\text{Im}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ is equal to
(a) 1 (b) –1 (c) 2 (d) –2

20. ω is an imaginary cube root of unity. If $(1+\omega^2)^m = (1+\omega^4)^m$, then least positive integral value of m is
(a) 6 (b) 5 (c) 4 (d) 3

21. The maximum value of $|z|$ when z satisfies the condition $\left|z + \frac{2}{z}\right| = 2$ is
(a) $\sqrt{3}-1$ (b) $\sqrt{3}+1$ (c) $\sqrt{3}$ (d) $\sqrt{2}+\sqrt{3}$

22. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is
(a) 0 (b) 1 (c) 2 (d) – 2

23. The value of $\left(\frac{-1+\sqrt{-3}}{2}\right)^{100} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{100}$ is
(a) 2 (b) 0 (c) – 1 (d) 5

24. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is
(a) -2^{15} (b) $2^{15}i$ (c) $-2^{15}i$ (d) 6^5

25. The true statement is
(a) $1 - i < 1 + i$ (b) $2i + 1 > -2i + 1$
(c) $2i > 1$ (d) None of these

26. The value of $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+i\sqrt{2})-(\sqrt{3}-i\sqrt{2})} =$
(a) $-\frac{\sqrt{7}}{2}i$ (b) $2\sqrt{7}i$ (c) $\frac{7}{2}i$ (d) $\frac{-7\sqrt{2}i}{2}$

27. The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer is
(a) 2 (b) 4 (c) 8 (d) 16

28. Given $z = \frac{q+ir}{1+p}$, then $\frac{p+iq}{1+r} = \frac{1+iz}{1-iz}$, if
(a) $p^2 + q^2 + r^2 = 1$ (b) $p^2 + q^2 + r^2 = 2$
(c) $p^2 + q^2 - r^2 = 1$ (d) None of these

29. If $f(x) = x^4 - 4x^3 + 4x^2 + 8x + 44$, then $f(3 + 2i)$ is
(a) 0 (b) –1 (c) 5 (d) 4

30. The value of $\frac{i^{148}+i^{146}+i^{144}+i^{142}+i^{140}}{i^{138}+i^{136}+i^{134}+i^{132}+i^{130}} - 1 =$
(a) –1 (b) –2 (c) –3 (d) –4

KCET Exam Archive 10 Years' PYQs (2014-2023)

1. If 1, ω, ω^2 are three cube roots of unity, then $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$ is ______
(a) 2 (b) 4 (c) 1 (d) 3 *(2015)*

2. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to
(a) 1 (b) 3 (c) 0 (d) 2 *(2015)*

3. The simplified form of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
(a) 0 (b) 1 (c) –1 (d) i *(2016)*

4. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then the least positive integral value of m is
(a) 4 (b) 1 (c) 2 (d) 3 *(2017)*

5. If $\left(\frac{1-i}{1+i}\right)^{96} = a + ib$, then (a, b) is
(a) (1, 1) (b) (1, 0)
(c) (0, 1) (d) (0, –1) *(2018)*

6. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then
(a) $x = 4n + 1 ; n \in N$ (b) $x = 2n + 1 ; n \in N$
(c) $x = 2n ; n \in N$ (d) $x = 4n ; n \in N$ *(2021)*

7. If $3x + i(4x - y) = 6 - i$ where x and y are real numbers, then the values of x and y are respectively,
(a) 3, 9 (b) 2, 9
(c) 2, 4 (d) 3, 4 *(2022)*

8. The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{4}{\sqrt{2}}$ (c) $\frac{\sqrt{2}}{4}$ (d) $\frac{2}{\sqrt{2}}$ *(2023)*

Hints & Explanations

Self Test - 1

1. **(a) :** $\frac{a^2+ib^2}{a^2-ib^2} = \frac{(a^2+ib^2)(a^2+ib^2)}{(a^2-ib^2)(a^2+ib^2)}$
$= \frac{a^4+i^2b^4+2ia^2b^2}{a^4-i^2b^4} = \frac{a^4-b^4+i2a^2b^2}{a^4+b^4}$
$= \frac{a^4-b^4}{a^4+b^4} + i\frac{2a^2b^2}{a^4+b^4}$

Hence, real part is $\frac{a^4-b^4}{a^4+b^4}$ and imaginary part is $\frac{2a^2b^2}{a^4+b^4}$.

2. **(a) :** The given equation can be rewritten as
$(x^4 - 8x^2) + i(2x - y) = 9 + i(2y - 5)$
$\Rightarrow x^4 - 8x^2 = 9, 2x - y = 2y - 5$
$\Rightarrow (x^2 - 9)(x^2 + 1) = 0 \Rightarrow x = \pm 3$ $(\because x^2 \neq -1)$
$\therefore$ At $x = 3, y = 11/3$ and at $x = -3, y = -1/3$

3. **(a) :** We have, $(3x - 7) + 2iy = -5y + (5 + x)i$
Equating real and imaginary parts, we get
$3x - 7 = -5y$ and $2y = 5 + x$
$\Rightarrow 3x + 5y = 7$ and $x - 2y = -5$
Solving these two equations, we get
$x = -1, y = 2$

4. **(c) :** We have $z_1 = z_2$
$\Rightarrow (1 + i)y^2 + (6 + i) = (2 + i)x$
$\Rightarrow (y^2 + 6) + i(y^2 + 1) = 2x + ix$

Comparing the real and imaginary parts on both sides, we get

$y^2 + 6 = 2x$ and $y^2 + 1 = x$

$\Rightarrow (y^2 + 1) + 5 = 2x$

$\Rightarrow x + 5 = 2x \quad (\because y^2 + 1 = x)$

$\Rightarrow x = 5$ and $y^2 + 1 = 5 \Rightarrow y = \pm 2$

$\therefore$ Set of values of (x, y) are $(5, 2), (5 - 2)$

5. **(d) :** We have, $z_1 = z_2$

$3 + iy = x + 4i$

Comparing real and imaginary parts on both sides, we get

$x = 3$ and $y = 4$

$\therefore x + y = 3 + 4 = 7$

Self Test - 2

1. **(c) :** Given $(3x - 2iy)(2 + i)^2 = 10(1 + i)$

$\Rightarrow (3x - 2iy)(4 + 4i + i^2) = 10 + 10i$

$\Rightarrow (3x - 2iy)(3 + 4i) = 10 + 10i$

$\Rightarrow (9x - 6yi + 12xi - 8i^2y) = 10 + 10i$

$\Rightarrow 9x + 8y + i(12x - 6y) = 10 + i \,.\, 10$

Equating real and imaginary parts, we get

$9x + 8y = 10 \quad$ ...(i)

and $12x - 6y = 10 \quad$ ...(ii)

Solving (i) and (ii), we get $x = \frac{14}{15}, y = \frac{1}{5}$

2. **(c) :** We have, $x + iy = (1 + 2i)(2 + 3i)(3 + 4i)$

$= (2 + 3i + 4i + 6i^2)(3 + 4i) = (-4 + 7i)(3 + 4i)$

$= -12 - 16i + 21i + 28i^2 = -40 + 5i$

Equating the real and the imaginary parts, we get

$x = -40, y = 5$

Now, $x^2 + y^2 = (-40)^2 + (5)^2 = 1600 + 25 = 1625$

3. **(b) :** $x - iy = z = (p + iq)^3 = p^3 - 3pq^2 + i(3p^2q - q^3)$

$\therefore x = p^3 - 3pq^2, y = -3p^2q + q^3$

$\frac{x}{p} + \frac{y}{q} = p^2 - 3q^2 - 3p^2 + q^2 = -2(p^2 + q^2)$

$\therefore \left(\frac{x}{p} + \frac{y}{q}\right) \div (p^2 + q^2) = -2$

4. **(a) :** $\frac{(1+i)^3}{(1-i)^3} - \frac{(1-i)^3}{(1+i)^3} = x + iy$

$\Rightarrow \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^3 - \left[\frac{(1-i)(1-i)}{(1+i)(1-i)}\right]^3 = x + iy$

$\Rightarrow \left(\frac{1+2i+i^2}{2}\right)^3 - \left(\frac{1+i^2-2i}{2}\right)^3 = x + iy$

$\Rightarrow x = 0, y = -2$

5. **(c) :** $\frac{(1+i)^2}{2-i} = x + iy$

or $x + iy = \frac{1-1+2i}{2-i} = \frac{2i}{2-i} \times \frac{2+i}{2+i} = \frac{-2}{5} + \frac{4}{5}i$

Comparing Real and Imaginary parts, we get

$x = \frac{-2}{5}, y = \frac{4}{5} \quad \therefore x + y = \frac{-2}{5} + \frac{4}{5} = \frac{2}{5}.$

6. **(d) :** $\frac{11-3i}{1+i} - i\alpha = \frac{11-3i-i\alpha+\alpha}{1+i} \times \frac{1-i}{1-i}$

$= \frac{11-3i-i\alpha+\alpha-11i-3-\alpha-i\alpha}{2}$

$= \frac{11-3-14i-2i\alpha}{2}$

Since, Im $(z - i\alpha) = 0$

$\therefore -14 - 2\alpha = 0, \alpha = -7$

7. **(d) :** $\frac{(1+i)^2}{i(2i-1)} = \frac{1+i^2+2i}{i(2i-1)} = \frac{2i}{i(2i-1)}$

$= \frac{2(2i+1)}{4i^2-1} = \frac{4i+2}{-4-1} = -\frac{4}{5}i - \frac{2}{5}$

$\therefore$ Imaginary part of $\frac{(1+i)^2}{i(2i-1)} = -\frac{4}{5}.$

8. **(a) :** We have,

$\frac{\sin 60° + i\cos 60°}{\cos 15° - i\sin 15°} \times \frac{\cos 15° + i\sin 15°}{\cos 15° + i\sin 15°}$

$= \frac{(\sin 60° \cos 15° - \cos 60° \sin 15°) + i(\cos 60° \cos 15° + \sin 60° \sin 15°)}{\cos^2 15° + \sin^2 15°}$

$= \sin(60° - 15°) + i\cos(60° - 15°)$

$= \sin 45° + i\cos 45° = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

9. **(c) :** We have, $x = \frac{-1+i\sqrt{3}}{2}$

$\therefore x^2 + x + 1 = \left(\frac{-1+i\sqrt{3}}{2}\right)^2 + \left(\frac{-1+i\sqrt{3}}{2}\right) + 1$

$= \frac{1-3-i2\sqrt{3}}{4} + \left(\frac{-1+i\sqrt{3}}{2}\right) + 1 = 0$

10. **(c) :** $(1 - i) + (-1 - 6i) = 1 - i - 1 - 6i$

$= (1 - 1) - i(1 + 6) = -7i = 0 + (-7i)$

Self Test - 3

1. **(d) :** Let $z = 3 - 2i$

$\bar{z} = 3 + 2i$

$z\bar{z} = (3 - 2i)(3 + 2i) = 9 - 4i^2 = 9 + 4 = 13$

2. **(b) :** Let $z = 2 - 3i$

$\bar{z} = 2 + 3i, |z|^2 = (2)^2 + (-3)^2 = 4 + 9 = 13$

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{2+3i}{13} = \frac{2}{13} + \frac{3i}{13}$

3. **(d) :** $\bar{z} = \frac{1}{i-1}$

We have $z = \overline{(\bar{z})}$ giving $z = \frac{1}{\bar{i}-1} = \frac{1}{-i-1} = \frac{-1}{i+1}$

4. **(c) :** $z = \frac{1-i}{3+i} + \frac{4i}{5} = \frac{1}{5} + \frac{2}{5}i$

$|z| = \sqrt{\frac{1}{25} + \frac{4}{25}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ unit.

5. **(b) :** Let $z = \frac{3+4i}{4-5i}$

We have to calculate z^{-1} *i.e.*, $\frac{1}{z}$

$\therefore \quad z^{-1} = \frac{1}{z} = \frac{4-5i}{3+4i} \times \frac{3-4i}{3-4i} = \frac{12-15i-16i+20i^2}{9-16i^2}$

$= \frac{-8}{25} - \frac{31}{25}i$

$\therefore \quad z^{-1} = \left(-\frac{8}{25}, -\frac{31}{25}\right)$

6. **(a) :** $|x_1 z_1 - y_1 z_2|^2 + |y_1 z_1 + x_1 z_2|^2$

$= |x_1 z_1|^2 + |y_1 z_2|^2 - 2\text{Re}(x_1 y_1 z_1 \bar{z}_2)$
$\quad + |y_1 z_1|^2 + |x_1 z_2|^2 + 2\text{Re}(x_1 y_1 z_1 \bar{z}_2)$

$= x_1^2 |z_1|^2 + y_1^2 |z_2|^2 + y_1^2 |z_1|^2 + x_1^2 |z_2|^2$

$= 2(x_1^2 + y_1^2)(4^2) = 32(x_1^2 + y_1^2)$

7. **(c) :** Let $z_1 = 3 + pi$ and $z_2 = \frac{3}{4} + pi$

Given that $|z_1| = 2|z_2|$

$\Rightarrow \sqrt{9+p^2} = 2\sqrt{\frac{9}{16} + p^2}$

On squaring both sides, we get

$9 + p^2 = 4\left(\frac{9}{16} + p^2\right) \Rightarrow 4p^2 - p^2 = 9 - \frac{9}{4}$

$\Rightarrow 3p^2 = \frac{27}{4} \Rightarrow p^2 = \frac{9}{4} \Rightarrow p = \pm\frac{3}{2}$

8. **(b) :** If $\frac{z-1}{z+1}$ be purely imaginary then let

$\frac{z-1}{z+1} = \frac{i}{k}$ (k is any non zero real number)

$\therefore \frac{z+1}{z-1} = \frac{k}{i} \Rightarrow \frac{2z}{2} = \frac{k+i}{k-i} \Rightarrow |z| = \left|\frac{k+i}{k-i}\right| = 1$

Self Test - 4

1. **(b) :** $z = -1 + i$, z lies in II quadrant.

2. **(b) :** Let $z = a + ib$
then in III quadrant $a < 0, b < 0$

Its conjugate $\bar{z} = \overline{a+ib} = a - ib = a + (-ib)$
$= a + ik$ where $k = -b$

$\Rightarrow \quad a < 0, k > 0$

$\Rightarrow \quad \bar{z}$ lies in II quadrant.

3. **(b) :** Since $z = x + iy$ lies in the third quadrant, so $x < 0$ and $y < 0$.

Taking, $\frac{\bar{z}}{z} = \frac{x-iy}{x+iy} = \frac{(x-iy)(x-iy)}{(x+iy)(x-iy)}$

$= \frac{x^2 - y^2 - 2ixy}{x^2 + y^2}$

or $\quad \frac{\bar{z}}{z} = \frac{x^2-y^2}{x^2+y^2} - \frac{2ixy}{x^2+y^2}$

Since $\frac{\bar{z}}{z}$ lies in third quadrant,

So, its real and imaginary parts are less than 0.

$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} < 0$ and $\frac{-2xy}{x^2+y^2} < 0$

$\Rightarrow \quad x^2 - y^2 < 0$ and $-2xy < 0$ $\qquad (\because x^2 + y^2 > 0)$

$\Rightarrow \quad x^2 < y^2$ and $xy > 0$

So, $\quad x < y < 0$.

4. **(a) :** A, B, C are $1, \omega, \omega^2$

or $A = (1, 0)$, $B = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $C = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Clearly $AB = BC = CA = \sqrt{3}$

$\therefore \quad$ Triangle is equilateral.

5. **(c) :** $z = -3 - 2i$

Mirror image of $-3 - 2i$ along x-axis is $-3 + 2i$

KCET Connect

1. **(a) :** $(x + 2y) + (2x - 3y)i + 4i = 5$

$\therefore \quad (x + 2y) + (2x - 3y)i = 5 - 4i$

Equating real and imaginary parts, we get

$x + 2y = 5 \qquad$ ...(i)

and $2x - 3y = -4 \qquad$ ...(ii)

Equation (i) × 2 – equation (ii), gives

$7y = 14 \quad \therefore \quad y = 2$

Substituting $y = 2$ in (i), we get

$x + 2(2) = 5 \Rightarrow x + 4 = 5 \Rightarrow x = 1$

$\therefore \quad x = 1$ and $y = 2$

2. **(b) :** We have, $4x + i(3x - y) = 3 + i(-6)$

Equating the real and imaginary parts, we get

$4x = 3$, $3x - y = -6$

On solving simultaneously, gives

$x = \frac{3}{4}$ and $y = \frac{33}{4}$.

3. **(d)**

4. **(a) :** Consider, $2a + i4b = 0 + 2i$

Equating real and imaginary parts, we get

$2a = 0$ and $4b = 2$

$\Rightarrow \quad a = 0$ and $b = \frac{1}{2}$

5. **(b) :** We have, $z_1 = z_2$

$\Rightarrow \quad 2 - iy = x + 3i$

Equating real and imaginary parts, we get

$2 = x$ and $-y = 3$

$\Rightarrow \quad x = 2$ and $y = -3$

6. (d) : We have, $3 + yi - 2i = x - i$

$\Rightarrow \quad 3 + (y - 2)i = x - i$

Equating real and imaginary parts, we get

$x = 3$ and $y - 2 = -1 \Rightarrow y = 1$

7. (c) : We have, $(a + b) - i(3a + 2b) = 5 + 2i$

Equating real and imaginary parts, we get $a + b = 5$ and $-(3a + 2b) = 2$

Solving above two equations, we get $a = -12$, $b = 17$

$\therefore \quad a + b = -12 + 17 = 5$

8. (a) : We have,

$$x + iy = \frac{a+i}{a-i} = \frac{a+i}{a-i} \times \frac{a+i}{a+i} = \frac{(a+i)^2}{a^2 - i^2} = \frac{a^2 - 1 + 2ai}{a^2+1}$$

$$= \frac{a^2-1}{a^2+1} + \frac{2a}{a^2+1} i$$

$$\Rightarrow \quad x = \frac{a^2-1}{a^2+1} \text{ and } y = \frac{2a}{a^2+1}$$

$$\therefore \quad ay - 1 = \frac{2a^2}{a^2+1} - 1 = \frac{a^2-1}{a^2+1} = x$$

9. (c) : $(x + iy)^{1/3} = 2 + 3i \Rightarrow (x + iy) = (2 + 3i)^3$

$= (4 - 9 + 12i)(2 + 3i) = (-5 + 12i)(2 + 3i)$

$= -10 + 24i - 15i - 36 = -46 + 9i$

$\therefore \quad x = -46, y = 9$

$\therefore \quad 3x + 2y = 3 \times (-46) + 2 \times 9 = -138 + 18 = -120$

10. (c) : Now, $z = \dfrac{2+i}{ai-1} = \dfrac{(2+i)(ai+1)}{(ai)^2 - 1}$

$$= \frac{-1}{a^2+1}\{(2+i)\cdot(ai+1)\}$$

$$\text{Im}(z) = \frac{-1}{1+a^2}(1 + 2a)$$

But, imaginary part is zero.

So, $1 + 2a = 0 \Rightarrow a = -1/2$.

11. (c) : $\left[\dfrac{1-i}{1+i}\right]^n$

$$= \frac{(1-i)^n}{(1+i)^n} \times \frac{(1-i)^n}{(1-i)^n} = \frac{(1-i)^{2n}}{(1-i^2)^n} = \frac{[(1-i)^2]^n}{2^n}$$

$$= \frac{(1 + i^2 - 2i)^n}{2^n} = \frac{(-2i)^n}{2^n} = (-i)^n = (-1)^{n/2}$$

12. (d) : $(1 + i)^3 = 1 + i^3 + 3i(1 + i) = -2 + 2i$ ...(i)

$(1 - i)^3 = 1 - i^3 - 3i(1 - i) = -2 - 2i$ ...(ii)

So, $(1 + i)^3 + (1 - i)^3 = -4$ [From (i) & (ii)]

13. (d) : We have $z_1 = 2\sqrt{2}(1 + i)$

$\Rightarrow \quad z_1^2 = 8(1 + i)^2 = 8(1 + i^2 + 2i) = 16i$

Also, $z_2 = 1 + i\sqrt{3}$

$\Rightarrow \quad z_2^3 = (1 + i\sqrt{3})^3 = 1 + 3\sqrt{3}i^3 + 3\sqrt{3}i(1 + i\sqrt{3})$

So, $z_1^2 z_2^3 = 16 \times (-8)i = -128i$

14. (a) : We have, $z = \dfrac{2-i}{i}$

$$\Rightarrow \quad z^2 = \frac{(2-i)^2}{i^2} = -3 + 4i$$

$\therefore \quad \text{Re}(z^2) + \text{Im}(z^2) = -3 + 4 = 1$

15. (c) : $\left(\dfrac{1+i}{1-i}\right)^n (1-i)^2 = \left\{\dfrac{(1+i)^2}{1-i^2}\right\}^n (1 + i^2 - 2i)$

$$= \left\{\frac{1 - 1 + 2i}{2}\right\}^n (-2i) = -2i^{n+1}$$

16. (b) : Re $z = 0 \Rightarrow z = iy$, y is real and $z^2 = -y^2$ real.

$\therefore \quad \text{Im } z^2 = 0$

17. (b) : $\dfrac{x + (x-2)i}{3+i} + \dfrac{2y + i(1-3y)}{3-i} = i$

Multiplying by $(3 + i)(3 - i)$, we get

$(3 - i)(x + (x - 2)i) + (3 + i)(2y + i(1 - 3y)) = 10i$

Equating the real parts on both sides

$3x + x - 2 + 6y - (1 - 3y) = 0$

$\Rightarrow \quad 4x + 9y = 3$ ...(i)

Equating the imaginary parts, we get

$-x + 3(x - 2) + 2y + 3(1 - 3y) = 10$

$\Rightarrow \quad 2x - 7y = 13$ ...(ii)

Solving (i) and (ii), we get

$x = 3, y = -1, x + y = 2$.

18. (b) : We have, $x = 1 + 2i \Rightarrow x - 1 = 2i$

$\Rightarrow \quad x^2 - 2x + 5 = 0$

Now, $x^3 + 7x^2 - x + 16$

$= x(x^2 - 2x + 5) + 9(x^2 - 2x + 5) + (12x - 29)$

$= x(0) + 9(0) + 12x - 29 = 12(1 + 2i) - 29 = -17 + 24i$

19. (a) : $\dfrac{z_1}{z_2} = (2 + 3i)\left(\dfrac{1}{1+2i}\right) = (2 + 3i)\left(\dfrac{1}{5} - \dfrac{2}{5}i\right)$

$$= \left(\frac{2}{5} + \frac{6}{5}\right) + i\left(-\frac{4}{5} + \frac{3}{5}\right) = \frac{8}{5} - \frac{1}{5}i$$

20. (d) : $(1 + i)^2 = 1 + i^2 + 2i = 2i$

$$\therefore \quad \frac{(1+i)^2}{3-i} = \frac{2i(3+i)}{3^2 - i^2} = \frac{6i - 2}{10} = \frac{-1+3i}{5}$$

$\therefore \quad$ Real part $= \dfrac{-1}{5}$

21. (d) : $\dfrac{(1+i)^n}{(1-i)^{n-2}} = \dfrac{(1+i)^2(1+i)^{n-2}}{(1-i)^{n-2}}$

$$= (1+i)^2\left(\frac{1+i}{1-i}\right)^{n-2} = (1+i)^2\left(\frac{(1+i)(1+i)}{1^2+1^2}\right)^{n-2}$$

$$= (1+i)^2\left(\frac{1-1+2i}{2}\right)^{n-2} = (2i)(i)^{n-2}$$

$$= 2(i)^{n-1} = 2(-1)^{\frac{n-1}{2}}$$

So, least positive integer n for which $2(-1)^{\frac{n-1}{2}}$ is positive, is 1.

22. (b) : $z_1 \cdot z_2 = (3 + 2i)(1 - i) = 3 + 2i - 3i - 2i^2 = 5 - i$

23. (c) : $z_1 = 2 - i,\ z_2 = 3 + i,\ z_3 = \dfrac{1+i}{2-i}$

$z_1 - z_2 + z_3 = 2 - i - (3 + i) + \dfrac{1+i}{2-i}$

$= 2 - i - 3 - i + \dfrac{1+i}{2-i} = -1 - 2i + \dfrac{1+i}{2-i} \times \dfrac{2+i}{2+i}$

$= -1 - 2i + \dfrac{(1+i)(2+i)}{(2)^2 - (i)^2} = -1 - 2i + \dfrac{(2 + 2i + i^2 + i)}{4+1}$

$= \dfrac{-5 - 10i + 1 + 3i}{5} = \dfrac{-4 - 7i}{5}$

24. (b) : $z_1 = \dfrac{-3-2i}{2+3i},\ z_2 = \dfrac{1+2i}{2+3i}$

$(z_1 + z_2) = \left(\dfrac{-3-2i}{2+3i}\right) + \left(\dfrac{1+2i}{2+3i}\right) = \dfrac{-2}{2+3i}$

$(z_1 - z_2) = \dfrac{-3 - 2i - 1 - 2i}{2+3i} = \dfrac{-4-4i}{2+3i} = \dfrac{-4(1+i)}{2+3i}$

$(z_1 + z_2)(z_1 - z_2) = \dfrac{-2}{(2+3i)} \times \dfrac{(-4)(1+i)}{(2+3i)} = \dfrac{8(1+i)}{(2+3i)^2}$

$= \dfrac{8(1+i)}{4 - 9 + 12i} = \dfrac{8(1+i)}{12i - 5} = \dfrac{8(1+i)}{12i - 5} \times \dfrac{(5+12i)}{(5+12i)} = \dfrac{56 - 136i}{169}$

25. (b) : $z_1 = \dfrac{1}{3+i}$

$\dfrac{1}{z_1} = \dfrac{1}{3+i} \times \dfrac{3-i}{3-i} = \dfrac{3-i}{3^2 - i^2} = \dfrac{3-i}{10}$

$z_1 + \dfrac{1}{z_1} = \dfrac{1}{3+i} + \dfrac{3-i}{10} = \dfrac{3-i}{10} + \dfrac{3-i}{10} = \dfrac{6-2i}{10} = \dfrac{3}{5} - \dfrac{i}{5}$

26. (c) : $z_1 = 3x - iy;\ z_2 = 5x + 3iy$

$\dfrac{z_1}{z_2} = \dfrac{3x - iy}{5x + i3y} \times \dfrac{5x - i3y}{5x - i3y}$

$= \dfrac{15x^2 - 5xyi - 9xyi - 3y^2}{(5x)^2 - (i3y)^2} = \dfrac{15x^2 - 3y^2 - 14xyi}{25x^2 + 9y^2}$

$\Rightarrow \dfrac{Ax^2 - By^2}{25x^2 + 9y^2} - i\dfrac{Cxy}{25x^2 + 9y^2} = \dfrac{15x^2 - 3y^2}{25x^2 + 9y^2} - i\dfrac{14xy}{25x^2 + 9y^2}$

$\therefore\ A = 15, B = 3, C = 14$

$A - B + C = 15 - 3 + 14 = 26$

27. (a) : $z = 3 + i$

$z - 2 = 3 + i - 2 = 1 + i$

$(z - 2)^2 = (1 + i)^2 = 1 + i^2 + 2i = 1 - 1 + 2i = 2i$

$(z-2)^2 + \dfrac{1}{(z-2)^2} = 2i + \dfrac{1}{2i} = 2i + \dfrac{i}{2i^2} = 2i - \dfrac{i}{2} = \dfrac{3}{2}i$

28. (b) : $z = (1 + i)^2 - (2 + i) = 1 + i^2 + 2i - 2 - i$

$= 1 - 1 + 2i - 2 - i = i - 2$

So, $\dfrac{z+1}{z-1} = \dfrac{i-2+1}{i-2-1} = \dfrac{i-1}{i-3}$

29. (d) : Let $z = \left(\dfrac{1+i}{1-i}\right)^n$

$= \left(\dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}\right)^n = \left(\dfrac{1+i^2+2i}{1+1}\right)^n = \left(\dfrac{2i}{2}\right)^n = (i)^n$

If n is even, then z is real.

30. (b)

31. (a) : Consider, $\dfrac{1+i}{1-i} = \dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i} = i$

$\therefore\ \left(\dfrac{1+i}{1-i}\right)^{100} = i^{100} = 1$

32. (a)

33. (a) : Given that, $\left(\dfrac{1+i}{1-i}\right)^3 - \left(\dfrac{1-i}{1+i}\right)^3 = x + iy$ ...(i)

$\therefore\ \left(\dfrac{1+i}{1-i}\right)^3 = \left(\dfrac{1+i}{1-i} \times \dfrac{1+i}{1+i}\right)^3$

$= \left(\dfrac{(1+i)^2}{1-i^2}\right)^3 = \left(\dfrac{1+i^2+2i}{2}\right)^3 = i^3 = -i$ ...(ii)

Similarly, $\left(\dfrac{1-i}{1+i}\right)^3 = i$ ...(iii)

Using (ii) and (iii) in (i), we get

$-i - i = x + iy$

$\Rightarrow\ -2i = x + iy$

On comparing real and imaginary parts, we get

$x = 0$ and $y = -2\ \Rightarrow\ (x, y) = (0, -2)$

34. (b) : We have, $(1 + i)^{2n} = (1 - i)^{2n} \Rightarrow \left(\dfrac{1+i}{1-i}\right)^{2n} = 1$

$\Rightarrow\ (i)^{2n} = 1$, which is possible if $n = 2$ $(\because\ i^4 = 1)$

35. (b)

36. (c) : $i^{-99} = \dfrac{1}{i^{99}} = \dfrac{1}{i^{98}i} = \dfrac{1}{(i^2)^{49} \cdot i} = \dfrac{1}{(-1).i} = \dfrac{i}{(-1).i^2} = i$

37. (b)

38. (d) : Let $z = \dfrac{i^4 - i^2}{\sqrt{-5}}$

$\Rightarrow\ z = \dfrac{i^2(i^2 - 1)}{\sqrt{5}i} = \dfrac{i^2(-1-1)}{i\sqrt{5}} = \dfrac{-2i}{\sqrt{5}}$

39. (d) : We have, $\dfrac{i^{4n+1} - i^{4n-1}}{2}$

$= \dfrac{i^{4n}.i - i^{4n} \cdot i^{(-1)}}{2} = \dfrac{(i - i^{-1})}{2}$ $[\because\ i^4 = 1]$

$= \dfrac{1}{2}\left(i - \dfrac{1}{i}\right) = \dfrac{1}{2}\left(i - \dfrac{1 \times i}{i \times i}\right) = \dfrac{1}{2}(i + i) = i$

40. (c) : We have, $i^5 + i^6 + i^7 + i^8$

$= i^4[i + i^2 + i^3 + i^4]$

$= 1[i - 1 - i + 1]$ $[\because\ i^2 = -1, i^3 = -i, i^4 = 1]$

$= 0$

41. (b) : $\dfrac{i^4 + i^9 + i^{16}}{2 - i^8 + i^{10} + i^3} = a + ib$

or $a + ib = \dfrac{1 + i + 1}{2 - 1 + i^2 + (-i)} = \dfrac{2+i}{1-1-i} = \dfrac{2+i}{-i}$

$\therefore\ (a, b) \equiv (-1, 2)$

42. **(a) :** $i^{10}+i^{11}+i^{12}+i^{13}=0$

and $\left(\frac{1}{i^{11}}+\frac{1}{i^{12}}+\frac{1}{i^{13}}+\frac{1}{i^{14}}\right)=0$

$\therefore$ Remaining expression $=i^{14}+\frac{1}{i^{15}}=i^2+\frac{1}{i^3}=-1+i$

43. **(d) :** Since, $i^2=-1$, $i^3=-i$ and $i^4=1$

$$\therefore \sum_{n=1}^{13}(i^n+i^{n+1})=(i+i^2)+(i^2+i^3)+(i^3+i^4)+\ldots+(i^{13}+i^{14})$$

$$=i+2(i^2+i^3+i^4+\ldots+i^{13})+i^{14}$$

$$=i+2[3(i+i^2+i^3+i^4)]+i^{14}$$

$$=i+0-1=i-1$$

44. **(a) :** We have, $z=i^9+i^{19}=(i^2)^4\cdot i+(i^2)^9\cdot i$

$$=i+(-i)=0=0+0i$$

45. **(c) :** We have, $(-\sqrt{-1})^{4n+3}=(-i)^{4n+3}=(-i)^{4n}(-i)^3$

$$=\{(-i)^4\}^n(-i^3)=1\times -i^3=i$$

46. **(b) :** $\sqrt{-3}=i\sqrt{3}$, $\sqrt{-6}=i\sqrt{6}$

So, $\sqrt{(-3)}\sqrt{(-6)}=i^2 3\sqrt{2}=-3\sqrt{2}$

47. **(b) :** $\bar{z}_1=3+i$

$z_1=\bar{\bar{z}}_1=\overline{3+i}=3-i$

$z_2=\bar{\bar{z}}_2=\overline{-5-i}=-5+i$

$\therefore\ z_1+z_2=3-i-5+i=-2$

$\therefore\ |z_1+z_2|=|-2|=2$

48. **(c) :** Let $z=\frac{1}{\sqrt{a}-i\sqrt{b}}$

$\therefore\quad z\times\frac{1}{z}=1$

$\therefore\ \frac{1}{z}=\sqrt{a}-i\sqrt{b}$

49. **(b) :** Let $z=\frac{\sqrt{a}-\sqrt{bi}}{\sqrt{x}+\sqrt{yi}}$

$$|z|=\left|\frac{\sqrt{a}-\sqrt{bi}}{\sqrt{x}+\sqrt{yi}}\right|=\frac{|\sqrt{a}-\sqrt{bi}|}{|\sqrt{x}+\sqrt{yi}|}=\frac{\sqrt{a+b}}{\sqrt{x+y}}=\sqrt{\frac{a+b}{x+y}}$$

50. **(b) :** Let $z=\frac{2+i}{4i+(1+i)^2}=\frac{2+i}{4i+1+i^2+2i}=\frac{2+i}{6i}$

$$|z|=\left|\frac{2+i}{6i}\right|=\frac{|2+i|}{|6i|}=\frac{\sqrt{2^2+1^2}}{\sqrt{6^2}}=\frac{\sqrt{5}}{6}$$

51. **(b) :** Let $z=\frac{a-b^2i}{a^2+bi}$

$$z=\frac{a-b^2i}{a^2+bi}\times\left(\frac{a^2-bi}{a^2-bi}\right)$$

$$=\frac{(a-b^2i)(a^2-bi)}{(a^2+bi)(a^2-bi)}=\frac{a^3-a^2b^2i-abi-b^3}{(a^2)^2+b^2}$$

$$=\frac{a^3-b^3-i(ab+a^2b^2)}{a^4+b^2}$$

$$\therefore\ \bar{z}=\frac{a^3-b^3+i(ab+a^2b^2)}{a^4+b^2}$$

52. **(a) :** $|\sqrt{3}-\sqrt{-5}|=|\sqrt{3}-\sqrt{5i^2}|=|\sqrt{3}-\sqrt{5}i|$

$$=\sqrt{(\sqrt{3})^2+(-\sqrt{5})^2}=\sqrt{8}$$

53. **(b) :** We have, $2^{101}z=(\sqrt{3}+i)^{104}$

Taking modulus on both sides, we get

$\Rightarrow\ 2^{101}|z|=|(\sqrt{3}+i)^{104}|\Rightarrow\ 2^{101}|z|=(\sqrt{3+1})^{104}$

$\Rightarrow\ 2^{101}|z|=2^{104}\quad\therefore\ |z|=2^3=8$

54. **(c) :** Let $z=2+\sqrt{7}i$

Reciprocal of $z=\frac{\bar{z}}{|z|^2}=\frac{2-\sqrt{7}i}{(4+7)}=\frac{2}{11}-\frac{\sqrt{7}}{11}i$

55. **(b) :** We have, $z_1=\frac{\sqrt{3}+\sqrt{2}i}{\sqrt{2}-\sqrt{3}i}$

$$\Rightarrow\quad z_1=\frac{\sqrt{3}+\sqrt{2}i}{\sqrt{2}-\sqrt{3}i}\times\frac{\sqrt{2}+\sqrt{3}i}{\sqrt{2}+\sqrt{3}i}$$

$$z_1=\frac{\sqrt{6}+2i+3i-\sqrt{6}}{2+3}=\frac{5i}{5}=i$$

$$z_2=\frac{\sqrt{5}+\sqrt{7}i}{\sqrt{7}-\sqrt{5}i}\times\frac{\sqrt{7}+\sqrt{5}i}{\sqrt{7}+\sqrt{5}i}$$

$$=\frac{\sqrt{35}+7i+5i-\sqrt{35}}{7+5}=\frac{12i}{12}=i$$

Now, $z_1+z_2=i+i=2i$

$|\overline{z_1+z_2}|=|-2i|=2$

56. **(b)**

57. **(b)**

58. **(b) :** $\frac{z-\sqrt{2}}{z+\sqrt{2}}=\frac{i}{k}$, ($k$ is any non zero real number)

$$\Rightarrow\ \frac{z+\sqrt{2}}{z-\sqrt{2}}=\frac{k}{i}\ \Rightarrow\ \frac{z+\sqrt{2}+z-\sqrt{2}}{z+\sqrt{2}-z+\sqrt{2}}=\frac{k+i}{k-1}$$

$$\Rightarrow\ \frac{2z}{2\sqrt{2}}=\frac{k+i}{k-i}\ \Rightarrow\ \left|\frac{z}{\sqrt{2}}\right|=\left|\frac{k+i}{k-i}\right|$$

$$\Rightarrow\ |z|=\sqrt{2}$$

59. **(d) :** $z=\frac{8+i}{\sqrt{2}-i}$

$$z\bar{z}=|z|^2=\left|\frac{8+i}{\sqrt{2}-i}\right|^2=\left(\frac{\sqrt{8^2+1^2}}{\sqrt{(\sqrt{2})^2+1^2}}\right)^2=\left(\frac{\sqrt{65}}{\sqrt{3}}\right)^2=\frac{65}{3}$$

60. **(c) :** $z+\frac{1}{z}=1+i$

$$\left(z+\frac{1}{z}\right)^3=(1+i)^3$$

$\Rightarrow \quad z^3 + \dfrac{1}{z^3} + 3z \times \dfrac{1}{z}\left(z + \dfrac{1}{z}\right) = 1 + i^3 + 3 \cdot 1^2 \cdot i + 3 \cdot 1 \cdot i^2$

$\Rightarrow \quad z^3 + \dfrac{1}{z^3} + 3(1+i) = 1 - i + 3i - 3$

$\Rightarrow \quad z^3 + \dfrac{1}{z^3} = -2 + 2i - 3 - 3i = -5 - i$

$\therefore \quad \left|z^3 + \dfrac{1}{z^3}\right| = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$

61. (c) : $z = \dfrac{5-4i}{3+2i}$

$z - i\beta = \dfrac{5-4i}{3+2i} - i\beta = \dfrac{5 - 4i - i\beta(3+2i)}{3+2i}$

$= \dfrac{5 - 4i - 3i\beta + 2\beta}{3+2i}$

$|z - i\beta| = \left|\dfrac{(5+2\beta) - i(4+3\beta)}{3+2i}\right|$

$\Rightarrow \quad 3\sqrt{5} = \dfrac{|5+2\beta) - i(4+3\beta)|}{|3+2i|}$

$\Rightarrow \quad 3\sqrt{5} = \dfrac{\sqrt{(5+2\beta)^2 + (4+3\beta)^2}}{\sqrt{3^2+2^2}}$

$\Rightarrow \quad 3\sqrt{5} = \dfrac{\sqrt{25 + 4\beta^2 + 20\beta + 16 + 9\beta^2 + 24\beta}}{\sqrt{13}}$

$\Rightarrow \quad 3\sqrt{5} = \dfrac{\sqrt{41 + 13\beta^2 + 44\beta}}{\sqrt{13}}$

$\Rightarrow \quad 45 \times 13 = 41 + 13\beta^2 + 44\beta$

$\Rightarrow \quad 13\beta^2 + 44\beta = 585 - 41 = 544$

62. (c) : $z_1 = 1 + i$; $z_2 = 2 - 2i$, $z_3 = -3 + i$

$z_1 + z_2 + z_3 = 1 + i + 2 - 2i - 3 + i = 0$

$\therefore \quad z_1^3 + z_2^3 + z_3^3 = 3z_1z_2z_3 = 3(1+i)(2-2i)(-3+i)$

$= 3[(1+i)(-6 + 6i + 2i + 2)]$

$= 3[(1+i)(-4+8i) = 12(1+i)(-1+2i)$

$= 12(-1 - i + 2i - 2) = 12(-3 + i) = -36 + 12i$

63. (b) : As given, $|z_1| = |z_2| = \left|\dfrac{1}{z_1} + \dfrac{1}{z_2}\right| = \left|\dfrac{z_1+z_2}{z_1z_2}\right| = 6$

Now, $\left|\dfrac{z_1+z_2}{z_1z_2}\right| = 6$

$\Rightarrow \quad |z_1 + z_2| = 6|z_1||z_2| = 6 \times 6 \times 6 = 216$

64. (b) : $\sqrt{i} = \sqrt{\dfrac{2i}{2}} = \sqrt{\dfrac{1+2i-1}{2}} = \sqrt{\dfrac{1+2i+i^2}{2}}$

$= \sqrt{\dfrac{(1+i)^2}{2}} = \dfrac{1+i}{\sqrt{2}}$

$\sqrt{-i} = \sqrt{\dfrac{-2i}{2}} = \sqrt{\dfrac{1-2i+i^2}{2}} = \sqrt{\dfrac{(1-i)^2}{2}} = \dfrac{1-i}{\sqrt{2}}$

$\sqrt{i} + 2\sqrt{-i} = \dfrac{1+i}{\sqrt{2}} + \dfrac{2(1-i)}{\sqrt{2}} + \dfrac{1+i+2-2i}{\sqrt{2}} = \dfrac{3-i}{\sqrt{2}}$

65. (b): We have, $(-\sqrt{-1})^{4n+4} + (i^{49} + i^{-257})^{10}$

$= (-i)^{4n} \cdot (-i)^4 + \left[i + \dfrac{1}{(i^3)^{85}} \times \dfrac{1}{i^2}\right]^{10}$

$= 1 \cdot 1 + [i - i]^{10} = 1$

66. (b) : $z = \dfrac{1+i}{1-i} - \dfrac{1-i}{1+i} = \dfrac{(1+i)(1+i) - (1-i)(1-i)}{(1+i)(1-i)}$

$= \dfrac{(1 + i^2 + 2i) - (1 + i^2 - 2i)}{1+1} = \dfrac{2i+2i}{2} = \dfrac{4i}{2} = 2i$

$\therefore \quad z^2 + z\bar{z} = (2i)^2 + (2i)(-2i) = 4i^2 - 4i^2 = 0$

67. (b) : $A + iB = \dfrac{1+\sqrt{2}i}{1-\sqrt{2}i} \times \dfrac{1+\sqrt{2}i}{1+\sqrt{2}i} = \dfrac{(1+\sqrt{2}i)^2}{1+2}$

$= \dfrac{1 + (\sqrt{2}i)^2 + 2 \cdot 1 \cdot \sqrt{2}i}{3} = \dfrac{1 - 2 + 2\sqrt{2}i}{3} = \dfrac{-1 + 2\sqrt{2}i}{3}$

$\therefore \quad A = -\dfrac{1}{3}, B = \dfrac{2\sqrt{2}}{3}$

$\therefore \quad AB = \left(-\dfrac{1}{3}\right) \times \left(\dfrac{2\sqrt{2}i}{3}\right) = \dfrac{-2\sqrt{2}}{9}$

68. (d) : The two complex number are

$A = x + iy$, $B = \alpha + i\beta$

Quotient $\dfrac{A}{B} = \dfrac{x+iy}{\alpha + i\beta}$,

$\left|\dfrac{A}{B}\right| = \left|\dfrac{x+iy}{\alpha+i\beta}\right| = \dfrac{\sqrt{x^2+y^2}}{\sqrt{\alpha^2+\beta^2}} = \dfrac{|A|}{|B|}$

Hence, modulus of the Quotient is equal to the quotient of their moduli.

69. (d) : Let $z_1 = a_1 + ib_1$, $z_2 = a_2 + ib_2$

$|z_1| = |z_2|$

$\sqrt{a_1^2 + b_1^2} = \sqrt{a_2^2 + b_2^2}$

It is true many values of a_1, a_2, and b_1, b_2.

So, a_1 must not equal to a_2 and b_1 must not equal to b_2.

70. (c) : We have, $3x = 6 + 15i$

$\Rightarrow \quad x = \dfrac{6+15i}{3} = 2 + 5i$

$x^3 + x^2 - 8x + 72 + 107 = x^2(x+1) - 8(x-9) + 107$

$= (2+5i)^2[(2 + 5i + 1)] - 8[2 + 5i - 9] + 107$

$= (4 + 25i^2 + 20i)(3 + 5i) + 8(7 - 5i) + 107$

$= (4 - 25 + 20i)(3 + 5i) + 56 - 40i + 107$

$= (-21 + 20i)(3 + 5i) + 56 - 40i + 107$

$= -63 + 60i - 105i - 100 + 56 - 40i + 107$

$= -107 - 85i + 107 = -85i$

$|x^3 + x^2 - 8x + 179| = |-85i| = 85$

71. **(a) :** We have, $(1+i)z = (1-i)\bar{z}$

$\Rightarrow \frac{z}{\bar{z}} = \frac{(1-i)}{(1+i)} \Rightarrow \frac{z}{\bar{z}} = \frac{(1-i)(1-i)}{(1+i)(1-i)}$

$\Rightarrow \frac{z}{\bar{z}} = \frac{(1-i)^2}{1-i^2} = \frac{1+i^2-2i}{1-i^2} = -i$

$\Rightarrow z = -i\bar{z}$

72. **(b) :** Let $z = x+iy$

Now, $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}, \ z \neq -1$

$= \frac{x-1+iy}{x+1+iy} = \frac{(x-1+iy)(x+1-iy)}{(x+1+iy)(x+1-iy)}$

$= \frac{(x^2-1)+iy(x+1)-iy(x-1)-i^2y^2}{(x+1)^2-(iy)^2}$

$\Rightarrow \frac{z-1}{z+1} = \frac{(x^2-1)+y^2+i[y(x+1)-y(x-1)]}{(x+1)^2+y^2}$

Since, $\frac{z-1}{z+1}$ is purely imaginary number.

$\therefore \frac{(x^2-1)+y^2}{(x+1)^2+y^2} = 0 \Rightarrow x^2-1+y^2 = 0$

$\Rightarrow x^2+y^2 = 1 \Rightarrow |z| = 1$

73. **(a) :** Let $z = x+iy$

then, $(z+3)(\bar{z}+3) = (x+iy+3)(x-iy+3)$

$= ((x+3)+iy)((x+3)-iy)$

$= (x+3)^2-(iy)^2 = (x+3)^2+y^2$

$= |x+3+iy|^2 = |z+3|^2$

74. **(a) :** We have, $\alpha - i\beta = \frac{3-4ix}{3+4ix}$ ...(i)

Taking conjugate of (i) on both sides, we get

$\alpha + i\beta = \left[\overline{\left(\frac{3-4ix}{3+4ix}\right)}\right] = \frac{\overline{(3-4ix)}}{\overline{(3+4ix)}} = \frac{3+4ix}{3-4ix}$ ...(ii)

Multiplying (i) and (ii), we get

$(\alpha - i\beta)(\alpha + i\beta) = 1$

$\Rightarrow \alpha^2+\beta^2 = 1$

75. **(d) :** We have $a+ib = c+id$

$\Rightarrow |a+ib| = |c+id|$

$\Rightarrow \sqrt{a^2+b^2} = \sqrt{c^2+d^2} \Rightarrow a^2+b^2 = c^2+d^2$

76. **(b) :** Let $z = x+iy$

Then $|z| = |x+iy| = \sqrt{x^2+y^2}$

$\Rightarrow |z|^2 = x^2+y^2$ ...(i)

and $z^2 = (x+iy)^2 = x^2+i^2y^2+2ixy$

$\Rightarrow z^2 = x^2-y^2+2ixy$

$\Rightarrow |z^2| = \sqrt{(x^2-y^2)^2+(2xy)^2}$

$\Rightarrow |z^2| = \sqrt{x^4+y^4-2x^2y^2+4x^2y^2}$

$\Rightarrow |z^2| = \sqrt{x^4+y^4+2x^2y^2} = \sqrt{(x^2+y^2)^2}$

$\Rightarrow |z^2| = x^2+y^2$ ...(ii)

From (i) and (ii), $|z|^2 = |z^2|$

77. **(a) :** We have, $z = 1+2i$ ...(i)

then $|z| = \sqrt{1+4} = \sqrt{5}$ ...(ii)

Now, $f(z) = \frac{7-z}{1-z^2} = \frac{7-1-2i}{1-(1+2i)^2}$ (from (i))

$= \frac{6-2i}{1-1-4i^2-4i} = \frac{6-2i}{4-4i}$

$= \frac{(3-i)(2+2i)}{(2-2i)(2+2i)} = \frac{6-2i+6i-2i^2}{4-4i^2}$

$= \frac{6+4i+2}{4+4} = \frac{8+4i}{8} = 1+\frac{1}{2}i$

Since, $f(z) = 1+\frac{1}{2}i$

$\therefore |f(z)| = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{4+1}{4}} = \frac{\sqrt{5}}{2} = \frac{|z|}{2}$ (From (ii))

78. **(b) :** Multiplicative inverse of $(4-3i) = \frac{1}{4-3i}$

$= \frac{4+3i}{16+9} = \frac{4}{25}+\frac{3}{25}i$

79. **(d) :** Let $z = x+iy$

$\therefore |z| = \sqrt{x^2+y^2} = 2 \Rightarrow x^2+y^2 = 4$

Now, $\frac{z-\alpha}{z+\alpha} = \frac{x+iy-\alpha}{x+iy+\alpha} = \frac{(x-\alpha)+iy}{(x+\alpha)+iy} \times \frac{(x+\alpha)-iy}{(x+\alpha)-iy}$

$= \frac{(x^2+y^2-\alpha^2)}{(x+\alpha)^2+y^2} + \frac{i\,2\alpha y}{(x+\alpha)^2+y^2}$

$\Rightarrow \frac{x^2+y^2-\alpha^2}{(x+\alpha)^2+y^2} = 0 \quad \left[\because \frac{z-\alpha}{z+\alpha} \text{ is purely imaginary}\right]$

$\Rightarrow x^2+y^2-\alpha^2 = 0 \Rightarrow \alpha^2 = 4 \Rightarrow \alpha = \pm 2$

80. **(d) :** Let $\bar{z} = \frac{1}{i-1}$

Since $z = \overline{(\bar{z})} \Rightarrow z = \frac{1}{\bar{i}-1} = \frac{1}{-i-1} = \frac{-1}{i+1}$

81. **(d) :** Let $z = x+iy \quad \therefore |z| = \sqrt{x^2+y^2}$

Now $|z|+z = 3+i$

$\therefore \sqrt{x^2+y^2}+x+iy = 3+i$

$\Rightarrow x+\sqrt{x^2+y^2} = 3$ and $y = 1$

$\Rightarrow x+\sqrt{x^2+1} = 3 \Rightarrow \sqrt{x^2+1} = 3-x$

$\Rightarrow x^2+1 = (3-x)^2 \Rightarrow x = 4/3$

$\therefore |z| = \sqrt{\frac{16}{9}+1} = \frac{5}{3}$

82. **(d) :** We have, $\frac{(1+i)^2}{1-i} = \frac{1+i^2+2i}{1-i} = \frac{1-1+2i}{1-i}$

$= \frac{2i}{1-i} \times \frac{1+i}{1+i} = \frac{2i(1+i)}{1-(i)^2} = \frac{2i(1+i)}{1-(-1)}$

$= \frac{2i(1+i)}{2} = i + i^2 = i - 1.$

$\therefore$ Required conjugate is $-i - 1$

83. **(c)**

84. **(d)** : Let $z = x + iy$ be any complex number.

$z = -\bar{z}$ (given)

$\Rightarrow x + iy = -(x - iy) \Rightarrow 2x = 0 \Rightarrow x = 0$

$\therefore z = 0 + iy = iy$ is purely imaginary.

85. **(b)** : $(\sqrt{5} + \sqrt{3}i)^{33} = 2^{49} z$

Taking modulus on both sides

$2^{49} |z| = \left|\sqrt{5} + \sqrt{3}i\right|^{33}$

$2^{49} |z| = (\sqrt{5+3})^{33} = (\sqrt{8})^{33} = 2^{33}\, 2^{33/2}$

or $|z| = \frac{2^{33}\, 2^{33/2}}{2^{49}} = \frac{2^{33}\, 2^{33/2}}{2^{33} \cdot 2^{16}} = 2^{\frac{33}{2} - 16} = 2^{1/2} = \sqrt{2}$

86. **(b)** : Let $\frac{2z_1}{3z_2} = pi$ (p is any non zero real number)

$\therefore \left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \left|\frac{\frac{z_1}{z_2} - 1}{\frac{z_1}{z_2} + 1}\right|$ [By dividing N^r and D^r by z_2]

$= \left|\frac{\frac{3pi}{2} - 1}{\frac{3pi}{2} + 1}\right| = \left|\frac{3pi - 2}{3pi + 2}\right| = 1$

[$\because$ The modulus of z and its conjugate modulus $\bar{z}$ have the same value]

87. **(c)** : Let $z = x + iy$, $\bar{z} = x - iy$

So, $z + \bar{z} = 2x, z - \bar{z} = 2iy$

$(3 + i)(z + \bar{z}) - (2 + i)(z - \bar{z}) + 14i = 0$

$\Rightarrow (3 + i)(2x) - (2 + i)(2iy) + 14i = 0$

$\Rightarrow (3x + y) + i(x - 2y + 7) = 0$

$\Rightarrow 3x + y = 0$ ...(i)

and $x - 2y + 7 = 0$ ...(ii)

Solving (i) and (ii), we have

$x = -1, y = 3$

So $z = -1 + 3i$, $\bar{z} = -1 - 3i$

$\therefore z\bar{z} = (-1 + 3i)(-1 - 3i) = 10$

88. **(d)** : $x + iy = \sqrt{\frac{a + ib}{c + id}}$ ...(i)

Taking conjugate, we get

$x - iy = \sqrt{\frac{a - ib}{c - id}}$ ...(ii)

Multiplying (i) and (ii), we get

$x^2 + y^2 = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$

89. **(a)** : We have, $|Z| = 1$ and $Z + \frac{1}{Z} = x + iy$

Let $Z = P + iQ$

$\therefore \sqrt{P^2 + Q^2} = 1 \Rightarrow P^2 + Q^2 = 1$

Now, $Z + \frac{1}{Z} = x + iy$

$\therefore P + iQ + \frac{1}{P + iQ} = x + iy$

$\Rightarrow P + iQ + \frac{P - iQ}{P^2 + Q^2} = x + iy$

$\Rightarrow P + iQ + P - iQ = x + iy$ ($\because P^2 + Q^2 = 1$)

$\Rightarrow 2P = x + iy$

Equating coefficients of real and imaginary parts, we get

$x = 2P$ and $y = 0$

$\therefore xy = 0$

90. **(b)** : $iz^3 + z^2 - z + i = 0$

Dividing both side by i and using $\frac{1}{i} = -i$.

We have, $z^3 - iz^2 + iz + 1 = 0$

$\Rightarrow z^2(z - i) + i(z - i) = 0$ ($\because i^2 = -1$)

$\Rightarrow (z - i)(z^2 + i) = 0 \therefore z = i$ or $z^2 = -i$

$\therefore |z| = |i| = 1$ and $|z^2| = |z|^2 = 1$

$\therefore |z| = 1$

91. **(d)** : $\because z = \frac{4}{1 - i} \quad \therefore \bar{z} = \frac{4}{1 + i}$

92. **(c)** : $\overline{(x + iy)(1 - 2i)} = 1 + i \Rightarrow (x - iy)(1 + 2i) = 1 + i,$

$\Rightarrow x - iy = \frac{1 + i}{1 + 2i} \Rightarrow x + iy = \frac{1 - i}{1 - 2i}$

93. **(b)** : $\text{Re}(1 + iy)^3 = 1 - 3y^2 = -26$

$\Rightarrow y^2 = 9 \Rightarrow |y| = 3$

94. **(b)** : $\sqrt{x^2 + y^2} = -y + 1$

$\Rightarrow x^2 + y^2 = y^2 - 2y + 1 \Rightarrow x^2 = 1 - 2y$

95. **(d)** : $f(z) = 1 + z + z^2$.

$\text{Re}(f(z)) = 1 + x^2 - y^2 + x$

$\text{Re}(\overline{f(z)}) = 0 \Rightarrow x^2 - y^2 + x + 1 = 0$

96. **(a)** : Let $z = x + iy$

$\therefore |z| = 5 \Rightarrow \sqrt{x^2 + y^2} = 5 \Rightarrow x^2 + y^2 = 25$

Now, $w = \frac{z - 5}{z + 5} = \frac{x + iy - 5}{x + iy + 5} = \frac{(x - 5) + iy}{(x + 5) + iy}$

On rationalizing the denominator, we get

$\frac{((x - 5) + iy)((x + 5) - iy)}{(x + 5)^2 + (y)^2} = \frac{x^2 - 25 + y^2 + 10yi}{(x + 5)^2 + y^2}$

$= \frac{10yi}{(x + 5)^2 + y^2}$ [Using $x^2 + y^2 = 25$]

$= 0 + \frac{10yi}{(x + 5)^2 + y^2}$

$\therefore \text{Re}(w) = 0$

97. **(a)** : Let $z_1 = a + ib, z_2 = c + id, z_3 = e + if$

$z_1 \text{ Im}(\bar{z}_2 z_3) = (a + ib) \text{ Im}((c - id)(e + if)) = (a + ib)(cf - de)$

Similarly, $z_2 \operatorname{Im}(\bar{z}_3 z_1) = (c + id)(eb - af)$

and $z_3 \operatorname{Im}(\bar{z}_1 z_2) = (e + if)(ad - cb)$

Now, $z_1 \operatorname{Im}(\bar{z}_2 z_3) + z_2 \operatorname{Im}(\bar{z}_3 z_1) + z_3 \operatorname{Im}(\bar{z}_1 z_2)$

$= (acf - dae + ceb - caf + aed - ceb) + i(bcf - bde + deb - daf + afd - cfb)$

$= 0$

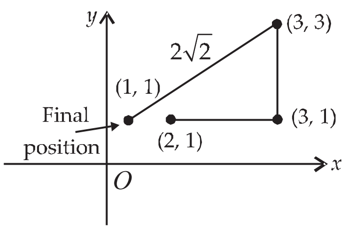
98. **(c) :** We have, $\left|\dfrac{1+2i}{1-(1-i)^2}\right| = \left|\dfrac{1+2i}{1+2i}\right| = 1$

99. **(d) :** $\dfrac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)} = \dfrac{(1-i\sqrt{3})(1+i)\times(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)}$

$= \dfrac{(\sqrt{3}-3i-i-\sqrt{3})(1+i)}{3+1} = \dfrac{-4i(1+i)}{4} = -i+1 = 1-i$

Clearly, above point lies in quadrant IV.

100. **(a) :**



Hence, the final position of the point is represented by $1 + i$.

KCET Ready

1. **(d) :** $z^2 + z + 1 = 0 \Rightarrow z = \omega, \omega^2$

Thus, $\omega + \omega^2 = -1$ and $\omega^3 = 1$

$\therefore \left(z+\dfrac{1}{z}\right)^2 + \left(z^2+\dfrac{1}{z^2}\right)^2 + \ldots + \left(z^6+\dfrac{1}{z^6}\right)^2$

$= 4(\omega+\omega^2)^2 + 2(\omega^3+\omega^3)^2$ $\quad (\because z = \omega)$

$= 4(-1)^2 + 2(2^2) = 4 + 8 = 12$

2. **(d) :** $(x-1)^3 + 8 = 0$ $\quad$ ...(*)

$x = -1$ satisfies $(x-1)^3 + 8 = 0$ *i.e.*, $(-2)^3 + 8 = 0$

$\Rightarrow 0 = 0$

Similarly, for $1 - 2\omega$, we have $(x-1)^3 + 8 = 0$

$\Rightarrow (1 - 2\omega - 1)^3 + 8 = 0 \Rightarrow (-2\omega)^3 + 8 = 0$

$\Rightarrow -8 + 8 = 0$

and for $1 - 2\omega^2$, we have $(1 - 2\omega^2 - 1)^3 + 8 = 0$

$\Rightarrow \omega^6(-8) + (8) = 0 \Rightarrow 0 = 0$

$\therefore -1, 1-2\omega, 1-2\omega^2$ are roots of $(x-1)^3 + 8 = 0$ and on the other hand the other roots does not satisfy the equation $(x-1)^3 + 8 = 0$.

3. **(c) :** $\left(i^{18}+\dfrac{1}{i^{25}}\right)^3 = \left(-1+\dfrac{1}{i}\right)^3$

$= -1 + \dfrac{1}{i^3} - \dfrac{3}{i^2} + \dfrac{3}{i} = -1 - \dfrac{1}{i} + 3 + \dfrac{3}{i} = 2 + \dfrac{2}{i}\times\dfrac{i}{i}$

$= 2 - 2i$

4. **(a) :** We know that sum, difference and product of two complex numbers is also a complex number.

$\therefore Z_1 + Z_2$ is always a complex number.

5. **(d) :** Given, $\left(-2-\dfrac{1}{3}i\right)^3 = \dfrac{x+iy}{27}$

$\Rightarrow \dfrac{-(6+i)^3}{27} = \dfrac{x+iy}{27} \Rightarrow -(198+107i) = x+iy$

$\Rightarrow x = -198$ and $y = -107 \therefore y - x = 198 - 107 = 91$

6. **(c) :** Given, $\dfrac{2z-n}{2z+n} = 2i - 1$

Let $z = x + 10i$ $\quad [\because \operatorname{Im}(z) = 10 \text{ (Given)}]$

$\therefore \dfrac{2(x+10i)-n}{2(x+10i)+n} = 2i - 1$

$\Rightarrow 2x + 20i - n = (2i-1)(2x + 20i + n)$

$\Rightarrow (2x - n) + 20i = 4xi - 40 + 2ni - 2x - 20i - n$

$\Rightarrow (2x - n) + 20i = -(n + 2x + 40) + i(4x + 2n - 20)$

Comparing real and imaginary parts, we get

$2x - n = -n - 2x - 40 \Rightarrow x = -10$

and $4x + 2n - 20 = 20 \Rightarrow 4(-10) + 2n = 40 \Rightarrow n = 40$

7. **(d)**

8. **(c) :** $|z_1| = |z_2| = |z_3| = \ldots\ldots = |z_n| = 1$

$\Rightarrow z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = \ldots\ldots = z_n\bar{z}_n = 1$

$\Rightarrow \bar{z}_1 = \dfrac{1}{z_1}, \bar{z}_2 = \dfrac{1}{z_2}, \bar{z}_3 = \dfrac{1}{z_3}, \ldots\ldots, \bar{z}_n = \dfrac{1}{z_n}.$

Now, $|z_1 + z_2 + z_3 + \ldots + z_n| = |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n|$

$= \left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} + \ldots + \dfrac{1}{z_n}\right|$

9. **(d) :** Let $\omega = x + iy$. If $\dfrac{\omega - \bar{\omega}z}{1-z}$ is real then

$\dfrac{x+iy-(x-iy)z}{1-z}$ is real

$\dfrac{x(1-z)}{1-z} + \dfrac{iy(1+z)}{1-z}$ is real $\Rightarrow \dfrac{y(1+z)}{1-z} = 0 \Rightarrow z = -1$

$\because y \neq 0$ and $1 - z \neq 0$; $|z| = 1$.

10. **(d) :** We have, $\dfrac{(1+i)(2+3i)(3-4i)}{(2-3i)(1-i)(3+4i)} = a + ib$

Taking modulus on both sides, we get

$\left|\dfrac{(1+i)(2+3i)(3-4i)}{(2-3i)(1-i)(3+4i)}\right| = |a+ib|$

$\Rightarrow \dfrac{\sqrt{1+1}\times\sqrt{4+9}\times\sqrt{9+16}}{\sqrt{4+9}\times\sqrt{1+1}\times\sqrt{9+16}} = \sqrt{a^2+b^2} \Rightarrow a^2 + b^2 = 1$

11. **(c) :** We have, $\left|z - \dfrac{3}{z}\right| = 2$

Now, $|z| = \left|\left(z-\dfrac{3}{z}\right)+\dfrac{3}{z}\right| \le \left|z-\dfrac{3}{z}\right| + \left|\dfrac{3}{z}\right| = 2 + \left|\dfrac{3}{z}\right|$

$\Rightarrow |z| - \dfrac{3}{|z|} \le 2 \Rightarrow |z|^2 - 2|z| - 3 \le 0$

$\Rightarrow (|z| + 1)(|z| - 3) \le 0 \Rightarrow |z| \ge -1, |z| \le 3.$

So, greatest value of $|z|$ is 3.

12. (a) : $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ ...(i)

$\because \quad |z|^2 = z\cdot\bar{z} = 1 \Rightarrow z = \frac{1}{\bar{z}}$

$\therefore \quad |z_1+z_2+z_3| = \left|\frac{1}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right| = \left|\overline{\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}}\right|$

$= \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ [From (i)]

13. (b) : We have $b + ic = (1 + a)z$

i.e. $z = \frac{b+ic}{1+a}$ *i.e.* $iz = \frac{-c+ib}{1+a}$

Therefore, we have

$$\frac{1+iz}{1-iz} = \frac{1+a-c+ib}{1+a+c-ib} = \frac{(1+a-c+ib)(1+a+c+ib)}{(1+a+c-ib)(1+a+c+ib)}$$

$$= \frac{(1+a)^2-c^2-b^2+i2b(1+a)}{(1+a+c)^2+b^2} \quad \text{...(i)}$$

Now, we have

$(1 + a)^2 - c^2 - b^2 + i2b(1 + a)$

$= (a^2 + 2a + 1) + (a^2 + b^2 - 1) - b^2 + i2b(1 + a)$

[Using $c^2 = 1 - a^2 - b^2$]

$= 2a^2 + 2a + i2b(1 + a) = 2(1 + a)(a + ib)$

and $(1 + a + c)^2 + b^2 = a^2 + b^2 + c^2 + 1 + 2a + 2c + 2ac$

$= 2(1 + a + c + ac)$ [Using $a^2 + b^2 + c^2 = 1$]

$= 2(1 + a)(1 + c)$

Putting, the above values in equation (i), we have

$$\frac{1+iz}{1-iz} = \frac{2(1+a)(a+ib)}{2(1+a)(1+c)} = \frac{a+ib}{1+c}$$

14. (a) : $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}+2 = \frac{1+i^2+1+i^2+1}{i^2+1+i^2+1+i^2}+2$

$= \frac{1-1+1-1+1}{-1+1-1+1-1}+2 = -1+2 = 1$

15. (b) : We have, $z = (3\sqrt{7}+4i)^2(3\sqrt{7}-4i)^3$

$= (3\sqrt{7}+4i)^2(3\sqrt{7}-4i)^2(3\sqrt{7}-4i)$

$= ((3\sqrt{7})^2-(4i)^2)^2(3\sqrt{7}-4i) = (79)^2(3\sqrt{7}-4i)$

$\Rightarrow \quad \text{Re}(z) = (79)^2(3\sqrt{7})$

16. (d) : We have, $|z| = \left|z - \frac{8}{z} + \frac{8}{z}\right|$

i.e. $|z| \le \left|z-\frac{8}{z}\right| + \left|\frac{8}{z}\right|$ *i.e.* $|z| \le 2 + \frac{8}{|z|}$

i.e. $|z|^2 - 2|z| - 8 \le 0$ *i.e.* $(|z| + 2)(|z| - 4) \le 0$

i.e. $-2 \le |z| \le 4$

17. (c) : We have,

$(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)(1 - \omega^8 + \omega^{16})$

$= (1 + \omega^2 - \omega)(1 - \omega^2 + \omega)(1 - \omega + \omega^2)(1 - \omega^2 + \omega)$

[$\because \quad \omega^4 = \omega^3\cdot\omega = \omega$; $\omega^8 = (\omega^3)^2\cdot\omega^2 = \omega^2$; $\omega^{16} = (\omega^3)^5\cdot\omega$ $= \omega$ and $\omega^3 = 1$]

$= (-\omega-\omega)(-\omega^2-\omega^2)(-\omega-\omega)(-\omega^2-\omega^2)$

$= (-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2) = 16\cdot\omega^6$

$= 16(\omega^3)^2 = 16(1)^2 = 16$

18. (d) : We have $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

[$\because \quad 1 + \omega + \omega^2 = 0$]

$= (-2)^7(\omega^2)^7 = -128\omega^2$

19. (c) : We have,

$$\frac{z_1z_2}{\bar{z}_1} = \frac{(1-i)(-2+4i)}{1+i} = \frac{-2+2i+4i+4}{1+i}$$

$$= \frac{(2+6i)(1-i)}{1-i^2} = \frac{8+4i}{2} = 4+2i$$

$\therefore \quad \text{Im}\left(\frac{z_1z_2}{\bar{z}_1}\right) = 2$

20. (d) : $(1 + \omega^2)^m = (1 + \omega^4)^m$

$\therefore \quad (-\omega)^m = (-\omega^2)^m$ [$\because \quad 1 + \omega + \omega^2 = 0, \omega^3 = 1$]

or $\quad \omega^m = \omega^{2m}$

which is only satisfied for $m = 3$, as $\omega^3 = \omega^6 = 1$.

21. (b) : We have, $|z| = \left|z+\frac{2}{z}-\frac{2}{z}\right| \le \left|z+\frac{2}{z}\right| + \frac{2}{|z|}$

$\Rightarrow \quad |z| \le 2 + \frac{2}{|z|} \Rightarrow |z|^2 \le 2|z| + 2$

$\Rightarrow \quad |z|^2 - 2|z| + 1 \le 1 + 2 \Rightarrow (|z| - 1)^2 \le 3$

$\Rightarrow \quad -\sqrt{3} \le |z| - 1 \le \sqrt{3} \Rightarrow 1 - \sqrt{3} \le |z| \le 1 + \sqrt{3}$

22. (c) : The given expression may be written as

$$= \left[\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{1}{2}-\frac{\sqrt{3}}{2}i}\right]^6 + \left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\frac{1}{2}+\frac{\sqrt{3}}{2}i}\right]^6$$

$$= \left[\frac{-\left(\frac{-1-\sqrt{3}\,i}{2}\right)}{-\left(\frac{-1+\sqrt{3}\,i}{2}\right)}\right]^6 + \left[\frac{-\left(\frac{-1+\sqrt{3}\,i}{2}\right)}{-\left(\frac{-1-\sqrt{3}\,i}{2}\right)}\right]^6$$

$$= \left(\frac{\omega^2}{\omega}\right)^6 + \left(\frac{\omega}{\omega^2}\right)^6 = \omega^6 + \frac{1}{\omega^6} = 1 + 1 = 2 \quad (\because \ \omega^3 = 1)$$

23. (c) : Let, $\frac{-1+\sqrt{3}\,i}{2} = \omega$

Then, $\frac{-1-\sqrt{3}\,i}{2} = \omega^2$

$$\therefore \quad \left(\frac{-1+\sqrt{-3}}{2}\right)^{100} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{100}$$

$= \omega^{100} + (\omega^2)^{100} = \omega^{100} + \omega^{200}$

$= \omega + \omega^2$

$= -1$ [$\because \quad 1 + \omega + \omega^2 = 0$]

24. **(c)** : We have, $\left(\dfrac{-1+i\sqrt{3}}{1-i}\right)^{30}$

$= \left(\dfrac{2\omega}{1-i}\right)^{30}$ $\quad\left[\text{Since, } \omega = \dfrac{-1+\sqrt{3}i}{2}\right]$

$= \dfrac{2^{30}\cdot\omega^{30}}{((1-i)^2)^{15}} = \dfrac{2^{30}(\omega^3)^{10}}{(1+i^2-2i)^{15}} = \dfrac{2^{30}(1)}{(-2i)^{15}} = \dfrac{2^{30}}{(-1)^{15}2^{15}i^{15}} = -2^{15}i$

25. **(d)** : As complex numbers are not comparable.

26. **(d)** : $\dfrac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+i\sqrt{2})-(\sqrt{3}-i\sqrt{2})}$

$= \dfrac{9+5}{2\sqrt{2}i} = \dfrac{14}{2\sqrt{2}}(-i) = \dfrac{-7\sqrt{2}\,i}{2}$

27. **(c)** : $\left(\dfrac{2i}{1+i}\right)^n = \left(\dfrac{2i(1-i)}{2}\right)^n = (1+i)^n$

Now, from options,

When $n = 2$, $(1+i)^n = 2i$

When $n = 4$, $(1+i)^n = -4$

When $n = 8$, $(1+i)^n = 16$, a positive integer.

28. **(a)** : We have, $z = \dfrac{q+ir}{1+p} \Rightarrow \dfrac{iz}{1} = \dfrac{-r+iq}{1+p}$

$\Rightarrow \dfrac{1+iz}{1-iz} = \dfrac{1+p-r+iq}{1+p+r-iq}$

$\Rightarrow \dfrac{p+iq}{1+r} = \dfrac{(1+p-r)+iq}{(1+p+r)-iq} \quad \left(\because \dfrac{1+iz}{1-iz} = \dfrac{p+iq}{1+r} \text{ (Given)}\right)$

$\Rightarrow p(1+p+r) + q^2 + i(q(1+p+r) - qp)$

$= (1+r)(1+p-r) + iq(1+r)$

$\Rightarrow p^2 + q^2 + r^2 = 1$

29. **(c)** : $(3+2i)^2 = 9 + 4i^2 + 12i = 5 + 12i$

$\therefore (3+2i)^3 = (3+2i)(5+12i) = -9 + 46i$

and $(3+2i)^4 = (5+12i)^2 = 25 - 144 + 120i = -119 + 120i$

$\therefore f(3+2i) = (3+2i)^4 - 4(3+2i)^3 + 4(3+2i)^2 + 8(3+2i) + 44$

$= -119 + 120i - 4(-9+46i) + 4(5+12i) + 8(3+2i) + 44$

$= -119 + 120i + 36 - 184i + 20 + 48i + 24 + 16i + 44$

$= 5$

30. **(b)**

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1. **(b)** : Since 1, ω, ω^2 are cube roots of unity

$\therefore 1 + \omega + \omega^2 = 0$

Now, $(1-\omega+\omega^2)(1+\omega-\omega^2)$

$= (-\omega-\omega)(-\omega^2-\omega^2)$

$= (-2\omega)(-2\omega^2) = 4\omega^3 = 4$

2. **(d)** : We have, $z = \dfrac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$

Taking Modulus on both sides, we get

$|z| = \dfrac{|\sqrt{3}+i|^3|3i+4|^2}{|8+6i|^2}$

$= \dfrac{(\sqrt{3+1})^3(\sqrt{9+16})^2}{(\sqrt{64+36})^2} = \dfrac{8\times 25}{100} = 2$

3. **(a)** : We have, $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$= i^n + i^{n+1} + i^n\cdot i^2 + i^{n+1}\cdot i^2$

$= i^n + i^{n+1} - i^n - i^{n+1}$ $\quad [\because i^2 = -1]$

$= 0$

4. **(a)** : Given, $\left(\dfrac{1+i}{1-i}\right)^m = 1 \Rightarrow i^m = i^4 \Rightarrow m = 4$

5. **(b)** : Given, $\dfrac{1-i}{1+i} = \dfrac{1-i}{1+i}\times\dfrac{1-i}{1-i} = \dfrac{(1-i)^2}{1-i^2} = -i$

$\therefore \left(\dfrac{1-i}{1+i}\right)^{96} = (-i)^{96} = 1 = 1 + 0i$ $\quad$ ...(i)

Comparing (i) with $a + ib$, we get $a = 1$, $b = 0$

So, the value of $(a, b) = (1, 0)$

6. **(d)** : We have, $\left(\dfrac{1+i}{1-i}\right)^x = 1$

$\Rightarrow \left(\dfrac{(1+i)^2}{1-i^2}\right)^x = 1 \Rightarrow \left(\dfrac{2i}{2}\right)^x = 1$

$\Rightarrow i^x = 1 \Rightarrow x = 4n, n \in N$

7. **(b)** : We have, $3x + i(4x - y) = 6 - i$

On comparing real and imaginary parts, we get

$3x = 6$ and $4x - y = -1$

$\Rightarrow x = 2$ and $4(2) - y = -1 \Rightarrow x = 2$ and $y = 9$

8. **(c)** : Let $z = \dfrac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$

$\Rightarrow z = \dfrac{(1+i)^3(1+3i)^2}{4(1-3i)(1+3i)(1-i)(1+i)}$

$= \dfrac{(2i-2)(6i-8)}{4\times(1+9)(1+1)} = \dfrac{2\times 2(i-1)(3i-4)}{4\times 10\times 2}$

$= \dfrac{1}{20}[1-7i]$

$\therefore |z| = \dfrac{1}{20}|1-7i| = \dfrac{\sqrt{1+49}}{20} = \dfrac{\sqrt{2}}{4}$

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