PUC I YEAR MATHEMATICS (ANNUAL EXAM-2024)

DEPUTY DIRECTOR, DEPARTMENT OF SCHOOL EDUCATION (PRE-UNIVERSITY)

ANNUAL EXAMINATION – 2024 (SOLUTION)

PUC I YEAR KARNATAKA STATE LEVEL QUESTION PAPER

Time: 3 Hrs 15 Min **Subject : Mathematics (35)** **Max Marks: 80**

Instructions: (1) The question paper has five Parts namely A, B, C, D and E. Answer all the parts.
(2) Part-A has 15 multiple choice questions, 5 fill in the blanks questions

PART-A

I. Answer ALL the multiple choice questions : $(15 \times 1 = 15)$

1. **The interval form of the $\{x : x \in R, -4 < x \leq 6\}$ set is**
   A) $[-4,6]$. B) $(-4,6]$
   C) $(-4,6)$ C) $[-4,6)$
   Ans: B) $(-4,6]$

2. **If the set 'A' has 3 elements and the set 'B' has 3 elements then the number of elements in $A \times B$ are**
   A) 9 B) 6
   C) 3 D) 27
   Ans: A) 9

3. **The radian measure of 240^0 is**
   A) $\frac{\pi}{3}$ B) $\frac{3\pi}{4}$
   C) $\frac{\pi}{4}$ D) $\frac{4\pi}{3}$
   Ans: D) $\frac{4\pi}{3}$

4. **The simplest form of the complex numer i^{-35} is**
   A) i B) $-i$
   C) 1 D) -1
   Ans: A) i

5. **The solution set of the ineuality $30x < 200$ when $x \in N$ is**
   A) $\{1,2,3,4,5,6\}$ B) $\{0,1,2,3,4,5,6\}$
   C) $\{1,2,3,4,5,6,7\}$ D) $\{\dots, -2, -1, 0, 1, 2, \dots\}$
   Ans: A) $\{1,2,3,4,5,6\}$

6. **If $^nc_9 = {}^nc_8$ then $^nc_{17}$ is**
   A) 17 B) 7
   C) 1 D) 10
   Ans: C) 1

7. **In the expansion of $(a+b)^n$, the sum of the indices of 'a' and 'b' is**

A) $n+1$ B) $2n$

C) $n-1$ D) n

Ans: The Question is incomplete

(Now, the question is like this, see below)

In each term of the expansion of $(a+b)^n$, the sum of the indices of 'a' and 'b' is

A) $n+1$ B) $2n$

C) $n-1$ D) n

Ans: D) n

8. **The 4th term of the sequence defined by $a_n = \frac{n}{n+1}$ is**

A) $\frac{5}{4}$ B) $\frac{4}{5}$

C) $\frac{4}{3}$ D) $\frac{3}{4}$

Ans: B) $\frac{4}{5}$

9. **Equation of a line parallel to x-axis and passing through the point $(-2, 3)$ is**

A) $x = 3$ B) $x = -2$

C) $y = 3$ D) $y = -2$

Ans: C) $y = 3$

10. **Equation of a circle with centre $(0, 0)$ and radius 'r' units is is**

A) $(x-a)^2 + (y-b)^2 = r^2$ B) $x^2 + y^2 = 1$

C) $x^2 + y^2 = r^2$ D) $(x+a)^2 + (y+b)^2 = r^2$

Ans: C) $x^2 + y^2 = r^2$

11. **The length of the Latus rectum of the hyperola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is**

A) $\frac{2b}{a^2}$, B) $\frac{2a}{b^2}$,

C) $\frac{a}{2b^2}$, D) $\frac{2b^2}{a}$,

Ans: D) $\frac{2b^2}{a}$

12. **The Octant in which the points $(-4, 2, -5)$ lie**

A) II, B) IV ,

C) V , D) VI,

Ans: D) VI

13. **The value of $\lim\limits_{x\to 0} \frac{\cos x}{\pi - x}$ is**

A) π B) $\frac{1}{\pi}$

C) 0 D) limit does not exists

Ans: B) $\frac{1}{\pi}$

14. **The mean value of the following data : 4,7,8,9,10,12,13,17 is**

A) 10 B) 9

C) 8 D) 12

Ans: A) 10

15. The probability of drawing a club card from a well shuffled deck of 52 cards is

A) $\frac{1}{13}$ B) $\frac{1}{52}$

C) $\frac{1}{4}$ D) $\frac{1}{2}$

Ans: C) $\frac{1}{4}$

II. Fill in the blanks by choosing the appropriate answer from those give in brackets : $(5 \times 1 = 5)$

$(\mathbf{42}, \mathbf{-1}, \sqrt{\mathbf{3}}, \mathbf{1}, \mathbf{0}, \mathbf{20})$

16. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ then the value of y is ………………

Ans: 1

17. The value of $sin(n\pi)$ is......................... where $n \in Z$.

Ans: 0

18. The value of $\frac{7!}{5!}$ is ………………

Ans: 42

19. The slope of the line making inclination of 60^0 with the positive direction of x-axis is ………………

Ans: $\sqrt{3}$

20. The derivative of $x^2 - 2$ w.r.t x at $x = 10$ is ………………

Ans: 20

PART-B

Answer ANY SIX questions : $(6 \times 2 = 12)$

21. Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$ Find $V - B$ and $B - V$

Ans: $V - B = \{a, e, i, o, u\} - \{a, i, k, u\} = \{e, o\}$

$B - V = \{a, i, k, u\} - \{a, e, i, o, u\} = \{k\}$

22. Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Ans: Yes, $A \subset B$ Because, every element of A are the element of B

$A \cup B = \{a, b, c\} = B$

23. Prove that $sin^2\frac{\pi}{6} + cos^2\frac{\pi}{3} - tan^2\frac{\pi}{4} = -\frac{1}{2}$

Ans: LHS $= sin^2\frac{\pi}{6} + cos^2\frac{\pi}{3} - tan^2\frac{\pi}{4}$ w.k.t $sin\frac{\pi}{6} = \frac{1}{2}$, $cos\frac{\pi}{3} = \frac{1}{2}$, $tan\frac{\pi}{4} = 1$,

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$

$= \frac{1}{4} + \frac{1}{4} - 1$

$= \frac{1+1-4}{4} = \frac{-2}{4}$

$= -\frac{1}{2}$ = RHS

24. Find the multiplicative inverse of the complex number $Z = \sqrt{5} + 3i$

Ans: Given Complex number is $Z = \sqrt{5} + 3i$

Modulus $|Z| = \sqrt{(\sqrt{5})^2 + (3)^2} = \sqrt{14}$

Conjugate $\bar{Z} = \sqrt{5} - 3i$

Multiplicative inverse $Z^{-1} = \frac{\sqrt{5}-3i}{(\sqrt{14})^2} = \frac{\sqrt{5}-3i}{14}$

25. Express $3(7+i7)+i(7+i7)$ in the form $a+ib$

Ans: $3(7+i7)+i(7+i7) = 21+21i+7i+7i^2$ $\therefore\ i^2=-1$

$= 21+21i+7i-7$

$= 14+28i$

26. Solve $5x-3 \geq 3x-5$ and show the graph of solution on number line.

Ans: $5x-3 \geq 3x-5$ adding 3 on both side

$5x-3+3 \geq 3x-5+3$

$5x \geq 3x-2$ subtracting $3x$ on both side

$5x-3x \geq 3x-2-3x$

$2x \geq -2$ dividing 2 on both side

$\frac{2x}{2} \geq \frac{-2}{2}$

$x \geq -1$

$x \in [-1, \infty)$

27. How many 3-digit even numbers can be formed from the digits 1,2,3,4,6,7 if no digit is repeated?

Ans: Given digits are 1,2,3,4,6,7

Total number of digits = 6 (There are 3 even numbers out of 6 numbers)

To make 3 digit even numbers :

4	5	3

Unit's place can be filled with 3 different ways

Ten's place can be filled with 5 different ways

Hundred's place can be filled with 4 different ways

Total Number of 3 digit even numbers $= 4 \times 5 \times 3 = 60$

28. Using binomial theorem, evaluate $(99)^3$

Ans: $(99)^3 = (100-1)^3$ comparing with $(a+b)^n$ we have $a=100$, $b=-1$ and $n=3$

$= {}^3C_0(100)^3(-1)^0 + {}^3C_1(100)^2(-1)^1 + {}^3C_2(100)^1(-1)^2 + {}^3C_3(100)^0(-1)^3$

$= 1(1000000) - 3(10000) + 3(100) - 1(1)$

$= 1000000 - 30000 + 300 - 1$

$= 970299$

29. Find the equation of the line intersecting the x-axis at a distance of 3 units to the left of origin with slope – 2.

Ans: The point on x-axis at a distance of 3 units to the left of origin is $(-3,0)$ with slope -2

By using point slope form $y-y_1 = m(x-x_1)$ where slope $(x_1, y_1) = (-3,0)$ and $m=-2$

$y-0 = -2(x+3)$

$y = -2x-6$

$2x+y+6=0$

30. Evaluate : $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

Ans: put $1+x=y$ if $x\to 0$ then $y \to 1$

Then $x = y-1$

Now, $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} = \lim_{y\to 1} \frac{\sqrt{y}-1}{y-1}$

$= \lim_{y\to 1} \frac{y^{1/2}-1^{1/2}}{y-1}$

$= \frac{1}{2} 1^{1/2-1}$

$= \frac{1}{2}(1^{-1/2}) = \frac{1}{2}$

31. A die is thrown. Describe the following events:

i) A: a number less than 4 ii) B: a number not less than 3

Ans : $S = \{1,2,3,4,5,6\}$

$A = \{1,2,3\}$

$B = \{3,4,5,6\}$

PART-C

Answer ANY SIX questions : $(\mathbf{6 \times 3 = 18})$

32. Let $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$ and $B = \{2,3,5,7\}$ verify that $(A \cup B)' = A' \cap B'$

Ans: $A' = U - A$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,4,6,8\}$

$= \{1,3,5,7,9\}$

$B' = U - B$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,3,5,7\}$

$= \{1,4,6,8,9\}$

$A \cup B = \{2,4,6,8\} \cup \{2,3,5,7\}$

$= \{2,3,4,5,6,7,8\}$

Now, $(A \cup B)' = U - (A \cup B)$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,3,4,5,6,7,8\}$

$= \{1,9\}$ ………… (1)

Now, $A' \cap B' = \{1,3,5,7,9\} \cap \{1,4,6,8,9\}$

$= \{1,9\}$ ………… (2)

From (1) and (2), we have $(A \cup B)' = A' \cap B'$

33. Let $f(x) = x^2$ and $g(x) = 2x + 1$be two real valued functions. Find $(f + g)(x)$, $(f - g)(x)$ and $(fg)(x)$

Ans: i)$(f + g)(x) = f(x) + g(x)$

$= (x^2) + (2x + 1)$

$= x^2 + 2x + 1$

ii)$(f - g)(x) = f(x) - g(x)$

$= (x^2) - (2x + 1)$

$= x^2 - 2x - 1$

iii)$(fg)(x) = f(x) \cdot g(x)$

$= (x^2) \cdot (2x + 1)$

$= 2x^3 + x^2$

34. Prove that $sin3x = 3sinx - 4sin^3x$

Ans: $sin(x + y) = sinx \cdot cosy + cosx \cdot siny$ replacing y by 2x

$sin(x + 2x) = sinx \cdot cos2x + cosx \cdot sin2x$

$sin3x = sinx(1 - 2sin^2x) + cosx(2sinxcosx)$

$sin3x = sinx - 2sin^3x + 2sinxcos^2x$

$sin3x = sinx - 2sin^3x + 2sinx(1 - sin^2x)$

$sin3x = sinx - 2sin^3x + 2sinx - 2sin^3x$

$sin3x = 3sinx - 4sin^3x$

35. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$.

Ans: $x + iy = \frac{a+ib}{a-ib}$………………(1)

Conjugate of eq(1) $x - iy = \frac{a-ib}{a+ib}$ ……….(2)

Multiplying (1) and (2)

$(x + iy) \cdot (x + iy) = (\frac{a+ib}{a-ib}) \cdot (\frac{a-ib}{a+ib})$

$x^2 - i^2y^2 = \frac{a^2-i^2b^2}{a^2-i^2b^2}$ w.k.t $i^2 = -1$

$x^2 + y^2 = \frac{a^2+b^2}{a^2+b^2}$

$x^2 + y^2 = 1$

OR Given Complex number

$x + iy = \frac{a+ib}{a-ib}$

$|x + iy| = \frac{|a+ib|}{|a-ib|}$

$\sqrt{x^2 + y^2} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$

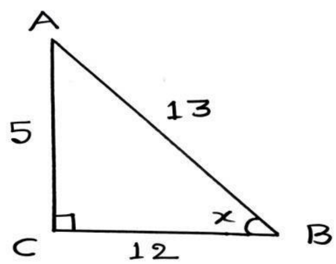
$\sqrt{x^2 + y^2} = 1$

$x^2 + y^2 = 1$

36. If $tanx = -\frac{5}{12}$, x lies in second quadrant, find other five trigonometric functions

Ans: $tanx = -\frac{5}{12}$

By Using Pythagorus theorem



$AC^2 + BC^2 = AB^2$

$12^2 + 5^2 = AB^2$

$144 + 25 = AB^2$

$AB^2 = 169$

$AB = 13$

x lies in second quadrant

(In second Quadrant sinx and cosecx are positive remaining are negative)

$sinx = \frac{5}{13}$, $cosecx = \frac{13}{5}$

$cosx = -\frac{12}{13}$, $secx = -\frac{13}{12}$

$tanx = -\frac{5}{12}$, $cotx = -\frac{12}{5}$

37. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

Ans: Let first odd natural number $= x$, Let second odd natural number $= x + 2$

Then the order pair is $(x, x+2)$

both of which are larger than 10 : $x > 10$ and $x + 2 > 10$

and their sum is less than 40

$x + (x + 2) < 40$

$2x + 2 < 40$

$2x < 38$

$x < 19$

Order pairs are (11,13), (13,15), (15,17), (17,19)

38. If A.M and G.M of two positive numbers 'a' and 'b' are 10 and 8, respectively. Find the numbers

Ans: $A.M = \frac{a+b}{2} = 10$ and $G.M = \sqrt{ab} = 8$

$a + b = 20$, $ab = 64$

$b = 20 - a$, $a(20 - a) = 64$

$20a - a^2 = 64$

$a^2 - 20a + 64 = 0$

$(a - 16)(a - 4) = 0$

$a = 16$ or $a = 4$

If $a = 16$ then $b = 4$ and if $a = 4$ then $b = 16$

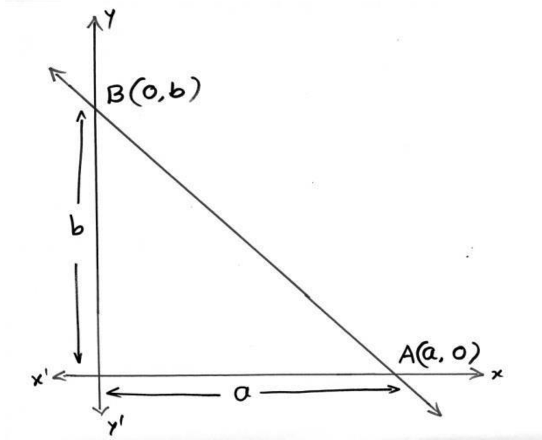
Therefore the numbers 'a' and 'b' are 16, 4 or 4,16

39. Derive the equation of line with x-intercept 'a' and y-intercept 'b' in the form of $\frac{x}{a}+\frac{y}{b}=1$

Ans: The line meets x axis at $A(a,0)$ here a is x-intercept

The line meets y axis at $B(0,b)$ here b is y-intercept

Equation of line passing through $A(a,0)$ and $B(0,b)$



$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$$
$$y-0=\left(\frac{b-0}{0-a}\right)(x-a)$$
$$y=-\frac{b}{a}(x-a)$$
$$ay=-b(x-a)$$
$$ay=-bx+ab$$
$bx+ay=ab$ dividing ab on both side
$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{ab}{ab}$$
$$\frac{x}{a}+\frac{y}{b}=1$$

40. Find the equation of parabola whose vertex is $(0,0)$, passing through the point $(2,3)$ and axis is along x axis.

Ans: The parabola is symmetric about positive x axis because the point $(2,3)$ lies in I Quadrant

Therefore equation of parabola is $y^2=4ax$.............(1)

Eq (1) passing through $(2,3)$: $(3)^2=4a(2)$
$$9=8a$$
$$a=\frac{9}{8}$$

Then eq(1) becomes $y^2=4ax$
$$y^2=4\left(\frac{9}{8}\right)x$$
$$y^2=\frac{9}{2}x$$
$$2y^2=9x$$

41. Show that the points $P(-2,3,5)$, $Q(1,2,3)$ and $R(7,0,-1)$ are collinear

Ans: $P(-2,3,5)$, $Q(1,2,3)$ and $R(7,0,-1)$
$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Distance between P and Q
$$PQ=\sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$$
$$=\sqrt{(3)^2+(-1)^2+(-2)^2}$$
$$=\sqrt{9+1+4}$$
$=\sqrt{14}$ units ....(1)

Distance between Q and R
$$QR=\sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$$
$$=\sqrt{(6)^2+(-2)^2+(-4)^2}$$
$$=\sqrt{36+4+16}$$
$=\sqrt{56}=2\sqrt{14}$ units............... (2)

Distance between P and R
$$PR=\sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$$
$$=\sqrt{(9)^2+(-3)^2+(-6)^2}$$
$$=\sqrt{81+9+36}$$
$=\sqrt{126}=3\sqrt{14}$ units................(3)

From (1), (2) and (3) we have $PR=PQ+QR$

Therefore the given three points are collinear

42. Find the derivative of $y = sinx$ with respect to x from first principle method

Ans: $y = sinx$

$\frac{dy}{dx} = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$

$\frac{dy}{dx} = \lim_{h\to0} \frac{sin(x+h)-sinx}{h}$, w.k.t the formula : $sinA + sinB = 2sin\left(\frac{A+B}{2}\right) cos\left(\frac{A-B}{2}\right)$

$\frac{dy}{dx} = \lim_{h\to0} \frac{2cos\left(\frac{(x+h)+x}{2}\right)sin\left(\frac{(x+h)-x}{2}\right)}{h}$

$\frac{dy}{dx} = \lim_{h\to0} \frac{2cos(x+\frac{h}{2})sin(\frac{h}{2})}{h}$

$\frac{dy}{dx} = \lim_{h\to0} \frac{cos(x+\frac{h}{2})sin(\frac{h}{2})}{\frac{h}{2}}$

$\frac{dy}{dx} = \lim_{h\to0} cos\left(x+\frac{h}{2}\right) \cdot \lim_{h\to0} \frac{sin(\frac{h}{2})}{\frac{h}{2}}$ w.k.t $\lim_{x\to0} \frac{sinx}{x} = 1$

$\frac{dy}{dx} = cos\left(x+\frac{0}{2}\right) \cdot 1$

$\frac{dy}{dx} = cosx \cdot 1$

$\frac{d}{dx}(sinx) = cosx$

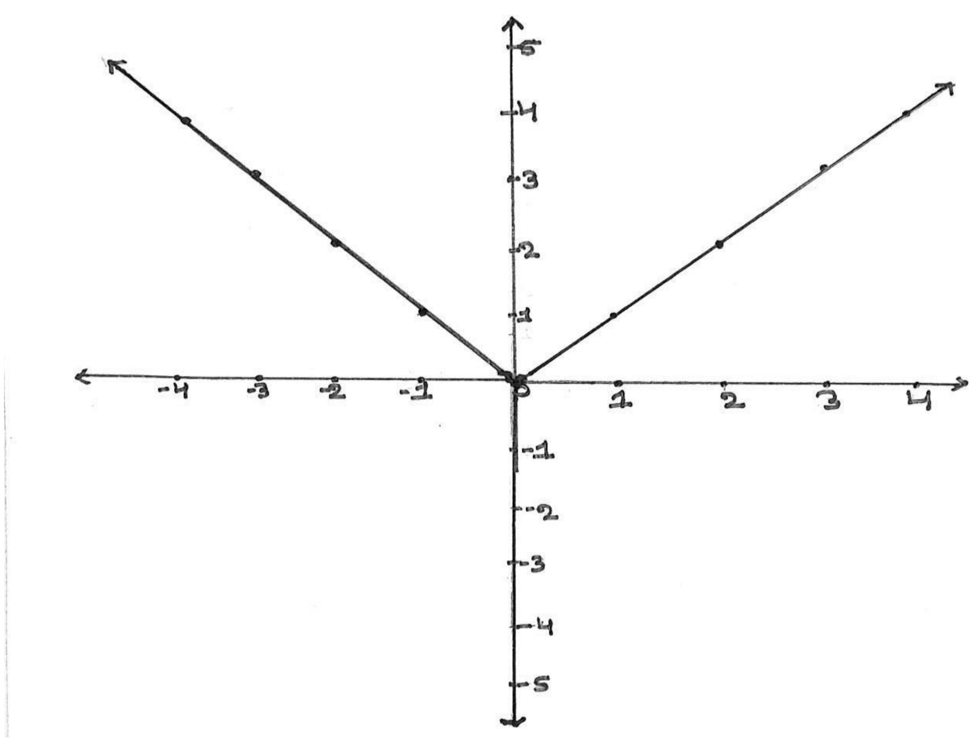
PART-D

Answer any four questions : $(\mathbf{5 \times 4 = 20})$

43. Define modulus function. Draw the graph of it. Also its write domain and range

Ans: Let R be the set of real numbers. The function $f: R \to R$ defined by

$f(x) = |x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$, $\forall\ x \in R$ such function is called modulus function.



Domain = R

Range = $R_+ \cup \{0\}$ or $[0, \infty)$

44. Prove that $\frac{cos4x+cos3x+cos2x}{sin4x+sin3x+sin2x} = cot3x$

Ans: w.k.t the formula : $cosA + cosB = 2cos\left(\frac{A+B}{2}\right) cos\left(\frac{A-B}{2}\right)$

w.k.t the formula : $sinA + sinB = 2sin\left(\frac{A+B}{2}\right) cos\left(\frac{A-B}{2}\right)$

$LHS = \frac{cos4x+cos3x+cos2x}{sin4x+sin3x+sin2x}$

$= \frac{(cos4x+cos2x)+cos3x}{(sin4x+sin2x)+sin3x}$

$= \frac{2cos\left(\frac{4x+2x}{2}\right)cos\left(\frac{4x-2x}{2}\right)+cos3x}{2sin\left(\frac{4x+2x}{2}\right)cos\left(\frac{4x-2x}{2}\right)+sin3x}$

$= \frac{2cos3xcosx+cos3x}{2sin3xcosx+sin3x}$

$= \frac{2cos3x(cosx+1)}{2sin3x(cosx+1)}$

$= \frac{cos3x}{sin3x}$

$= cot3x = RHS$

45. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has : (i) no girl ? (iii) at least 3 girls ?

Ans : In a group selected members = 5

The number of girls = 4

The number of boys = 7

(i) no girl

Number of ways = ${}^4C_0 \times {}^7C_5$

$= \frac{4!}{4!0!} \times \frac{7!}{2!5!}$

$= \frac{4!}{4!0!} \times \frac{7\times6\times5!}{2\times1\times5!}$

$= \frac{7\times6}{2} = 21$

(i) at least 3 girls

Number of ways = ${}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

$= \frac{4!}{1!3!} \times \frac{7!}{5!2!} + \frac{4!}{0!4!} \times \frac{7!}{6!1!}$

$= \frac{4\times3!}{1\times3!} \times \frac{7\times6\times5!}{5!\times2\times1} + \frac{4!}{0!4!} \times \frac{7\times6!}{6!\times1}$

$= \frac{4}{1} \times \frac{7\times6}{2} + \frac{1}{1} \times \frac{7}{1}$

$= \frac{4}{1} \times \frac{7\times6}{2} + \frac{1}{1} \times \frac{7}{1}$

$= 84 + 7 = 91$

46. State and prove 'Binomial Theorem' for positive integral index 'n'

Ans: Statement: For all a,b real numbers and n is a positive integer then

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \cdots \dots \dots \dots . + {}^nC_nb^n$

Proof: By using Mathematical induction

i) $n = 1, P(1)$: $LHS = (a + b)^1 = a + b, RHS = {}^1C_0a^1 + {}^1C_1b^1 = a + b$

Therefore LHS=RHS, The result is true for $P(1)$

ii) $n = k, P(k)$: $(a + b)^k = {}^kC_0a^k + {}^kC_1a^{k-1}b + {}^kC_2a^{k-2}b^2 + \cdots \dots \dots \dots . + {}^kC_kb^k$ …….(1)

The result is true for $P(k)$

iii) $n = k + 1, P(k + 1)$:

$(a + b)^{k+1} = (a + b)^k(a + b)^1$

$= (a + b)^1(a + b)^k$

$= (a + b)({}^kC_0a^k + {}^kC_1a^{k-1}b + {}^kC_2a^{k-2}b^2 + \cdots \dots \dots \dots . + {}^kC_kb^k)$

$= {}^kC_0a^{k+1} + {}^kC_1a^kb + {}^kC_2a^{k-1}b^2 + \cdots \dots\dots\dots . + {}^kC_kab^k$

$+ {}^kC_0a^kb + {}^kC_1a^{k-1}b^2 + {}^kC_2a^{k-2}b^3 + \cdots \dots\dots\dots . + {}^kC_kb^{k+1}$

$= {}^{k+1}C_0a^{k+1} + a^kb({}^kC_1 + {}^kC_0) + a^{k-1}b^2({}^kC_2 + {}^kC_1) + \cdots \dots\dots\dots\dots + {}^kC_kb^{k+1}$

$= {}^{k+1}C_0a^{k+1} + a^kb({}^{k+1}C_1) + a^{k-1}b^2({}^{k+1}C_2) + \cdots \dots\dots\dots . + {}^{k+1}C_{k+1}b^{k+1}$

$= {}^{k+1}C_0a^{k+1} + {}^{k+1}C_1a^kb + {}^{k+1}C_2a^{k-1}b^2 + \cdots \dots\dots\dots . + {}^{k+1}C_{k+1}b^{k+1}$

The result is true for $P(k+1)$, whenever $P(k)$ is true

Therefore $P(n)$ is true for all natural numbers n

Note: ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

${}^kC_1 + {}^kC_0 = {}^{k+1}C_1$

${}^kC_2 + {}^kC_1 = {}^{k+1}C_2$, and ${}^kC_k = {}^{k+1}C_{k+1} = 1$, ${}^kC_0 = {}^{k+1}C_0 = 1$

47. Prove that the length of the perpendicular from a point (x_1, y_1) to the line $Ax + By + C = 0$ is $d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Ans: The line $Ax + By + C = 0$ meets x axis at $Q\left(-\frac{C}{A}, 0\right)$ and y axis at $R\left(0, -\frac{C}{B}\right)$

$P(x_1, y_1)$ is a point at distance of d units from $Ax + By + C = 0$

In ΔPQR, we have

Area of $\Delta PQR = \frac{1}{2} PM \cdot QR$

$PM = \frac{2(\text{Area of } \Delta PQR)}{QR}$ ……..(1)

Area of triangle formed by (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$Area\ of\ \Delta PQR = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Area of triangle formed by $P(x_1, y_1)$, $Q\left(-\frac{C}{A}, 0\right)$ and $R\left(0, -\frac{C}{B}\right)$

$Area\ of\ \Delta PQR = \frac{1}{2}\left|x_1\left(0 + \frac{C}{B}\right) - \frac{C}{A}\left(-\frac{C}{B} - y_1\right) + 0\,(y_1 - 0)\right|$

$Area\ of\ \Delta PQR = \frac{1}{2}\left|\frac{Cx_1}{B} + \frac{Cy_1}{A} + \frac{C^2}{AB}\right|$

$Area\ of\ \Delta PQR = \frac{1}{2}\left|\frac{ACx_1+BCy_1+C^2}{AB}\right|$

$2(Area\ of\ \Delta PQR)$

$= \left|\frac{C}{AB}\right| |Ax_1 + By_1 + C|$ ……….(2)

Distance Between Q and R is

$QR = \sqrt{\left(0 + \frac{C}{A}\right)^2 + \left(-\frac{C}{B} - 0\right)^2}$

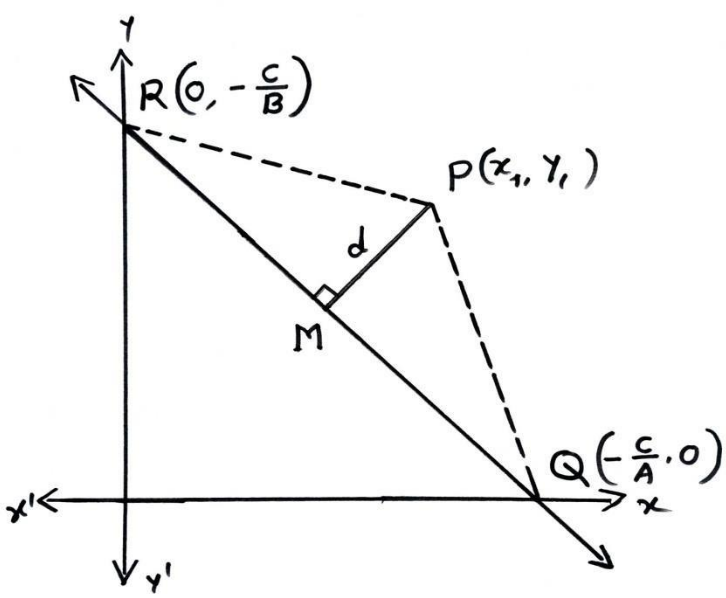
$QR = \sqrt{\frac{C^2}{A^2} + \frac{C^2}{B^2}}$

$QR = \left|\frac{C}{AB}\right| \sqrt{A^2 + B^2}$ …………(3)

substitute eq(2)and eq(3) in eq(1)

$PM = \frac{\left|\frac{C}{AB}\right| |Ax_1+By_1+C|}{\left|\frac{C}{AB}\right|\sqrt{A^2+B^2}}$

$d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$



48. Prove geometrically that $\lim_{x\to 0}\frac{\sin x}{x} = 1$, where x being measured in radians

Ans: $A\hat{O}B = x$ (x being measured in radian)
$OA = OC$ radius of circle
AB and CD are perpendicular to OA then join AC
We get, $\frac{1}{2}Area\ of\ \Delta AOC < \frac{1}{2}Area\ of\ Sector\ AOC < \frac{1}{2}Area\ of\ \Delta AOB$

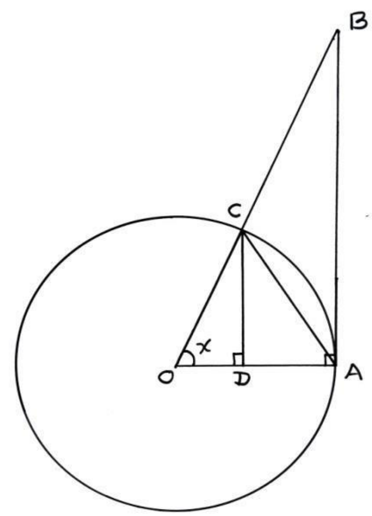
$$\frac{1}{2}(OA)(CD) < \frac{1}{2}(OA)^2x < \frac{1}{2}(OA)(AB)$$

$$(CD) < (OA)x < (AB) \text{.................}(1)$$

In ΔDOC
$\sin x = \frac{CD}{OC}$
$CD = (OC)\sin x$

In ΔAOB
$\tan x = \frac{AB}{OA}$
$AB = (OA)\tan x$

Then equation (1) Becomes



$(OC)\sin x < (OA)x < (OA)\tan x$
$(OA)\sin x < (OA)x < (OA)\tan x$
$\sin x < x < \tan x$ dividing $\sin x$
$\frac{\sin x}{\sin x} < \frac{x}{\sin x} < \frac{\tan x}{\sin x}$
$1 < \frac{x}{\sin x} < \frac{\sin x}{\cos x}\cdot\frac{1}{\sin x}$
$1 < \frac{x}{\sin x} < \frac{1}{\sin x}$ taking reciprocal
$1 > \frac{\sin x}{x} > \cos x$ apply limit as $x \to 0$
$\lim_{x\to 0} 1 > \lim_{x\to 0}\frac{\sin x}{x} > \lim_{x\to 0}\cos x$
$1 > \lim_{x\to 0}\frac{\sin x}{x} > \cos(0)$
$1 > \lim_{x\to 0}\frac{\sin x}{x} > 1$
Therefore $\lim_{x\to 0}\frac{\sin x}{x} = 1$

49. Find the mean deviation about mean for the following data

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Ans:

x_i	f_i	f_ix_i	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	$N=\sum f_i = 25$	$\sum f_ix_i = 350$		$\sum f_i\lvert x_i-\bar{x}\rvert = 158$

$N = \sum f_i = 25$
Now $\bar{x} = \frac{1}{N}\sum f_i x_i$
$= \frac{1}{25}(350)$
$= 14$
Now $MD(\bar{x}) = \frac{1}{N}\sum f_i |x_i - \bar{x}|$
$= \frac{1}{25}(158)$
$= 6.25$

50. A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the probability that it will be (i) red, (ii) yellow, (iii) blue, (iv) not blue

Ans: Total discs $= 9$

$R =$Red discs $= 4$

$B =$Blue discs $= 3$

$Y =$Yellow discs $= 2$

$P(R) = \frac{4}{9}$

$P(Y) = \frac{2}{9}$

$P(B) = \frac{3}{9} = \frac{1}{3}$

$P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

PART-E

Answer the following questions :

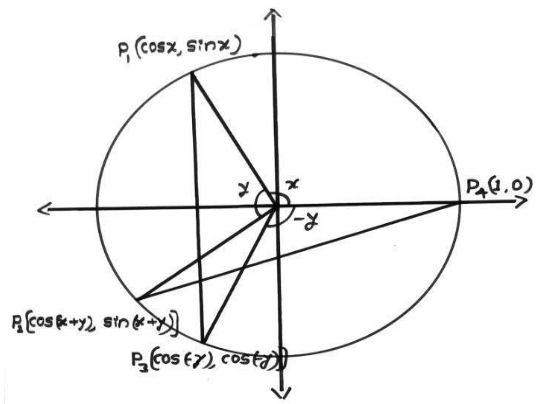
51. Prove geometrically that $\boldsymbol{cos(x+y) = cosx \cdot cosy - sinx \cdot siny}$ **(6M)**

Ans: $P_1\hat{O}P_4 = x$

$P_1\hat{O}P_2 = y$

$P_2\hat{O}P_4 = x + y$

$P_3\hat{O}P_4 = -y$



Coordinates of points P_1, P_2, P_3 and P_4

$P_1(cosx, sinx)$

$P_2(cos(x+y), sin(x+y))$

$P_3(cos(-y), sin(-y))$

$P_4(1,0)$

Distance between two points (x_1, y_1) and (x_2, y_2)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between P_1P_3

$P_1P_3 = \sqrt{(cosx - cos(-y))^2 + (sinx - sin(-y))^2}$ squaring on both side

$(P_1P_3)^2 = (cosx - cosy)^2 + (sinx + siny)^2$ $\quad cos(-\theta) = cos\theta, sin(-\theta) = -sin\theta$

$(P_1P_3)^2 = cos^2x + cos^2y - 2cosx \cdot cosy + sin^2x + sin^2y + 2sinx \cdot siny$

$(P_1P_3)^2 = (cos^2x + sin^2x) + (cos^2y + sin^2y) - 2[cosx \cdot cosy - sinx \cdot siny]$

$(P_1P_3)^2 = 1 + 1 - 2[cosx \cdot cosy - sinx \cdot siny]$

$(P_1P_3)^2 = 2 - 2[cosx \cdot cosy - sinx \cdot siny]$......................(1)

Distance between P_2P_4

$P_2P_4 = \sqrt{(cos(x+y) - 1)^2 + (sin(x+y) - 0)^2}$ squaring on both side

$(P_2P_4)^2 = (cos(x+y) - 1)^2 + sin^2(x+y)$

$(P_2P_4)^2 = cos^2(x+y) + 1 - 2cos(x+y) + sin^2(x+y)$

$(P_2P_4)^2 = (cos^2(x+y) + sin^2(x+y)) + 1 - 2cos(x+y)$

$(P_2P_4)^2 = 1 + 1 - 2cos(x+y)$

$(P_2P_4)^2 = 2 - 2cos(x+y)$......................(2)

Now $\Delta P_1OP_3 \cong \Delta P_2OP_4$ (Congruent Triangle)

Corresponding sides and corresponding angles are equal

Therefore $P_2P_4 = P_1P_3$ squaring on both side

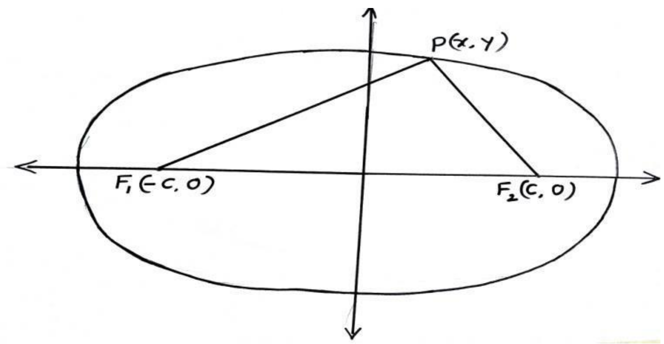
$(P_2P_4)^2 = (P_1P_3)^2$ from (1) and (2)

$2 - 2cos(x+y) = 2 - 2[cosx \cdot cosy - sinx \cdot siny]$

$cos(x+y) = cosx \cdot cosy - sinx \cdot siny$

OR

Derive equation of ellipse in the standard form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (6M)



Ans: Foci : $(\pm c, 0)$

$P(x, y)$ is a point on ellipse

Major axis $= 2a$

Minor axis $= 2b$

Relationship between a,b and c

$a^2 = c^2 + b^2$

$PF_1 = \sqrt{(x+c)^2 + y^2}$

$PF_2 = \sqrt{(x-c)^2 + y^2}$

Now By using definition of ellipse $PF_1 + PF_2 = 2a$

$\sqrt{(x+c)^2+y^2} + \sqrt{(x-c)^2+y^2} = 2a$

$\sqrt{(x+c)^2+y^2} = 2a - \sqrt{(x-c)^2+y^2}$

squaring on both side

$\left(\sqrt{(x+c)^2+y^2}\right)^2 = \left(2a - \sqrt{(x-c)^2+y^2}\right)^2$

$(x+c)^2 + y^2 = 4a^2 + [(x-c)^2 + y^2] - 4a\sqrt{(x-c)^2+y^2}$

$x^2 + c^2 + 2cx + y^2 = 4a^2 + x^2 + c^2 - 2cx + y^2 - 4a\sqrt{(x-c)^2+y^2}$

$4a\sqrt{(x-c)^2+y^2} = 4a^2 - 4cx$

$\sqrt{(x-c)^2+y^2} = a - \frac{cx}{a}$

squaring on both side

$\left(\sqrt{(x-c)^2+y^2}\right)^2 = \left(a - \frac{cx}{a}\right)^2$

$(x-c)^2 + y^2 = a^2 + \frac{c^2x^2}{a^2} - 2cx$

$x^2 + c^2 - 2cx + y^2 = a^2 + \frac{c^2x^2}{a^2} - 2cx$

$x^2 + c^2 + y^2 = a^2 + \frac{c^2x^2}{a^2}$

$x^2 - \frac{c^2x^2}{a^2} + y^2 = a^2 - c^2$

$\frac{x^2(a^2-c^2)}{a^2} + y^2 = (a^2 - c^2)$ dividing $(a^2 - c^2)$ on both side

$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$ w.k.t $a^2 - c^2 = b^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Conversely, if $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the equation of ellipse then $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$

$PF_1 = \sqrt{(x+c)^2+y^2} = a + \frac{cx}{a}$

$PF_2 = \sqrt{(x-c)^2+y^2} = a - \frac{cx}{a}$

Then $PF_1 + PF_2 = a + \frac{cx}{a} + a - \frac{cx}{a}$

$PF_1 + PF_2 = 2a$

Therefore equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

52. Find the sum of 'n' terms of sequence : $8, 88, 888, \ldots$ **(4M)**

Ans: $S_n = 8 + 88 + 888 + 8888 + \cdots$

$S_n = 8[1 + 11 + 111 + 1111 + \cdots]$

$S_n = \frac{8}{9} \times 9\left[1 + 11 + 111 + 1111 + \cdots\right]$

$S_n = \frac{8}{9}[9 + 99 + 999 + 9999 + \cdots]$

$S_n = \frac{8}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + (10000 - 1) + \cdots]$

$S_n = \frac{8}{9}[(10 + 100 + 1000 + 10000 + \cdots) - (1 + 1 + 1 + 1 + \cdots)]$

$S_n = \frac{8}{9}[(10^1 + 10^2 + 10^3 + 10^4 + \cdots) - (1 + 1 + 1 + 1 + \cdots)]$

$S_n = \frac{8}{9}[\frac{10(10^n - 1)}{10 - 1} - n]$

$S_n = \frac{8}{9}[\frac{10(10^n - 1)}{9} - n]$

OR

Find the derivative of $\frac{cosx}{1+sinx}$ w.r.t x **(4M)**

Ans: $y = \frac{cosx}{1+sinx}$ diff w.r.t x

$\frac{dy}{dx} = \frac{(1+sinx)(-sinx) - cosx(0+cosx)}{(1+sinx)^2}$

$\frac{dy}{dx} = \frac{-sinx - sin^2x - cos^2x}{(1+sinx)^2}$

$\frac{dy}{dx} = \frac{-sinx - (sin^2x + cos^2x)}{(1+sinx)^2}$

$\frac{dy}{dx} = \frac{-sinx - 1}{(1+sinx)^2}$

$\frac{dy}{dx} = \frac{-1 - sinx}{(1+sinx)^2}$

$\frac{dy}{dx} = \frac{-(1+sinx)}{(1+sinx)^2}$

$\frac{dy}{dx} = \frac{-1}{(1+sinx)}$

AS PER NEW PATTERN 2023-2024

TOP SCORER POCKET MARKS PACKAGE

FEATURES OF THE BOOK

PUC II YEAR MATHEMATICS

- **Blue print** of the Question Paper and **Question Paper Pattern**
- Chapter wise detailed solutions of
  - **Multiple Choice Questions** (**MCQ**)
- Chapter wise detailed solutions of
  - **Fill in the Blanks (FB)**
- Chapter wise Question Papers (Test Papers)
  - **For FIRST UNIT TEST and SECOND UNIT TEST**
  - **PROJECTS/ACTIVITY/ASSIGNEMENT**
- **Passing Package and Scoring Package**
- Different Set of Question Papers (Prepared by experts)
  - **10 Set of SAMPLE QUESTION PAPER**
  - **10 Set of PRACTICE QUESTION PAPER**
- Chapter wise detailed solutions of All the Previous
  - **Annual Examination/ Supplementary Examination/**
  - **Preparatory Examination/ Expected questions**
- **CET Question Paper Analysis**
  - Chapter wise weightage
  - Year wise analysis
  - How to score more in CET

SPECIAL NOTE :

For Annual Examination, the most possible Questions are there in this **TOP SCORER POCKET MARKS PACKAGE** book. If you practice all the questions from this Booklet, you will get **100/100 marks** in Annual examination for sure. (Included theory and project/activity/assignment)

AS PER NEW PATTERN 2023-2024

In this booklet, TOP SCORER POCKET MARKS PACKAGE

QUESTION PAPERS	NOTATION	TOTAL
MODEL QUESTION PAPERS	MQP-01, MQP-02, MQP-03, MQP-04, MQP-05, MQP-06, MQP-07, MQP-08	**8**
ANNUAL EXAM QUESTION PAPERS	MARCH-2014, MARCH-2015, MARCH-2016, MARCH-2017, MARCH-2018, MARCH-2019 MARCH-2020, AUGUST-2021, MARCH-2022 MARCH-2023	**10**
SUPPLEMENTARY QUESTION PAPERS	JUNE-2014, JUNE-2015, JUNE-2016, JUNE-2017, JUNE-2018, JUNE-2019 JUNE-2020, SEPTEMBER-2022,JUNE-2023, AUGUST-2023	**10**
STATE LEVEL PREPARATORY QUESTION PAPERS	PQP-01, PQP-02, PQP-03, PQP-04, PQP-05, PQP-06, PQP-07, PQP-08, PQP-09, PQP-10,	**10**
DISTRICT LEVEL PREPARATORY QUESTION PAPERS	D-PQP-1, D-PQP-2, D-PQP-3, D-PQP-4 D-PQP-5, ... D-PQP-29, D-PQP-30	**30**
LATEST MODEL QUESTION PAPERS	2019MQP-1, 2019MQP-2, 2019MQP-3 2021MQP-1, 2021MQP-2, 2022MQP-1, 2023MQP-2 2024MQP-1	**8**
PRACTICE QUESTION PAPERS PREPARED BY EXPERTS BASED ON NEW PATTERN 2023-2024	EPQP-01, EPQP-02, EPQP-03, EPQP-04, EPQP-05, EPQP-06, EPQP-07, EPQP-08 EPQP-09, EPQP-10	**10**
SAMPLE QUESTION PAPERS PREPARED BY EXPERTS BASED ON NEW PATTERN 2023-2024	SQP-01, SQP-02, SQP-03, SQP-04, SQP-05, SQP-06, SQP-07, SQP-08, SQP-09, SQP-10,	**10**
MOST LIKELY EXPECTED QUESTIONS WITH ANSWERS PREPARED BY EXPARTS		**25**
TOTAL QUESTION PAPERS WITH CHAPTERWISE SOLUTION		**131**