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Karnataka Common Entrance Test

KCET

Mathematics

10

Practice Tests

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MATHEMATICS

Karnataka Common Entrance Test - KCET

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Practice Test 01

1. If the feasible region for a solution of linear inequations is bounded, it is called as:

(a) Concave Polygon
(b) Finite Region
(c) Convex Polygon
(d) None of the above

2. The corner points of the feasible region determined by the system of linear constraints are $(0,10),(5,5),(25,20)$ and $(0,30)$. Let $Z=px+qy$, where $p,q>0$. Condition on p and q so that the maximum of Z occurs at both the points $(25,20)$ and $(0,30)$ is:

(a) $5p=2q$ (b) $2p=5q$
(c) $p=2q$ (d) $5p=q$

3. Find the maximum value of $4x+7y$ with the conditions $3x+8y\le 24, y\le 2, x\ge 0$ and $y\ge 0$.

(a) 32 (b) 42
(c) 39 (d) 30

4. What is the number of different messages that can be represented by three a's and two b's?

(a) 7 (b) 8
(c) 9 (d) 10

5. If ${}^{n}C_{15}={}^{n}C_{8}$, then find the value of n.

(a) 24 (b) 23
(c) 21 (d) 20

6. $\mathrm{n}(\mathrm{n}+1)(\mathrm{n}+5)$ is a multiple of 3 is true for:

(a) All natural numbers $\mathrm{n}>5$
(b) Only natural number $3\le \mathrm{n}<15$
(c) All natural numbers n
(d) Only natural number $-3\le \mathrm{n}<5$

7. For all positive integrals $10^{\mathrm{n}}+3^{4\mathrm{n}+2}+8$ is divisible by:

(a) 8 (b) 9
(c) 7 (d) 6

8. The negation of the statement "The product of 3 and 4 is 9" is:

(a) It is false that the product of 3 and 4 is 9
(b) The product of 3 and 4 is 12
(c) The product of 3 and 4 is not 12
(d) It is false that the product of 3 and 4 is not 9

9. What is the value of the determinant $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15} \end{vmatrix}$ where $i=\sqrt{-1}$?

(a) 0 (b) -2
(c) $4i$ (d) $-4i$

10. The factorized form of the following determinant is:

$$\begin{vmatrix} 1 & l & l^2 \\ 1 & m & m^2 \\ 1 & n & n^2 \end{vmatrix}$$

(a) $(m-n)(n-1)(n)$
(b) $(m-l)(n-l)(n-m)$
(c) $(l-m)(n-l)(n-m)$
(d) $(m-1)(n-1)(n-1)$

11. Find the value of $y-x$ from the following equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

(a) 7 (b) -7
(c) 6 (d) -6

12. If $A=\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, find $(AB)^T$.

(a) $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & -4 \end{bmatrix}$

13. Form the differential equation of the following $y^2=a\left(b^2-x^2\right)$:

(a) $yy'-xyy''+x(y')^2=0$
(b) $yy'+xyy''+x(y')^2=0$
(c) $yy'+yy''-x(y')^2=0$
(d) $yy'-xyy''-x(y')^2=0$

14. The integrating factor of the differential equation $2y\dfrac{dx}{dy}+x=5y^2$ is, $(y\ne 0)$:

(a) $\sqrt{y}$ (b) y^2
(c) y (d) $\dfrac{1}{\sqrt{y}}$

15. Solve: $x\dfrac{dy}{dx}-y=x^2$ for $y(2)$, given $y(1)=1$

(a) 1 (b) 2
(c) 3 (d) 4

16. If $\lim_{x\to 0}\dfrac{\log(1+\sin x)}{x}=k$, the value of k is:

(a) 0 (b) $\dfrac{3}{2}$
(c) $\dfrac{-3}{2}$ (d) 1

17. If $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, then $\dfrac{dy}{dx}=?$

(a) $\dfrac{b^2x}{a^2y}$ (b) $-\dfrac{b^2x}{a^2y}$
(c) $-\dfrac{b^2y}{a^2x}$ (d) $\dfrac{b^2y}{a^2x}$

18. Evaluate the integral $\int_0^{\frac{\pi}{4}}\sin^3 2t\cos 2t\,dt$.

(a) $\frac{1}{8}$ (b) $\frac{2}{8}$
(c) $\frac{1}{7}$ (d) $\frac{1}{9}$

19. The area bounded by $y=\log x, x$-axis and ordinates $x=1, x=2$ is:

(a) $\frac{1}{2}(\log 2)^2$ sq. unit
(b) $\log\left(\frac{2}{e}\right)$ sq. unit
(c) $\log\left(\frac{4}{e}\right)$ sq. unit
(d) $\log 4$ sq. unit

20. Find the area under the curve between $\mathrm{y}=\mathrm{x}$ and $\mathrm{y}=2\mathrm{x}+6$.

(a) 72 (b) 18
(c) 36 (d) 54

21. The roots of the equation $x^2+\dfrac{x}{\sqrt{3}}+1=0$ are:

(a) Imaginary
(b) Real and equal
(c) Real and distinct
(d) Real and distinct

22. For which value(s) of k will the roots of $3x^2+3=2kx$ be real and equal?

(a) ± 2 (b) ± 4
(c) ± 3 (d) ± 5

23. The value of x for which $|x+1|+\sqrt{(x-1)}=0$

(a) 0 (b) 1
(c) -1 (d) No value of x

24. If $2(3x-4)-2<4x-2\ge 2x-4$; then the possible value of x can be:

(a) 2 (b) 5
(c) -4 (d) -5

25. How many three- digits numbers are there which are divisible by 9.

(a) 98 (b) 99
(c) 100 (d) 101

26. How many two-digit numbers are divisible by 4?

(a) 21 (b) 22
(c) 24 (d) 25

27. Find the radius of the circle, $5x^2+5y^2-20x-6y+15=0$.

(a) $\dfrac{\sqrt{34}}{5}$ (b) $\dfrac{\sqrt{37}}{5}$

(c) $\frac{6}{5}$ (d) $\sqrt{\frac{6}{5}}$

28. Find the equation of the parabola with vertex at the origin, the axis along the x-axis and passing through the point P(3, 4).

(a) $y^2 = \frac{4}{3}x$ (b) $y^2 = \frac{3}{16}x$

(c) $y^2 = -\frac{9}{16}x$ (d) $y^2 = \frac{16}{3}x$

29. Which one of the following is correct?

(a) The function is one-one into

(b) The function is many-one into

(c) The function is one-one onto

(d) The function is many-one onto

30. Which of the following functions, $f : R \to R$ is one-one?

(a) $\text{f(x)} = |\text{x}|, \forall \text{x} \in \text{R}$

(b) $f(x) = x^2, \forall x \in R$

(c) $f(x) = -x, \forall x \in R$

(d) None of these

31. Let X be a binomial random variable with mean 1 and variance $\frac{3}{4}$. The probability that X takes the value of 3 is:

(a) $\frac{3}{64}$ (b) $\frac{3}{16}$

(c) $\frac{27}{64}$ (d) $\frac{3}{4}$

32. What is the probability of getting a sum 9 from two throws of a dice?

(a) $\frac{1}{9}$ (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

33. If β is perpendicular to both $\vec{\alpha}$ and $\vec{\gamma}$ where $\vec{\alpha} = \hat{k}$ and $\vec{\gamma} = 2\hat{\text{i}} + 3\hat{\text{j}} + 4\text{k}$, then what is β equal to?

(a) $3\hat{\imath} + 2\hat{\jmath}$ (b) $-3\hat{\imath} + 2\hat{\jmath}$

(c) $2\hat{\imath} - 3\hat{\jmath}$ (d) $-2\hat{\imath} + 3\hat{\jmath}$

34. For any vector α, what is the value of $(\alpha.\hat{\imath})\hat{\imath} + (\alpha.\hat{\jmath})\hat{\jmath} + (\alpha.\hat{k})\hat{k}$

(a) 3α (b) 2α

(c) α (d) $-\alpha$

35. Find the coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$.

(a) 567 (b) 990

(c) 365 (d) 459

36. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^{15}$, the constant term, is:

(a) $^{15}C_9$ (b) 0

(c) - $^{15}C_9$ (d) 1

37. What is the equation of the right bisector of the lines segments joining $(1, 1)$ and $(2, 3)$?

(a) $2x + 4y - 11 = 0$

(b) $2x - 4y - 5 = 0$

(c) $2x - 4y - 11 = 0$

(d) $x - y + 1 = 0$

38. The angle between the lines x – 2y = y and y – 2x = 5 is:

(a) $\tan^{-1}(\frac{1}{4})$ (b) $\tan^{-1}(\frac{3}{5})$

(c) $\tan^{-1}(\frac{5}{4})$ (d) $\tan^{-1}(\frac{2}{3})$

39. What is $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ equal to?

(a) $\sin^{-1} x$ (b) $2\cot^{-1} x$

(c) $2\tan^{-1} x$ (d) $\tan^{-1} x$

40. If $x = \tan^{-1}(\frac{1}{5})$ then $\sin 2x$ is equal to?

(a) $\frac{4}{13}$

(b) $\frac{5}{13}$

(c) $\frac{12}{13}$

(d) None of the above

41. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x?

(a) 10 (b) 12

(c) 13 (d) 15

42. In any discrete series (when all values are not same) if x represent mean deviation about mean and y represent standard deviation, then which one of the following is correct?

(a) $y \geq x$ (b) $y \leq x$

(c) $x = y$ (d) $x < y$

43. A bag contains $2n + 1$ coins, n coins have tails on both sides, whereas $n + 1$ coins are fair. A coin is picked on random from the bag and tossed. If the probability that toss in tail is $\frac{31}{42}$, total numbers of coins in the bag are:

(a) 20 (b) 21

(c) 25 (d) 33

44. Weather Forecast Company makes a forecast of raining at 70%. Company's forecast are only correct 60% of the time. Find the probability of it correctly forecasting rain?

(a) $\frac{21}{50}$ (b) $\frac{23}{50}$

(c) $\frac{26}{50}$ (d) $\frac{29}{50}$

45. If $6\sin^2 x - 2\cos^2 x = 4$, then find the value of $\tan x$.

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) $\sqrt{5}$ (d) 0

46. If $\cos^{-1}\left(\frac{p}{a}\right) + \cos^{-1}\left(\frac{q}{b}\right) = \alpha$, then $\frac{p^2}{a^2} + k\cos\alpha + \frac{q^2}{b^2} = \sin^2\alpha$ where k is equal to:

(a) $-\frac{2pq}{ab}$ (b) $\frac{2pq}{ab}$

(c) $-\frac{pq}{ab}$ (d) $\frac{pq}{ab}$

47. Find the equation of tangent to the curve $y = \sqrt{5x - 3} - 2$, which is parallel to the line $4x - 2y + 3 = 0$?

(a) $80x - 40y + 103 = 0$

(b) $80x + 40y + 103 = 0$

(c) $80x + 40y - 103 = 0$

(d) $80x - 40y - 103 = 0$

48. Find the points on the curve $y = x^2$ at which the slope of the tangent is equal to the y coordinate of the point.

(a) $(0, 0)$ and $(2, 3)$

(b) $(0, 0)$ and $(3, 4)$

(c) $(0, 0)$ and $(2, 4)$

(d) $(0, 1)$ and $(2, 4)$

49. The 6^{th} coefficient in the expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$

(a) $\frac{4580}{17}$

(b) $-\frac{896}{27}$

(c) $\frac{5580}{17}$

(d) None of these

50. Evaluate $\lim_{x\to 0} \frac{\log(1+2x)}{\tan 2x}$

(a) -1 (b) 1

(c) 2 (d) 4

51. What is the value of $\lim_{x\to 0} \frac{\left(e^{4x^2} - 1\right)}{x \sin x}$?

(a) 1 (b) 6

(c) 4 (d) 8

52. The coordinates of the foot of the perpendicular drawn from the point A(1,0,3) to the join of the points B(4,7,1) and C(3,5,3) are:

(a) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

(b) $(5, 7, 17)$

(c) $\left(\frac{5}{3}, -\frac{7}{3}, \frac{17}{3}\right)$

(d) $\left(-\frac{5}{3}, \frac{7}{3}, -\frac{17}{3}\right)$

53. Three planes x + y = 0, y + z = 0, and x + z = 0:

(a) Meet in a line

(b) Meet in a unique point

(c) Meet taken two at a time in parallel lines

(d) None of these

54. A set containing n elements, has exactly ________ subsets.

(a) n^2 (b) 2^n

(c) n (d) $n+1$

55. The domain of the function $f(x)=\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$ is:

(a) $\{x \mid x<1\}$ (b) $\{x \mid x>-1\}$

(c) $[0,1]$ (d) $[-1,1]$

56. If P_2Q and R are three sets, then which of the following is correct?

(a) $P\cup(Q\cap R)=(P\cup Q)\cap(P\cap R)$

(b) $P\cap(Q\cup R)=(P\cup Q)\cap(P\cup R)$

(c) $P\cup(Q\cap R)=(P\cup Q)\cap(P\cup R)$

(d) $P\cap(Q\cup R)=(P\cap Q)\cap(P\cap R)$

57. Of the members of three athletic teams in a school, 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, 12 play football and cricket and 8 play all three games. The total number of members in the three athletic teams is:

(a) 76 (b) 49

(c) 43 (d) 41

58. If $f(2a-x)=f(x)$ and $\int_0^a f(x)dx=\lambda$ then $\int_0^{2a} f(x)dx$ is:

(a) 2λ (b) 0

(c) 2λ (d) λ

59. XY-plane divides the line joining the points $A(2,3,-5)$ and $B(-1,-2,-3)$ in the ratio:

(a) 2 : 1 internally

(b) 3 : 2 externally

(c) 5 : 3 internally

(d) 5 : 3 externally

60. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z-3=0$.

(a) $\sin^{-1}\left(\frac{8}{21}\right)$

(b) $\sin^{-1}\left(\frac{2}{21}\right)$

(c) $\sin^{-1}\left(\frac{5}{21}\right)$

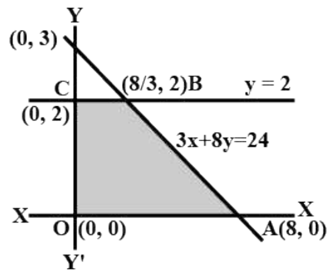
(d) None of these

// Hints and Solutions //

1(C). A bounded feasible region will have both a maximum value and a minimum value for the objective function. It is bounded if it can be enclosed in any shape. A convex polygon is a simple not self-intersecting closed shape in which no line segment between two points on the boundary ever goes outside the polygon.
So, the answer is convex polygon.

2(A). Maximum of Z occurs at $(25,20)$ and at $(0,30)$.
So, equating the vales of Z at these points, we get
$25p+20q=30q$
$25p=10q$
$\therefore 5p=2q$
This is the required relation.
Also as $p,q>0$, the value of Z is always positive and hence, is greater at $(25,20)$ and at $(0,30)$ than at $(0,10)$ and $(5,5)$.

3(A). Given condition is, $3x+8y\le 24$
$y\le 2, x\ge 0, y\ge 0$
The vertices of the feasible region are,
$(0,0),(8,0),\left(\frac{8}{3},0\right)$ and $(0,2)$
Find the value of Z at all points,
At O $(0,0)$, $Z=4\times 0+7\times 0=0$
At A $(8,0)$, $Z=4\times 8+7\times 0=32$
At B $\left(\frac{8}{3},0\right)$, $Z=4\times\frac{8}{3}+7\times 2=\frac{74}{3}$
At C $(0,2)$, $Z=4\times 0+7\times 2=14$
The maximum value of the objective function attains at $(8,0)$.
$Z=4x+7y$
The maximum value $=4\times 8+7\times 0=32$.



4(D). We know that:
Suppose a set of n objects has n_1 of one kind of object, n_2 of a second kind, n_3 of a third kind, and,
So, on with $n=n_1+n_2+n_3+\ldots+n_k$ then the number of distinguishable permutations of the n objects is:
$=\frac{n!}{n_1!\times n_2!\times n_3!\ldots\ldots n_k!}$
Given: Three a's and two b's

a	a	a	b	b

Total number $=3+2=5$
In the set of 5 words has 3 words of one kind and 2 words of the second kind.
Therefore, number of different messages that can be represented by three a's and two b's,
$=\frac{5!}{3!2!}=10$

5(B). Given that:
${}^nC_{15}={}^nC_8$
As we know that,
If ${}^nC_x={}^nC_y$, then,
$x+y=n$
Therefore,
$n=15+8=23$

6(C). Given:
$P(n): n(n+1)(n+5)$ is a multiple of 3.
For $n=1$
$n(n+1)(n+5)=1.2\cdot 6=12=3.4$
$P(n)$ is true for $n=1$
Suppose $p(k)$ is true for $n=k$
$k(k+1)(k+5)=3m$ (let) or
$k^3+6k^2+5k=3m$ ....(i)
Replacing k by $k+1$, we get
$(k+1)(k+2)(k+6)=k\left(k^2+8k+12\right)+\left(k^2+8k+12\right)$
$k^3+9k^2+20k+12=\left(k^3+6k^2+5k\right)+\left(3k^2+15k+12\right)$
$=3m+3k^2+15k+12$ [.......from (i)]
$=3\left(m+k^2+5k+4\right)$
$(k+1)(k+2)(k+6)$ is a multiple of 3 i.e., $P(k+1)$ is multiple of 3, if $P(k)$ is a multiple of 3 i.e., $P(k+1)$ is true whenever $P(k)$ is true.
So, $P(n)$ is true for all $n\in N$.

7(B). Given:
$10^n+3^{4n+2}+8$
Put $n=1$ in $10^n+3^{4n+2}+8$.
$\therefore 10^1+3^{4+2}+8=10+729+8$
$=747$
$747=3\times 3\times 83$
Prime factors of 747 are $3,3,83$.
From the given options we can say that $10^n+3^{4n+2}+8$ is divisible by 9 (3×3) for all positive values of n.

8(A). Given, the statement is:
The product of 3 and 4 is 9.
To find the negation of the statement, we find the opposite of the conclusion.
Then, the negation of the statement is:
It is false that the product of 3 and 4 is 9.

9(D). Given,
Determinant is $\begin{vmatrix} i & i^2 & i^3 \\ i^4 & i^6 & i^8 \\ i^9 & i^{12} & i^{15}\end{vmatrix}$.
Since, we have,
$i=\sqrt{-1}$
$\therefore i^2=-1, i^3=-i, i^4=1, i^6=-1, i^8=1, i^9=i, i^{12}=1,$
and $i^{15}=-i$
$=\begin{vmatrix} i & -1 & -i \\ 1 & -1 & 1 \\ i & 1 & -i\end{vmatrix}$
$=i(i-1)+1(-i-i)-i(1+i)$
$=i^2-i-2i-i-i^2$
$=-4i$

10(B). Given,
$\begin{vmatrix} 1 & l & l^2 \\ 1 & m & m^2 \\ 1 & n & n^2\end{vmatrix}$
Applying $R_2\to R_2-R_1$,

$\begin{vmatrix} 1 & l & l^2 \\ 0 & m-l & m^2-l^2 \\ 1 & n & n^2 \end{vmatrix}$

Applying $R_3 \to R_3 - R_1$,

$\begin{vmatrix} 1 & l & l^2 \\ 0 & m-l & m^2-l^2 \\ 0 & n-l & n^2-l^2 \end{vmatrix}$

$\begin{vmatrix} 1 & l & l^2 \\ 0 & m-l & (m-l)(m+l) \\ 0 & n-l & (n-l)(n+l) \end{vmatrix}$

$(m-l)(n-l)\begin{vmatrix} 1 & l & l^2 \\ 0 & 1 & (m+l) \\ 0 & 1 & (n+l) \end{vmatrix}$

Now, expanding from a_{11},

$(m-l)(n-l)\cdot 1.\begin{bmatrix} 1 & m+l \\ 1 & n+l \end{bmatrix}$

$=(m-l)(n-l)(n+l-m-l)$

$=(m-l)(n-l)(n-m)$

11(A). Given,

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

As we know that,
If two matrices A and B are equal then their corresponding elements are also equal.

$\therefore 2x+3=7$
$\Rightarrow 2x=7-3$
$\Rightarrow 2x=4$
$\Rightarrow x=2$
And $2y-4=14$
$\Rightarrow 2y=14+4$
$\Rightarrow 2y=18$
$\Rightarrow y=9$

Now,
$y-x=9-2=7$
So, the value of $y-x$ is 7 .

12(B). Given,

$A=\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & -1 \end{bmatrix}\times\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix} 1-3+0 \\ 3+6-5 \end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix} -2 \\ 4 \end{bmatrix}$

As we know,
The new matrix obtained by interchanging the rows and columns of the original matrix is called as the transpose of the matrix. It is denoted by A' or A^T.

$\therefore (AB)^T=\begin{bmatrix} -2 & 4 \end{bmatrix}$

13(D). Given,
$y^2=a\left(b^2-x^2\right)$
Differentiating w.r.t x
We get,
$2yy'=a(-2x)$
$\Rightarrow yy'=-ax$ .....(i)
Differentiating w.r.t x again
$yy''+(y')^2=-a$ .....(ii)
From (i) and (ii), we get,
$yy'=x\left(yy''+(y')^2\right)$
$\Rightarrow yy'-xyy''-x(y')^2=0$

14(A). Given,
$2y\frac{dx}{dy}+x=5y^2\ldots(i)$
Equation (i) can be simplified as,
$\frac{dx}{dy}+\frac{x}{2y}=\frac{5}{2}y$
On comparing eqn (i) with standard eqn,
$\frac{dx}{dy}+Px=Q$,
We get
$P=\frac{1}{2y}$ and $Q=\frac{5}{2}y$
Therefore,
$IF=e^{\int Pdy}=e^{\int\frac{1}{2y}dy}$
$\Rightarrow IF=e^{\frac{1}{2}\log y}=e^{\log y^{\frac{1}{2}}}$
$\Rightarrow IF=\sqrt{y}\quad\left(\because e^{a\log x}=x^a\right)$

15(D). Given,
$y(1)=1$
$x\frac{dy}{dx}-y=x^2$
$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=x$
It is linear differential equation is of first order.
$IF=e^{\frac{-1}{x}}dx$
$\Rightarrow IF=e^{-\ln x}$
$\Rightarrow IF=\frac{1}{x}$
Now,
$y\times(IF)=\int Q(IF)dx$
$\Rightarrow y\times\frac{1}{x}=\int x\times\frac{1}{x}dx$
$\Rightarrow \frac{y}{x}=\int dx$
Integrating,
$\frac{y}{x}=x+c$ (where c is integration constant)
$\Rightarrow \frac{1}{1}=1+c$
$\Rightarrow c=0$
$\frac{y}{x}=x$ Or $y=x^2$
For $y(2)$
$y=2^2$
$\Rightarrow y=4$

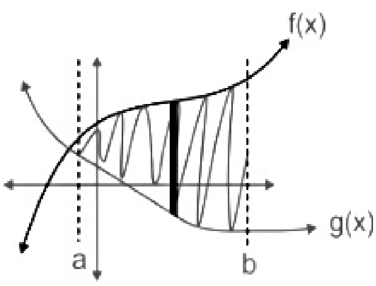
16(D). Given that:
$\lim_{x\to 0}\frac{\log(1+\sin x)}{x}=k$
Put x = 0, to check form
$\lim_{x\to 0}\frac{\log(1+\sin x)}{x}$
$=\frac{\log(1+0)}{0}$
$=\frac{0}{0}$
Applying L' hospital's rule as,
$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$

$\lim_{x\to 0}\frac{\log(1+\sin x)}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}(\log(1+\sin x))}{\frac{d}{dx}(x)}$
$=\lim_{x\to 0}\frac{\frac{1}{1+\sin x}\times\cos x}{1}$
$=\frac{1}{1+\sin 0}\times\cos 0$
$=1$
$\therefore k=1$

17(B). Given that:
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
Differentiating with respect to x, we get:
$\Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$
$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$
$\therefore \frac{dy}{dx}=-\frac{b^2x}{a^2y}$

18(A). Given, $\int_0^{\frac{\pi}{4}}\sin^3 2t\cos 2t dt$
Let,
$F(x)=\int\sin^3 2t\cos 2t dt$
Let $\sin 2t=u$
Differentiating w.r.t. t
$\frac{d(\sin 2t)}{dt}=\frac{du}{dt}$
$2\cos 2t=\frac{du}{dt}$
$dt=\frac{du}{2\cos 2t}$
Putting value of u and du in our integral
$\int\sin^3 2t\cos 2t dt=\int u^3\cos 2t\times\frac{du}{2\cos 2t}$
$=\frac{1}{2}\int u^3 du$
$=\frac{1}{2}\frac{u^{3+1}}{3+1}=\frac{1}{2}\frac{u^4}{4}=\frac{u^4}{8}$
Putting back $u=\sin 2t$
$=\frac{1}{8}\sin^4 2t$
Hence, $F(t)=\frac{1}{8}\sin^4 2t$
Now,
$\int_0^{\frac{\pi}{4}}\sin^3 2t\cos 2t=F\left(\frac{\pi}{4}\right)-F(0)$
$=\frac{1}{8}\sin^4 2\left(\frac{\pi}{4}\right)-\frac{1}{8}\sin^4 2(0)$
$=\frac{1}{8}\sin^4\frac{\pi}{2}-\frac{1}{8}\sin^4(0)$
$=\frac{1}{8}\times 1^4-\frac{1}{8}\times 0^4$
$=\frac{1}{8}\times 1-0$
$=\frac{1}{8}$

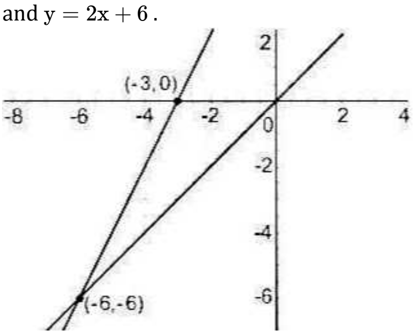
19(C). We know that:
Area bounded by function f(x) and g(x) is given as,



Area $=\int_a^b[f(x)-g(x)]dx=\int_a^b[$ Top $-$ bottom $]dx$

Given:
$y = \log x$
Then,
Area $= \int_1^2 \log x \, dx$
Applying by parts rule, we get:
$= [\log x \mathrm{x}]_1^2 - \int_1^2 \frac{1}{x} \times \mathrm{xdx}$
$= [\mathrm{x} \log \mathrm{x}]_1^2 - [\mathrm{x}]_1^2$
$= [2 \log 2 - \log 1] - [2 - 1]$
$= 2 \log 2 - 1$
$= \log 2^2 - \log \mathrm{e}$
$= \log 4 - \log \mathrm{e}$
$= \log \left(\frac{4}{e}\right)$ sq. unit

20(B). $\mathrm{y} = \mathrm{x}$ and $\mathrm{y} = 2\mathrm{x} + 6$
Finding a point of intersection:
$\Rightarrow \mathrm{x} = 2\mathrm{x} + 6$
$\Rightarrow \mathrm{x} = -6$ Thus, y = - 6.
Let us draw the graph of the curve $\mathrm{y} = \mathrm{x}$ and $\mathrm{y} = 2\mathrm{x} + 6$.



Let the enclosed area be A.
Using the formula of the area under the curve,
$\mathrm{A} = \int_a^b \mathrm{f(x)} - \mathrm{g(x)dx}$
$\Rightarrow \mathrm{A} = \int_{-6}^0 (2\mathrm{x} + 6 - \mathrm{x})\mathrm{dx}$
$= \int_{-6}^0 (\mathrm{x} + 6)\mathrm{dx}$
$= \left[\frac{\mathrm{x}^2}{2} + 6\mathrm{x}\right]_{-6}^0$
Substitute the limit to evaluate the area:
$\Rightarrow \mathrm{A} = 0 + 0 - \frac{36}{2} + 36 = 18$

21(A). Let us consider the standard form of a quadratic equation, $ax^2 + bx + c = 0$.
Discriminant $= D = b^2 - 4ac$
If the Discriminant > 0 then the roots are real and distinct.
If the Discriminant $= 0$ then the roots are real and equal.
If the Discriminant < 0 then the roots are Imaginary.
$x^2 + \frac{x}{\sqrt{3}} + 1 = 0$
$\Rightarrow \sqrt{3}x^2 + x + \sqrt{3} = 0$
Comparing this with the standard form $ax^2 + bx + c = 0$, we get $a = \sqrt{3}, b = 1$ and $c = \sqrt{3}$.
$\therefore D = b^2 - 4ac$
$= 1^2 - 4 \times \sqrt{3} \times \sqrt{3}$
$= 1 - 12 = -11$
$\therefore D < 0$
Thus, the roots are imaginary.

22(C). Quadratic equation is $ax^2 + bx + c$
Discriminant $D = b^2 - 4ac$
$D = 0$ means two real and both are identical roots.
Here, $3x^2 + 3 = 2kx$
$\Rightarrow 3x^2 - 2kx + 3 = 0$
Compare with standard form $ax^2 + bx + c$
$a = 3, b = -2k, c = 3$
Discriminant $\mathrm{D} = \mathrm{b}^2 - 4\mathrm{ac}$
$\Rightarrow \mathrm{D} = (-2k)^2 - 4(3)(3) = 4k^2 - 36$
For real and equal roots, $D = 0$.
$\therefore 4k^2 - 36 = 0$
$\Rightarrow 4\left(k^2 - 9\right) = 0$
$\Rightarrow k^2 - 9 = 0$
$\Rightarrow k = \pm 3$

23(D). Given, $|x + 1| + \sqrt{(x - 1)} = 0$, where each term is non-negative.
So, $|x + 1| = 0$ and $\sqrt{(x - 1)} = 0$ should be zero simultaneously.
i.e. $x = -1$ and $x = 1$, which is not possible.
So, there is no value of x for which each term is zero simultaneously.

24(A). Given:
$2(3x - 4) - 2 < 4x - 2 \geq 2x - 4$
First by solving the inequation: $2(3x - 4) - 2 < 4x - 2$ w e get,
$\Rightarrow 6x - 10 < 4x - 2$
$\Rightarrow 2x < 8$
$\Rightarrow x < 4$ .....(1)
Similarly, by solving the inequation $4x - 2 \geq 2x - 4$ we get,
$\Rightarrow 2x \geq -2$
$\Rightarrow x \geq -1$ .....(2)
From equation (1) and (2) we can say that $-1 \leq x < 4$
So, out of the given options the possible value which x can take is 2.

25(C). Three- digit numbers are divisible by 9 are:
108, 117, 126 999
Series of AP:
108,117, 126 999
T_n =999
a = 108
d = 117 - 108 = 9
As we know that,
$\mathrm{T}_n = a + (n - 1) d$
⇒ 999 = 108 + (n - 1) 9
⇒ 891 = (n - 1) 9
⇒ 99 = n - 1
⇒ n = 100

26(B). Two digit numbers which are divisible by 4 are 12, 16, 20, ..., 96 forms an AP with first term a = 12, common difference d = 4 and n th term a_n = 96.
⇒ a_n = a + (n - 1) × d
⇒ 12 + (n - 1) × 4 = 96
⇒ n = 22

27(A). As we know,
General form of the equation of a circle, $x^2 + y^2 + 2gx + 2fy + c = 0$
Centre is $(-g, -f)$ or $\left(\frac{-\text{coefficient of } x}{2}, \frac{-\text{coefficient of } y}{2}\right)$,
where g, f and c are constant.
Radius $= \sqrt{g^2 + f^2 - c}$
Given,
Equation of circle is $5x^2 + 5y^2 - 20x - 6y + 15 = 0$.
$x^2 + y^2 - 4x - \frac{6}{5}y + 3 = 0$ ...(i)
On compare eq. (i) with equation of circle, we get
$g = -2, f = \frac{-3}{5}$ and $c = 3$
As we know that,
Radius of circle $= \sqrt{g^2 + f^2 - c}$
Radius $= \sqrt{(-2)^2 + \left(\frac{-3}{5}\right)^2 - 3}$
$= \sqrt{4 + \frac{9}{25} - 3}$
$= \frac{\sqrt{34}}{5}$ units

28(D). As we know,
Equation of parabola having a vertex at the origin and along x-axis $y^2 = 4ax$
It is given that the vertex of the parabola is at the origin and its axis lies along the x-axis. So, its equation is
$y^2 = 4ax$ OR $y^2 = -4ax$
Since it passes through the point $P(3, 4)$, so it lies in the first quadrant.
$\therefore$ Its equation is $y^2 = 4ax$
Now, $P(3, 4)$ lies on it, so
$4^2 = 4a(3)$
$\Rightarrow 16 = 12a$
$\Rightarrow a = \frac{4}{3}$
So, the required equation is $y^2 = 4\left(\frac{4}{3}\right)x$.
$\therefore y^2 = \frac{16}{3}x$

29(D). Let $f(x)$ be any function.
$f(x)$ is onto if range of $f(x) =$ Co-domain
The function f is said to be many-one functions if there exist two or more than two different elements in X having the same image in Y .
Given function is:
$f : R \rightarrow \{0, 1\}$, such that:
$f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$
Co-domain $= \{0, 1\}$
Since, on taking a straight line parallel to the x -axis, the group of given function intersect it at many points.
$\Rightarrow f(x)$ is many-one.
Range of function is $\{0, 1\}$
As range of $f(x) =$ Co-domain
$\Rightarrow f(x)$ is onto.
Therefore, $f(x)$ is many-one onto.

30(C). Let us check for each option:
(A) Given:
$f(x) = |x|, \forall x \in R$
As we know that,
$f(x) = |x|$
$\Rightarrow f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$
So, $f(-1) = -(-1) = 1$ and $f(1) = 1$
$\Rightarrow f(-1) = f(1)$, but $-1 \neq 1$
$\therefore$ The property, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$,

does not hold true $\forall x_1, x_2 \in R$.
Therefore, the function $f(x)=|x|, \forall x \in R$ is not an injective function.
(B) Given:
$f(x)=x^2, \forall x \in R$
Let $x_1=1$ and $x_2=-1$
$\Rightarrow f(x_1)=x_1^2=1$
and, $f(x_2)=x_2^2=1$
$\Rightarrow f(x_1)=f(x_2)$, but $-1 \neq 1$
$\therefore$ The property, $f(x_1)=f(x_2) \Rightarrow x_1=x_2$, does not hold true $\forall x_1, x_2 \in R$.
Therefore, the function $f(x)=x^2, \forall x \in R$ is not an injective function.
(C) Given:
$f(x)=-x, \forall x \in R$
Let x_1 and x_2 be any two real numbers.
$\Rightarrow f(x_1)=-x_1$ and $f(x_2)=-x_2$
If $f(x_1)=f(x_2)$
$\Rightarrow -x_1=-x_2$
$\Rightarrow x_1=x_2$
$\therefore$ The property, $f(x_1)=f(x_2)$
$\Rightarrow x_1=x_2$, holds true $\forall x_1, x_2 \in R$
Therefore, the function $f(x)=-x, \forall x \in R$ is an injective function.

31(A). Given: Mean $=np=1$
Variance $=npq=\frac{3}{4}$
$\Rightarrow p=\frac{1}{4}, q=\frac{3}{4}, n=4$
Binomial distribution
$P(X=r)={}^nC_r p^r q^{n-r}$
$P(X=3)={}^4C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^{4-3} \Rightarrow$
$\frac{4\times3\times2\times1}{3\times2\times1}\times\frac{1}{64}\times\frac{3}{4} \Rightarrow 4\times\frac{1}{64}\times\frac{3}{4}$
$=\frac{3}{64}$

32(A). Given:
In two throws of a dice, total chances $n(S)=(6\times6)=36$
Let E is the event of getting a sum 9
$\therefore E=\{(3,6),(4,5),(5,4),(6,3)\}$
$\Rightarrow n(E)=4$
$\therefore P(E)=\frac{n(E)}{n(S)}$
$=\frac{4}{36} \Rightarrow \frac{1}{9}$

33(B). Given:
$\vec{\alpha}=k$ and $\vec{\gamma}=2i+3j+4k$
$\vec{\beta}$ is perpendicular to both $\vec{\alpha}$ and $\vec{\gamma}$
$\therefore \vec{\alpha}\times\vec{\gamma}=\vec{\beta}$
$$\vec{\alpha}\times\vec{\gamma}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ 2 & 3 & 4\end{vmatrix}$$
$=\hat{i}(0-3)-\hat{j}(0-2)+\hat{k}(0)$
$=-3\hat{i}+2\hat{j}$

34(C). Given:
α is any vector
Let $\alpha=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$
As we know that, if $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then
$\vec{a}.\vec{b}=a_1b_1+a_2b_2+a_3b_3$
$(\alpha.\hat{i})\hat{i}+(\alpha.\hat{j})\hat{j}+(\alpha.\hat{k})\hat{k}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$
$(\alpha.\hat{i})\hat{i}+(\alpha.\hat{j})\hat{j}+(\alpha.\hat{k})\hat{k}=\alpha$

35(B). Given:
$(1+x+x^2+x^3)^{11}$.
By expanding given equation using expansion formula we can get the coefficient x^4.
$1+x+x^2+x^3$
$\Rightarrow (1+x)+x^2(1+x)$
$\Rightarrow (1+x)(1+x^2)$
So,
$(1+x+x^2+x^3)^{11}=(1+x)^{11}(1+x^2)^{11}$
$=1+{}^{11}C_1x^2+{}^{11}C_2x^2+{}^{11}C_3x^3+{}^{11}C_4x^4$
$=1+{}^{11}C_1x^2+{}^{11}C_2x^4+\ldots\ldots$
To find term in from the product of two brackets on the right-hand-side, consider the following products terms as,
$=1\times{}^{11}C_2x^4+{}^{11}C_2x^2\times{}^{11}C_1x^2+{}^{11}C_4x^4$
$\Rightarrow \left[{}^{11}C_2+{}^{11}C_2\times{}^{11}C_1+{}^{11}C_4\right]x^4$
$\Rightarrow [55+605+330]x^4$
$\Rightarrow 990x^4$
So,
The coefficient of x^4 is 990.

36(C). Given:
$\left(x^3-\frac{1}{x^2}\right)^{15}$
Let $(r+1)^{th}$ term be the constant term in the expansion of $\left(x^3-\frac{1}{x^2}\right)^{15}$.
We know that in the binomial expansion of $(a+x)^n$, we have,
$T_{r+1}={}^nC_rx^{n-r}a^r$
$\therefore T_{r+1}={}^{15}C_r\left(x^3\right)^{15-r}\left(-\frac{1}{x^2}\right)^r$
$T_{r+1}={}^{15}C_rx^{45-5r}(-1)^r$ is independent of x. If:
$45-5r=0$
$\Rightarrow r=9$
Thus, 10^{th} term is independent of x and is given by
$T_{10}={}^{15}C_9(-1)^9$
$=-{}^{15}C_9$

37(A). Let the points be $A(1,1)$ and $B(2,3)$
Slope of line passing through two points (x_1,y_1) and (x_2,y_2) is given by:
$m=\frac{y_2-y_1}{x_2-x_1}$
Where,
$x_1=1, x_2=2, y_1=1, y_2=3$
Let the slope of line AB is m_1 and slope of perpendicular is m_2.
Slope of $AB\ (m_1)=\frac{3-1}{2-1}=2$
We know that when two lines are perpendicular then the product of their slope is -1.
$m_1 \times m_2 = -1$
Slope of perpendicular $(m_2)=\frac{-1}{2}$
Mid point of $AB=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
Mid point of $AB=\left(\frac{2+1}{2},\frac{3+1}{2}\right)$
$=\left(\frac{3}{2},2\right)$
So the equation of the right bisector of the lines is:
$y-2=\frac{-1}{2}\left(x-\frac{3}{2}\right)$
$\Rightarrow 4(y-2)=-(2x-3)$
$\Rightarrow 4y-8=-2x+3$
$\Rightarrow 2x+4y-11=0$

38(C). Given,
Lines are:
$x-2y=5\ldots\ldots\ldots(i)$
and $y-2x=5\ldots\ldots\ldots(ii)$
Let m_1 and m_2 are the slope of the given lines
From equation (i),
$x-5=2y$
$\Rightarrow y=\frac{x}{2}-\frac{5}{2}$
on comparing general equation of the line ($y=mx+c$) we get,
$m_1=\frac{1}{2}$
From equation (ii),
$y=2x+5$
$m_2=2$
Now,
Angle between the two lines is given by:
$\tan\theta=\left|\frac{(m_1+m_2)}{1+m_1\times m_2}\right|$
$\Rightarrow \tan\theta=\left|\frac{\left(\frac{1}{2}+2\right)}{\{1+\left(\frac{1}{2}\right)\times2\}}\right|$
$\Rightarrow \tan\theta=\left|\frac{\left(\frac{5}{2}\right)}{(1+1)}\right|$
$\Rightarrow \tan\theta=\left|\frac{\left(\frac{5}{2}\right)}{2}\right|$
$\Rightarrow \tan\theta=\frac{5}{4}$
$\Rightarrow \theta=\tan^{-1}\left(\frac{5}{4}\right)$

39(C). Given,
$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
Put $x=\tan\theta$
We have to find the value of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
Put $x=\tan\theta$
$\Rightarrow \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$
$\cos^{-1}\left(\frac{1-\tan^2\theta}{\sec^2\theta}\right)$
$(\because 1+\tan^2\theta=\sec^2\theta)$
$=\cos^{-1}\left(\cos^2\theta-\sin^2\theta\right)$
$=\cos^{-1}(\cos2\theta)$
$(\because \cos2\theta=\cos^2\theta-\sin^2\theta)$
$=2\theta \quad (\because \cos^{-1}\cos x=x)$
$=2\tan^{-1}x \quad (\because x=\tan\theta)$

40(B). Given,
$x=\tan^{-1}\left(\frac{1}{5}\right)$
$\tan x=\frac{1}{5}$
As we know that, $\sin2\theta=\frac{2\tan\theta}{1+\tan^2\theta}$
$\therefore \sin2x=\frac{2\tan x}{1+\tan^2x}$
$=\frac{2\times\frac{1}{5}}{1+\left(\frac{1}{5}\right)^2}$
$=\frac{\left(\frac{2}{3}\right)}{\frac{s+1}{25}}$
$=\frac{2}{5}\times\frac{25}{26}=\frac{10}{26}$

$= \frac{5}{13}$

41(D). Given scores 10, 12, 13, 15, 15, 13, 12, 10, x and mode = 15

The mode of the n observation is the number that has the highest frequency.

Frequency of score '12' = 2

Frequency of score '15' = 2

But for mode to be 15, x should be '15'.

42(D). Given: $x =$ M.D., $y =$ S.D

We know that,

M.D $= \frac{4}{5}$ S.D

Where, M.D is mean deviation and S.D is standard deviation

$x = \frac{4}{5} y$

$\therefore x < y$

43(B). Given:

$\Rightarrow$ Number of coins having tail on both sides $= n$

$\Rightarrow$ Number of fair coins $= n + 1$

According to question,

$\Rightarrow$ Probability of getting a tail $= \frac{31}{42}$

$\Rightarrow P$ (tail)

$= \frac{{}^nC_1}{{}^{2n+1}C_1} \times 1 + \frac{{}^{n+1}C_1}{{}^{2n+1}C_1} \times \frac{1}{2} = \frac{31}{42}$

$\Rightarrow \frac{n}{2n+1} + \frac{n+1}{2(2n+1)} = \frac{31}{42}$

$\Rightarrow (3n + 1) \times 21 = 31(2n + 1)$

$\Rightarrow 63n + 21 = 62n + 31$

$\Rightarrow n = 10$

$\therefore$ Total coins in bag $= 2n + 1 = 21$

44(A). Given:

Forecast of rain $= 70\%$

Correct probability $= 60\%$

$P(A) = \frac{70}{100} = \frac{7}{10}$

$P(B) = \frac{60}{100} = \frac{3}{5}$

Probability of two unrelated events happening together is equal to product of individual probabilities.

$\therefore$ Probability of correctly forecasting rain $P(A \cap B)$

$= P(A) \times P(B)$

$= \frac{7}{10} \times \frac{3}{5}$

$= \frac{21}{50}$

45(A). Given,

$6 \sin^2 x - 2 \cos^2 x = 4$

$\Rightarrow 6 \sin^2 x - 2 \cos^2 x = 4 \times 1$

As we know that,

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow 6 \sin^2 x - 2 \cos^2 x = 4 \left(\sin^2 x + \cos^2 x\right)$

$\Rightarrow 6 \sin^2 x - 2 \cos^2 x = 4 \sin^2 x + 4 \cos^2 x$

$\Rightarrow 6 \sin^2 x - 4 \sin^2 x = 4 \cos^2 x + 2 \cos^2 x$

$\Rightarrow 2 \sin^2 x = 6 \cos^2 x$

$\Rightarrow \tan^2 x = 3$

$\therefore \tan x = \sqrt{3}$

46(A). Given,

$\frac{p^2}{a^2} + k \cos \alpha + \frac{q^2}{b^2} = \sin^2 \alpha \ldots\ldots$ (i)

$\cos^{-1}\left(\frac{p}{a}\right) + \cos^{-1}\left(\frac{q}{b}\right) = \alpha$

As we know,

$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1 - x^2} \cdot \sqrt{1 - y^2}\right)$

$\cos^{-1}\left(\frac{pq}{ab} - \sqrt{1 - \frac{p^2}{a^2}}\sqrt{1 - \frac{q^2}{b^2}}\right) = \alpha$

$\cos \alpha = \left(\frac{pq}{ab} - \sqrt{1 - \frac{p^2}{a^2}}\sqrt{1 - \frac{q^2}{b^2}}\right)$

$\frac{pq}{ab} - \cos \alpha = \sqrt{1 - \frac{p^2}{a^2}}\sqrt{1 - \frac{q^2}{b^2}}$

Squaring both sides, we get

$(\frac{pq}{ab} - \cos \alpha)^2 = (\sqrt{1 - \frac{p^2}{a^2}}\sqrt{1 - \frac{q^2}{b^2}})^2$

$\frac{(pq)^2}{(ab)^2} + \cos^2 \alpha - 2\frac{pq}{ab}\cos \alpha = \left(1 - \frac{p^2}{a^2}\right)\left(1 - \frac{q^2}{b^2}\right)$

$\frac{(pq)^2}{(ab)^2} + \cos^2 \alpha - 2\frac{pq}{ab}\cos \alpha = 1 - \frac{p^2}{a^2} - \frac{q^2}{b^2} + \frac{(pq)^2}{(ab)^2}$

$\sin^2 \alpha = \frac{p^2}{a^2} + \frac{q^2}{b^2} - 2\frac{pq}{ab}\cos \alpha \ldots\ldots$ (ii)

Comparing equation (i) and (ii), we get

$k = -\frac{2pq}{ab}$

47(D). Given: Equation of curve is $y = \sqrt{5x - 3} - 2$ and the tangent to the curve $y = \sqrt{5x - 3} - 2$ is parallel to the line $4x - 2y + 3 = 0$

The given line $4x - 2y + 3 = 0$ can be re-written as:

$y = 2x + \left(\frac{3}{2}\right) = 0$

Now by comparing the above equation of line with $y = mx + c$ we get,

$m = 2$ and $c = \frac{3}{2}$

$\because$ The line $4x - 2y + 3 = 0$ is parallel to the tangent to the curve $y = \sqrt{5x - 3} - 2$

As we know that if two lines are parallel then their slope is same.

So, the slope of the tangent to the curve $y = \sqrt{5x - 3} - 2$ is $m = 2$

Let, the point of contact be (x_1, y_1)

As we know that slope of the tangent at any point say (x_1, y_1) to a curve is given by:

$m = \left[\frac{dy}{dx}\right]_{(x_1, y_1)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{5x-3}} \cdot 5 - 0 = \frac{5}{2\sqrt{5x-3}}$

$\Rightarrow \left[\frac{dy}{dx}\right]_{(x_1, y_1)} = \frac{5}{2\sqrt{5x_1-3}}$

$\because$ Slope of tangent to the curve $y = \sqrt{5x - 3} - 2$ is $m = 2$

$\Rightarrow 2 = \frac{5}{2\sqrt{5x_1-3}}$

By squaring both the sides of the above equation we get:

$4 = \frac{25}{4 \cdot (5x_1 - 3)}$

$\Rightarrow x_1 = \frac{73}{80}$

$\because (x_1, y_1)$ is point of conctact i.e., (x_1, y_1) will satisfy the equation of curve:

$y = \sqrt{5x - 3} - 2$

$\Rightarrow y_1 = \sqrt{5x_1 - 3} - 2$

By substituting $x_1 = \frac{73}{80}$ in the above equation we get:

$y_1 = -\frac{3}{4}$

So, the point of contact is: $\left(\frac{73}{80}, -\frac{3}{4}\right)$

As we know that equation of tangent at any point say (x_1, y_1) is given by:

$y - y_1 = \left[\frac{dy}{dx}\right]_{(x_1, y_1)} \cdot (x - x_1)$

$\Rightarrow y + \frac{3}{4} = 2 \cdot \left(x - \frac{73}{80}\right)$

$\Rightarrow 80x - 40y - 103 = 0$

So, the equation of tangent to the given curve at the point $\left(\frac{73}{80}, -\frac{3}{4}\right)$ is $80x - 40y - 103 = 0$

48(C). Slope of the curve $= \frac{dy}{dx}$

Given: Equation of the curve $y = x^2$ ... (1)

Let's find Slope of tangent at any point on curve (x, y)

$y = x^2$

Differentiating with respect to x, we get:

$\frac{dy}{dx} = 2x$

According to question, Slope of the tangent $= y$ -coordinate of the point

$2x = y$

$\Rightarrow 2x = x^2$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x(x - 2) = 0$

$\therefore x = 0$ or 2

Put the value of x in equation 1^{st}, we get:

$y = 0$ or 4

Therefore, the required points are $(0, 0)$ and $(2, 4)$

49(B). As we know the general term in $(a + b)^n$,

$T_{r+1} = {}^nC_r a^{n-r} b^r$

Given,

$\left(2x^2 - \frac{1}{3x^2}\right)^{10}$

Here, $a = 2x^2, b = \frac{-1}{3x^2}$ and $r = 5$

6^{th} coefficient $\rightarrow 5^{th}$ term

$T_6 = {}^{10}C_5 \left(2x^2\right)^5 \left(\frac{-1}{3x^2}\right)^5$

$= {}^{10}C_5 \times 2^5 \times \frac{-1}{3^5}$

$= -\frac{252 \times 32}{243}$

$= -\frac{896}{27}$

50(B). Given:

$\lim_{x \to 0} \frac{\log(1+2x)}{\tan 2x}$

Dividing and multiplying the numerator and denominator by $2x$, we get:

$= \lim_{x \to 0} \frac{\frac{\log(1+2x)}{2x} \times 2x}{\frac{\tan 2x}{2x} \times 2x}$

We know that:

$\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided $\lim_{x \to a} g(x) \neq 0$

$= \frac{\lim_{x \to 0} \frac{\log(1+2x)}{2x}}{\lim_{x \to 0} \frac{\tan 2x}{2x}}$

As we know that:
$\lim_{x\to0}\frac{\tan x}{x}=1$
and, $\lim_{x\to0}\frac{\log(1+x)}{x}=1$
Therefore,
$\lim_{x\to0}\frac{\tan 2x}{2x}=1$
and, $\lim_{x\to0}\frac{\log(1+2x)}{2x}=1$
Therefore,
$\lim_{x\to0}\frac{\log(1+2x)}{\tan 2x}=\frac{1}{1}=1$

51(C). Given that:
$\lim_{x\to0}\frac{\left(e^{4x^2}-1\right)}{x\sin x}$
Let,
$L=\lim_{x\to0}\frac{\left(e^{4x^2}-1\right)}{x\sin x}$
$L=\lim_{x\to0}\frac{\left(e^{4x^2}-1\right)}{x\sin x}\times\frac{4x}{4x}$
$L=\lim_{x\to0}\frac{\left(e^{4x^2}-1\right)}{4x^2}\times\left(\frac{x}{\sin x}\right)\times4$
We know that:
$\lim_{x\to0}\frac{e^x-1}{x}=1$
$\lim_{x\to0}\frac{\sin x}{x}=1$
$\therefore\lim_{x\to0}\frac{\left(e^{4x^2}-1\right)}{4x^2}=1$
and, $\lim_{x\to0}\left(\frac{x}{\sin x}\right)=1$
and,
Now,
$L=1\times1\times4=4$

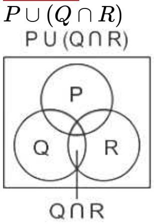
52(A). The given point is $P(1,0,3)$ and equation of line passing through $(4,7,1)$ and $(3,5,3)$ is given by,
$\frac{x-4}{1}=\frac{y-7}{2}=\frac{z-1}{-2}=k$ (let)...(1)
So, any point on this line is Q $(k+4,2k+7,-2k+1)$.
Now direction ratios of PQ are $k+3,2k+7,-2k-2$.
Also $PQ\perp(1)$
$\therefore 1(k+3)+2(2k+7)-2(-2k-2)=0$
$\Rightarrow k+3+4k+14+4k+2=0$
$\Rightarrow 9k+21=0$
$\Rightarrow 9k=-21$
$\Rightarrow k=\frac{-21}{7}$
$\Rightarrow k=\frac{-7}{3}$
Coordinates of Q are $x=\frac{-7}{3}+4,y=2\times\frac{-7}{3}+7,z=-2\times\frac{-7}{3}+1$
$\Rightarrow x=\frac{-7+12}{3},y=\frac{-14+21}{3},z=\frac{14+3}{3}$
$\Rightarrow x=\frac{5}{3},y=\frac{7}{3},z=\frac{17}{3}$
So, the coordinates of the foot of the perpendicular drawn from the point $A(1,0,3)$ to the join of the points $B(4,7,1)$ and $C(3,5,3)$ are $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$.

53(B). Given,
Three planes are
x + y = 0...(i)
y + z = 0...(ii)
x + z = 0...(iii)
Adding these three planes, we get
2(x + y + z) = 0
⇒ x + y + z = 0...(iv)
Putting value of (x + y) in (iv), we get
0 + z = 0
⇒ z = 0
Putting value of (y + z) in (iv), we get
x + 0 = 0
⇒ x = 0
Putting value of (x + z) in (iv), we get
y + 0 = 0
⇒ y = 0
So, (x, y, z) = (0, 0, 0)
So, the three planes meet in a unique point.

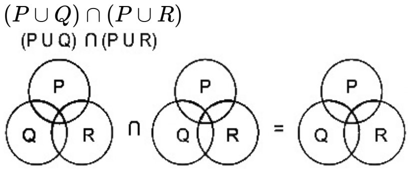
54(B). If a set containing n elements then number of elements in their subset = 2^n
For a given set A, a set B is a subset of set A if all elements of set B are also elements of set A. Set A is called the super-set of set B.
Null set "{ }" or "ϕ" is a subset of all sets.

55(D). Given,
$f(x)=\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$
Here, $1-x^2\geq0$
$\Rightarrow x^2-1\leq0$
$\Rightarrow (x-1)(x+1)\leq0$
when, $x-1=0\Rightarrow x=1$
when, $x+1=0\Rightarrow x=-1$
thus, domain of $x=[-1,1]$

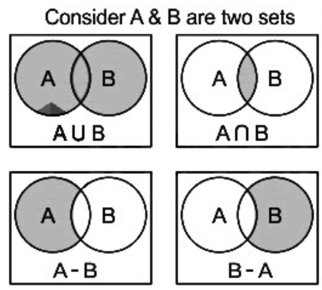
56(C). Given: P, Q, R are three sets
$P\cup(Q\cap R)$



$(P\cup Q)\cap(P\cup R)$



Venn Diagram
→ It's used to illustrate the logical relation.
= Ships between two or more sets or items.
⇒ They serve to graphically organize things, highlighting how the items are similar and different.
= widely used in mathematics, statistics, logic, teaching, linguistics, computer science and business.



Conclusion:
$P\cup(Q\cap R)=(P\cup Q)\cap(P\cup R)$

57(C). Let B, H, F denote the sets of members who are in the basket hall team, hockey team and football team respectively.
Given n(B) =21, n(H)= 2 6, n(F) = 29.
n(H∩B) = 14, n (H∩F) = 15, n(F∩B) = 12 and n(B ∩ H ∩ F) = 8.
We have to find n(B U H U F) i.e., The total number of members in the three athletic teams.
n(B U H U F) = n(B) + n(H).n(F) − n(B ∩ H) − n(H ∩ F) − n(F ∩ B) + n(B ∩ H ∩ F).
n(B U H U F) = (21 + 26 + 29) − (14 + 15 + 12) + 8 = 43

58(A). Given:
$f(2a-x)=f(x)$ ......(1)
$\int_0^a f(x)dx=\lambda$ .....(2)
Using the property (1)
$\int_0^{2a}f(x)dx=\int_0^a f(x)dx+\int_0^a f(2a-x)dx$
From equation (1)
$\Rightarrow\int_0^{2a}f(x)dx=\int_0^a f(x)dx+\int_0^a f(x)dx$
From equation (2)
$\Rightarrow\int_0^{2a}f(x)dx=\lambda+\lambda$
$\Rightarrow\int_0^{2a}f(x)dx=2\lambda$

59(D). Let XY plane divides the line joining the points $A(2,3,-5)$ and $B(-1,-2,-3)$ in the ratio $k:1$.
When the line segment is divided internally in the ratio $m:n$, we use the formula:
$\Leftrightarrow(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$
Using the section formula, the coordinate of the point of intersection is given by:
$\left(\frac{-k+2}{k+1},\frac{-2k+3}{k+1},\frac{-3k-5}{k+1}\right)$
As we know, on the XY plane Z-coordinate is zero.
Therefore, $\frac{-3k-5}{k+1}=0$
$\Rightarrow -3k-5=0$
$\Rightarrow -3k=5$
$\Rightarrow\frac{k}{1}=\frac{-5}{3}$
Therefore, the ratio is 5 : 3 externally.

60(A). Given:
Equation of line is $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and equation of plane is $10x+2y-11z-3=0$
As we know that the angle between the line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and the plane $a_2x+b_2y+c_2z+d=0$ is given by:

$\sin\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right)\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$

Here, $a_1 = 2, b_1 = 3, c_1 = 6, a_2 = 10, b_2 = 2$ and $c_2 = -11$

$\Rightarrow a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2 = 20 + 6 - 66 = -40$

$\Rightarrow \sqrt{a_1^2 + b_1^2 + c_1^2} = 7$ and $\sqrt{a_2^2 + b_2^2 + c_2^2} = 15$

$\Rightarrow \sin\theta = \frac{40}{7\times 15} = \frac{8}{21}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{8}{21}\right)$