Important Questions for Class 11

Physics

Chapter 9 - Mechanical Properties of Solids

Very Short Answer Questions **1 Mark**

1. The stretching of a coil spring is determined by its shear modulus. Why?

Ans: Stretching of coil spring is determined by shear modulus as when a coil spring is stretched, neither its length nor its volume or its shape changes.

2. The spherical ball contracts in volume by 0.1% when subjected to a uniform normal pressure of 100 atmosphere. Calculate the bulk modulus of material of the ball.

Ans: Volumetric strain is given by:

$$\text{Volumetric strain} = \frac{\Delta V}{V} = 0.1\% = \frac{0.1}{100} = 10^{-3}$$

Normal Stress is 100 atmosphere

$100 \times 10^5 = 10^7 \text{ N/m}^2$

Hence, the Bulk Modulus of the material of the ball will be:

$$K = \frac{\text{Normal Stress}}{\text{Volumetric Strain}} = \frac{10^7}{10^{-3}} = 10^{10} \text{ N/m}^2$$

3. State Hooke's law.

Ans: According to Hooke's law the extension produced in the wire is directly proportional to the load applied within the elastic limit.

$\Rightarrow \text{Stress} \propto \text{Strain}$

Hence, Stress $= E \times$ Strain

Where,

$E =$ Modulus of elasticity.

4. What are ductile and brittle materials?

Ans: The materials which show large plastic range beyond elastic limit are called ductile materials. Examples of ductile materials are copper, iron etc.

The materials which show very small plastic range beyond elastic limit are called ductile materials. Examples of brittle materials are cast Iron, glass etc.

5. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed N, what is the maximum load the cable can support?

Ans: In the above question it is given that:

Radius of the steel cable is $r = 1.5\text{ cm} = 0.015\text{ m}$.

Maximum allowable stress is $10^8\,\text{N/m}^2$.

Now maximum stress will be:

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross section}}$$

Hence,

$$\text{Maximum force} = \text{Maximum stress} \times \text{Area of cros-ssection}$$

So,

$$\text{Maximum force} = 10^8 \times \pi(0.015)^2 = 7.065 \times 10^4\,\text{N}$$

Therefore, the maximum load the cable can support is $7.065 \times 10^4\,\text{N}$.

6. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans: In the above question it is given that:

Hydraulic pressure exerted on the glass slab is $p = 10$ atm.

Bulk modulus of glass is $B = 37 \times 10^{9}\,N/m^{2}$.

Now,

Bulk modulus is given by:

$$B = \frac{p}{\frac{\Delta V}{V}}$$

Where,

$\frac{\Delta V}{V}$ is the fractional change in volume.

$$\frac{\Delta V}{V} = \frac{p}{B} = \frac{10 \times 1.013 \times 10^{5}}{37 \times 10^{9}} = 2.73 \times 10^{-5}$$

Therefore, the fractional change in volume of a glass slab is 2.73×10^{-5}.

Very Short Answer Questions **2 Marks**

1. Write the characteristics of displacement.

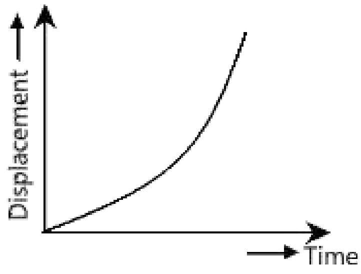
Ans: Following are the characteristics of displacement:

(1) Displacement is a vector quantity having both magnitude and direction.

(2) Displacement of a given body can be positive, negative or zero.

2. Draw displacement time graph for uniformly accelerated motion. What is its shape?

Ans: Following is the time graph for uniformly accelerated motion which is parabolic in shape.



3. Sameer went on his bike from Delhi to Gurgaon at a speed of 60km/hr and came back at a speed of 40km/hr. What is his average speed for the entire journey?

Ans: In the above question it is given that:

Speed of bike when Sameer travelled from Delhi to Gurgaon is $v_1 = 60\text{km} / \text{hr}$.

Come back speed is $v_2 = 40\text{km} / \text{hr}$.

Therefore, average speed will be:

$$\text{Average speed} = \frac{2v_1v_2}{v_1 + v_2} = \frac{2(60)(40)}{60 + 40} = 48\text{km} / \text{hr}$$

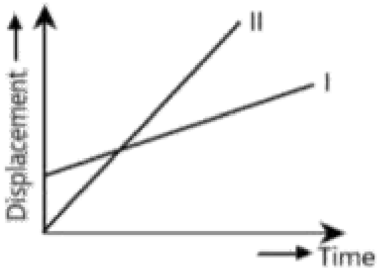
4. What causes variation in velocity of a particle?

Ans: Variation in velocity of a particle happens when:

(1) magnitude of velocity changes

(2) direction of motion changes.

5. Figure. Shows displacement – time curves I and II. What conclusions do you draw from these graphs?



Ans: From the graph given in the question we can conclude that:

(1) Both the curves represent uniform linear motion.

(2) Uniform velocity of II is more than the velocity of I because slope of curve (II) is greater.

6. Displacement of a particle is given by the expression $x = 3t^2 + 7t - 9$, where x is in meters and t is in seconds. What is acceleration?

Ans: Expression of Displacement a particle is given by $x = 3t^2 + 7t - 9$

Therefore,

$$v = \frac{dx}{dt} = 6t + 7$$

And

$a = \frac{dv}{dt} = 6 m/s^2$, which is the required acceleration.

7. A particle is thrown upwards. It attains a height (h) after 5 seconds and again after 9s comes back. What is the speed of the particle at a height h?

Ans: According to Newton's laws of motion:

$$s = ut + \frac{1}{2}at^2$$

The net displacement at 4s is zero as the particle comes to the same point at 9s where it was at 5s.

Hence,

$$0 = (u \times 4) - \frac{1}{2}(g)(4)^2$$

$$(u) \times 4 = \frac{1}{2}(g)(4)^2$$

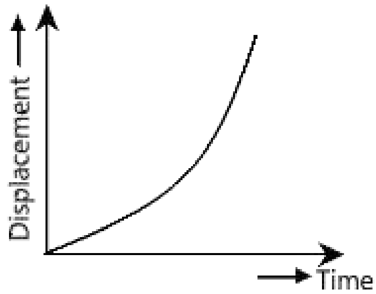
Hence,

$$u = 2 \times 9.8 = 19.6 m / s$$

Hence, the speed of the particle at a height h is $19.6 m / s$.

8. Draw displacement time graph for a uniformly accelerated motion? What is its shape?

Ans: Following is the displacement time graph for a uniformly accelerated motion:



9. The displacement x of a particle moving in one dimension under the action of constant force is related to the time by the equation $t = \sqrt{x} - 3$ where x is in meters and t is in seconds. Find the velocity of the particle at (1) t = 3s (2) t = 6s.

Ans: The given equation for displacement of particle is

$$t = \sqrt{x} - 3$$

$\Rightarrow x = (t+3)^2$

Now, velocity is calculated as:

$$v = \frac{dx}{dt} = 2(t+3)$$

Therefore,

(1) t = 3s

Ans: v at $t = 3s$ will be:

$$v = 2(3+3) = 12m/s$$

(2) t = 6s.

Ans: v at $t = 6s$ will be:

$$v = 2(6+3) = 18m/s$$

10. A balloon is ascending at the rate of 4.9m/s. A pocket is dropped from the balloon when situated at a height of 245m. How long does it take the packet to reach the ground? What is its final velocity?

Ans: In the above question it is given that:

Initial velocity, $u = 4.9m/s$

Height, $h = 245m$.

As the pocket is in freefall, $a = g = 9.8m/s^2$.

Therefore, using Newton's Laws of motion:

$$s = ut + \frac{1}{2}at^2$$

$$245 = -4.9t + \frac{1}{2}(9.8)t^2$$

$$4.9t^2 - 4.9t - 245 = 0$$

$\Rightarrow t = 7.6$ or -5.6

Hence, $t = 7.6s$

Now, $v = u + at$

$v = -4.9 + 9.8(7.6) = 69.6m/s$

Hence, it takes $7.6s$ for the packet to reach the ground and the final velocity is $69.6m/s$.

11. A car moving on a straight highway with speed of 126km/hr. is brought to a stop within a distance of 200m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop?

Ans: In the above question it is given that:

$u = 126km/r = 35m/s$,

$v = 0m/s$,

$s = 200m$ And

$t = ?$

Now,

We know that:

$$a = \frac{u^2 - v^2}{2s} = \frac{35^2 - 0}{2(200)} = -3.06m/s^2$$

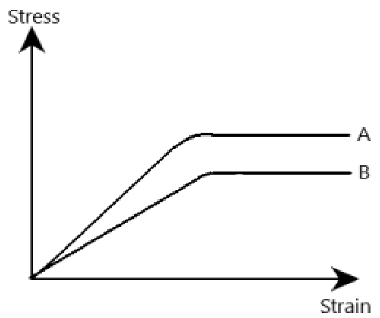
Now, $v = u + at$

Therefore, $0 = 35 - (3.06)t$

$t = 11.4s$

Hence, the retardation of the car is $-3.06m/s^2$ and it takes $11.4s$ for the car to stop.

12. In the following stress – strain curve, which has:



1) Greater young's Modulus 2) More Ductility 3) More Tensile strength.

Ans: From the graph given in the question:

1) Young's Modulus is the slope of stress – strain graph. As the slope of A is greater than that of B, therefore A has greater young's Modulus.
2) Ductility is defined as the extent of plastic deformation. From the graph, it is greater for A. Hence, A has more ductility.
3) Tensile strength is the direct measure of the required stress. From the graph it is clear that stress for A is greater. Hence A has more tensile strength.

13. A cube is subject to a pressure of $5\times10^{5}\,N/m^{2}$. Each side of the cube is shortened by 1%. Find: - 1) the volumetric strain 2) the bulk modulus of elasticity of the cube.

Ans: In the above question it is given that:

Pressure is $p=5\times10^{5}\,N/m^{2}$.

Consider the initial length of the cube to be l m.

Therefore, Initial volume will be $l^{3}\ m^{3}$.

Now change in length is $1\%=0.01l$.

Therefore, final length will be: $l-0.01l=0.99l$.

Hence, final volume be $0.99^{3}l^{3}\ m^{3}$.

Now, change in volume will be:

$$\Delta V = l^3 - 0.99^3 l^3 = \left(1 - \left(\frac{99}{100}\right)^3\right) l^3$$

1) the volumetric strain

Ans: Therefore, volumetric strain is given by:

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{\left(1-\left(\frac{99}{100}\right)^3\right) l^3}{l^3} = \left(1 - \left(\frac{99}{100}\right)^3\right)$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3}{100} = 0.03$$

2) the bulk modulus of elasticity of the cube.

Ans: $\text{Bulk modulus} = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{5\times10^5}{0.03} = 1.67\times10^7\ \text{N/m}^2$

14. If the potential energy is minimum at $r = r_0 = 0.74A^\circ$, is the force attractive or repulsive at $r = 0.5A^\circ$; $1.9A^\circ$ and α?

Ans: As the potential energy is minimum at $r = r_0 = 0.74A^\circ$, the

interatomic force between two atoms will be zero for $r = r_0 = 0.74A^\circ$.

1) For $r = 0.5A^\circ$ (Which is less than r_0 **),**

The force is repulsive.

2) For $r = 1.9A^\circ$ (Which is greater than r_0),

The force is attractive.

3) For $r = \alpha$, the force is zero.

15. A hollow shaft is found to be stronger than a solid shaft made of the same material. Why?

Ans. The torque required to produce a given twist in a hollow cylinder is greater than that required to produce in a solid cylinder of same length and material through the same angle. Hence, a hollow shaft is found to be stronger

than a solid shaft made of equal material.

16. Calculate the work done when a wire of length l and area of cross – section A is made of material of young's Modulus Y is stretched by an amount x?

Ans: We know that:

$$\text{Young's Modulus} = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$\Rightarrow Y = \frac{\frac{F}{A}}{\frac{l}{L}}$$

Hence,

$$Y = \frac{FL}{Al}$$

Where,

F is Force,

A is the Area,

l is the change in length,

L is the original length and

x is the change in length.

Hence, the average extension will be $\frac{x}{2}$.

We know that,

$$\text{Work Done} = \text{Force. Average extension}$$

And $Y = \dfrac{FL}{Al}$

Therefore,

$$\text{Work Done} = \left(\frac{YAl}{L}\right)\left(\frac{x}{2}\right)$$

As $x = l$

$$\text{Work Done} = \frac{YAx^2}{2L}$$

18. The length of a metal is l_1 , then the tension in it is T_1 and it is l_2 when tension is T_2. Find the original length of wire?

Ans: Consider l to be the original length of the material wire,

A to be the original length of metal wire,

Hence, the change in length in the first case will be $(l_1 - l)$

Change in length is second case will be $(l_2 - l)$.

We know that:

$$\text{Young's Modulus} = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$\Rightarrow Y = \frac{\frac{T}{A}}{\frac{\Delta l}{l}}$$

Hence, for first case:

$$Y = \frac{T_1}{A} \times \frac{l}{(l_1 - l)}$$

And for second case:

$$Y = \frac{T_2}{A} \times \frac{l}{(l_2 - l)}$$

As the Young's modulus is same for both cases,

$$\frac{T_1}{A} \times \frac{l}{(l_1 - l)} = \frac{T_2}{A} \times \frac{l}{(l_2 - l)}$$

Hence,

$$T_1(l_2 - l) = T_2(l_1 - l)$$

$$T_2 l_1 - T_1 l_2 = (T_2 - T_1)l$$

$\Rightarrow l = \dfrac{T_2 l_1 - T_1 l_2}{(T_2 - T_1)}$, which is the original length of the wire.

19. An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support?

Ans: We know that:

$$\text{Breaking load} = \text{Breaking Stress} \times \text{Area}$$

Hence, if cable is cut to half of its original length, there will be no change in its area of cross section. Therefore, there is no effect on the maximum load that the wire can support.

20. Define modulus of elasticity and write its various types.

Ans: The ratio of the stress to the corresponding strain produced within the elastic limit is defined as the modulus of elasticity (E).

$$E = \frac{\text{Stress}}{\text{Strain}}$$

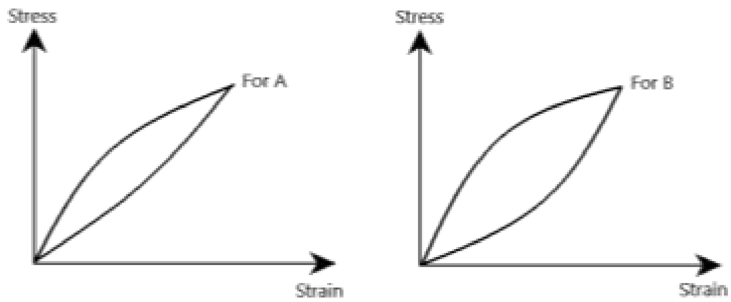
Types of Modulus of elasticity are:

1. Young's Modulus $= \frac{\text{Normal stress}}{\text{Longitudinal strain}}$,

2. Bulk Modulus $= \frac{\text{Normal stress}}{\text{Volumetric strain}}$,

3. Modulus of Rigidity $= \frac{\text{Tangential stress}}{\text{Shearing strain}}$

21. Two different types of rubber are found to have the stress – strain curves as shown in the figure stress



1) In what ways do these curves suffer from the stress- strain curve of a metal wire?

2) Which of the two rubbers A and B would you prefer to be installed in the working of a heavy machinery?

3) Which of these two rubbers would you choose for a car tyre?

Ans: From the figure given in question it is clear that:

1) Hooke's law is not obeyed as the curve is not a straight line. Thus, these curves are called elastic hysteresis as the materials do not retrace curves during unloading.

2) Rubber B is preferred as the area of loop B is greater than that of A. This states that A has more absorption power for vibrations which is useful in machinery.

3) As hysteresis loop is a direct measure of heat dissipation, rubber A is preferred over B, to minimize the heating in the car tyres.

22. Which is more elastic rubber or steel? Explain.

Ans: Consider,

Length of rubber and steel rod to be ‘l’ and their area to be ‘a’,

Y_r to be Young’s modulus of elasticity for rubber,

Y_s to be Young’s modulus of elasticity for steel,

When stretching force F is applied,

Extension in rubber is Δl_r and

Extension in steel is Δl_s.

Therefore,

$Y_r = \frac{Fl}{A\Delta l_r}$ And

$Y_s = \frac{Fl}{A\Delta l_s}$

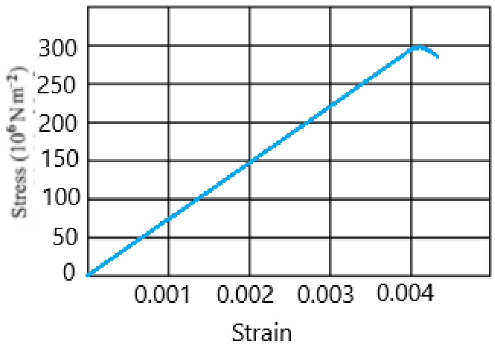
Now Δl_r is greater than Δl_s.

And, $Y \propto \frac{1}{\Delta l}$

Therefore, $Y_s > Y_r$

As modulus of elasticity measures elasticity of the material, Hence, steel is more elastic than rubber.

23. Figure 9.11 shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



Ans: Considering the graph given above:

(a) Young's modulus

Ans: when Stress $=150 \times 10^6 \ N/m^2$,

strain $= 0.002$.

Young's modulus, $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{150 \times 10^6}{0.002} = 7.5 \times 10^{10} N/m^2$

Therefore, Young's modulus for the given material is $7.5 \times 10^{10} N/m^2$.

(b) approximate yield strength for this material

Ans: Yield strength of a material is defined as the maximum stress the material can sustain without crossing the elastic limit. From the graph given, the approximate yield strength of the material is $300 \times 10^6 N/m^2$ i.e., $3 \times 10^8 N/m^2$

24. The stress-strain graphs for materials A and B are shown in Fig. 9.12.

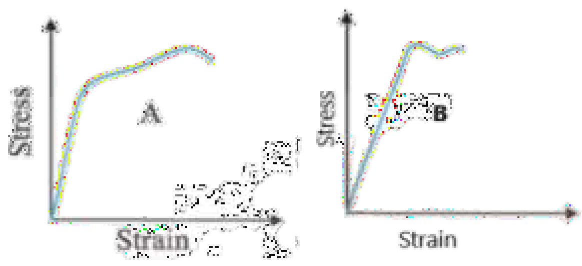


Fig 9.12

The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?

Ans: From the graph given in the question it is observed that for a given strain, the stress for material A is more than it is for material B.

$$\text{Young's modulus} \propto \text{Stress}$$

Therefore, A has greater Young's modulus.

(b) Which of the two is the stronger material?

Ans: The strength of a material is given by the amount of stress required for fracturing a material, corresponding to its fracture point. Fracture point is defined as the extreme point in a stress- strain curve. From the graph, material A can withstand more strain than material B. Therefore, material A is stronger than material B.

25. Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel

Ans: We know that for a given stress, the strain in rubber is more than it is in steel.

And Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}}$

Hence, for a constant stress,

$$Y \propto \frac{1}{\text{Strain}}$$

Therefore, Young's modulus for rubber is less than it is for steel.

Hence, statement (a) is false.

(b) The stretching of a coil is determined by its shear modulus.

Ans: Shear modulus is defined as the ratio of the applied stress to the change in the shape of a body. The stretching of a coil leads to the change in its shape. Thus, shear modulus of elasticity is involved in this process. Hence, statement (b) is true.

26. How much should the pressure on a litre of water be changed to compress it by 0.10%?

Ans: In the above question it is given that:

Volume of water is $V = 1\text{ L}$.

The water is to be compressed by 0.10%.

The fractional change is $\frac{\Delta V}{V} = \frac{0.1}{100 \times 1} = 10^{-3}$

Bulk modulus is given by $B = \frac{p}{\frac{\Delta V}{V}}$

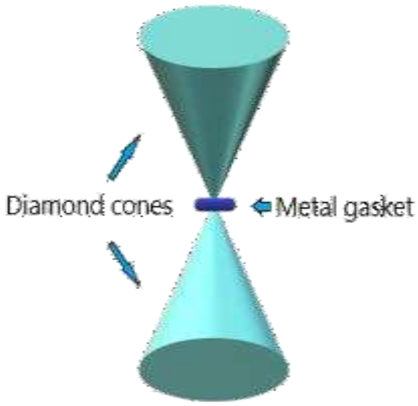
$$p = B \times \frac{\Delta V}{V}$$

We have $B = 2.2 \times 10^{9}\text{ N/m}^{2}$

$\Rightarrow p = 2.2 \times 10^{9} \times 10^{-3} = 2.2 \times 10^{6}\text{ N/m}^{2}$

Hence, the pressure on water should be $2.2 \times 10^{6}\text{ N/m}^{2}$.

27. Anvils made of single crystals of diamond, with the shape as shown in figure, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Ans: In the above question it is given that:

The diameter of the cones at the narrow ends is $d = 0.50\text{mm} = 0.5\times10^{-3}\text{m}$

Radius will be $r = 0.25\times10^{-3}\,\text{m}$

The Compressional force is $F = 50000\text{N}$.

Pressure at the tip of the anvil is given by:

$$P = \frac{\text{Force}}{\text{Area}} = \frac{50000}{\pi\left(0.25\times10^{-3}\right)^2} = 2.55\times10^{11}\text{Pa}$$

Clearly, the pressure at the tip of the anvil will be $2.55\times10^{11}\text{Pa}$.

28. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed $6.9\times10^{7}\,\text{Pa}$? Assume that each rivet is to carry one quarter of the load.

Ans: In the above question it is given that:

The diameter of the metal strip is $d = 6.0\text{mm} = 6.0\times10^{-3}\text{m}$.

Radius will be $r = 3.0\times10^{-3}\text{m}$.

Maximum shearing stress is $6.9\times10^{7}\,\text{Pa}$.

We know that:

$$\text{Maximum stress} = \frac{\text{Maximum load or force}}{\text{Area}}$$

$$\text{Maximum force} = \text{Maximum stress} \times \text{Area}$$

$$\text{Maximum force} = 6.9 \times 10^{7} \times \pi(r)^{2}$$

$$\text{Maximum force} = 6.9 \times 10^{7} \times \pi\left(3.0 \times 10^{-3}\right)^{2} = 1949.94\text{N}$$

Since each rivet is said to carry one quarter of the load;

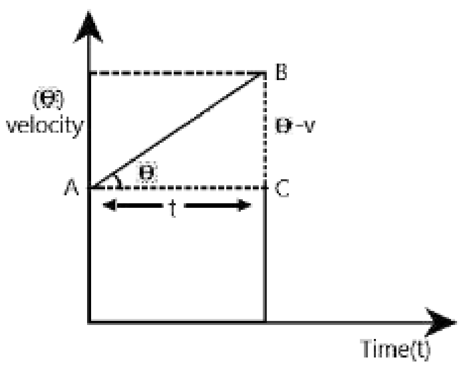
Maximum tension on each rivet is $1949.94 \times 4 = 7799.76\text{N}$

Clearly, the maximum tension that can be exerted is 7799.76N.

Short Answer Questions **3 Marks**

1. Define from the velocity time graph, v = u + at

Ans: Consider the graph given below



Slope of graph is $\tan\theta = \frac{u - v}{t}$

And $\tan\theta = a$

Hence, $at = u - v$

$v = u + at$

2. A particle is moving along a straight line and its position is given by the relation

$$x = \left(t^3 - 6t^2 - 15t + 40\right)m$$

Find

(a) The time at which velocity is zero.

(b) Position and displacement of the particle at that point.

(c) Acceleration

Ans: Given expression of position is $x = \left(t^3 - 6t^2 - 15t + 40\right)m$.

$$v = \frac{dx}{dt} = \left(3t^2 - 12t - 15\right)m/s \quad \text{and}$$

$$a = \frac{dv}{dt} = \left(6t - 12\right)m/s^2$$

(a) The time at which velocity is zero.

Ans: Calculating time at which velocity is zero,

$$\left(3t^2 - 12t - 15\right) = 0$$

$$t^2 - 4t - 5 = 0$$

$$\left(t - 5\right)\left(t + 1\right) = 0$$

$$\therefore t = 5, -1$$

Hence, $t = 5s$

(b) Position and displacement of the particle at that point.

Ans: Position at $t = 5s$ is given by

$$x = \left(5\right)^3 - 6\left(5\right)^2 - 15\left(5\right) + 40 = -60$$

Position at $t = 0s$ is given by

$x = (0)^3 - 6(0)^2 - 15(0) + 40 = 40$

Hence displacement $= x_5 - x_0 = -60 - 40 = -100\text{m}$

(c) Acceleration

Ans: Acceleration at $t = 5\text{s}$ is given by:

$a = 6(5) - 12 = 18\text{m}/\text{s}^2$

3. A police jeep on a petrol duty on national highway was moving with a speed of 54km/hr. in the same direction. It finds a thief rushing up in a car at a rate of 126km/hr in the same direction. Police sub – inspector fired at the car of the thief with his service revolver with a muzzle speed of 100m/s. With what speed will the bullet hit the thief's car?

Ans: In the above question it is given that:

$V_{PJ} = 54\text{km}/\text{hr} = 15\text{m}/\text{s}$

$V_{TC} = 126\text{km}/\text{hr} = 35\text{m}/\text{s}$

$v_b = 100\text{m}/\text{s}$

Hence,

Velocity of car w.r.t. police, $V_{CP} = 35 - 15 = 20\text{m}/\text{s}$

Velocity of bullet w.r.t. car, $V_{BC} = 100 - 20 = 80\text{m}/\text{s}$

Hence, bullet will hit the car with velocity $80\text{m}/\text{s}$.

4. Establish the relation $S_{nth} = u + \frac{a}{2}(2n - 1)$ where the letters have their usual meanings.

Ans: We have, $S_{nth} = u + \frac{a}{2}(2n - 1)$.

We know that $S_{nth} = S_n - S_{n-1}$

And

$$S_n = un + \frac{1}{2}an^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$S_{nth} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

Hence,

$$S_{nth} = u - \frac{1}{2}a + na$$

Therefore,

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

Hence proved.

5. A stone is dropped from the top of a cliff and is found to travel 44.1m diving the last second before it reaches the ground. What is the height of the cliff? g = 9.8m/s2

Ans: Consider the height of the cliff to be h m.

$u = 0 m/s$,

$a = g = 9.8 m/s^2$.

If n is the total time taken by the stone while falling,

$$S_{nth} = u + \frac{a}{2}(2n-1)$$

$$44.1 = 0 + \frac{9.8}{2}(2n-1)$$

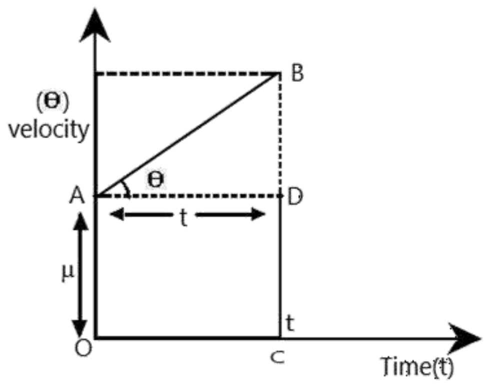
$$n = \frac{10}{2} = 5s$$

Now,

$$h = ut + \frac{1}{2}at^2$$

$$h = \frac{1}{2}(9.8)(5)^2 = 122.5m$$

6. Establish from the velocity time graph, a uniform accelerated motion.



Ans: Consider the graph given in the question.

The displacement of the particle is given by the area under the v-t graph.

$$S = \text{area OABC}$$

$$S = \text{area of rectangle AODC} + \text{area of ADB}$$

Hence,

$$S = (OA \times OC) + \left(\frac{1}{2} AD \times BD\right)$$

$$S = ut + \frac{1}{2}(AD) \times \left(\frac{AD \times DB}{AD}\right)$$

$$S = ut + \frac{1}{2}(AD)^2 \times \left(\frac{DB}{AD}\right)$$

$$S = ut + \frac{1}{2}(t)^2 \times \left(\frac{DB}{AD}\right)$$

$$S = ut + \frac{1}{2}(t)^2 \times (a)$$

$$\left[\text{As } a = \tan\theta = \frac{BD}{AD}\right]$$

Therefore,

$$S = ut + \frac{1}{2}at^2$$

7.

(a) Define the term relative velocity.

Ans: The relative velocity of any object A with respect to object B is termed as the time rate of change of position of A with respect to B.

(b) Write the expression for relative velocity of one moving with respect to another body when objects are moving in the same direction and are moving in opposite directions?

Ans: Consider two objects to be moving in the same direction,

Then,

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

Where,

$\vec{V}_A$ is the velocity of A.

$\vec{V}_B$ is the velocity of B.

$\vec{V}_{AB}$ is the velocity of A with respect to B.

Now consider two objects to be moving in the opposite direction, Then,

$$\vec{V}_{AB} = \vec{V}_A - \left(-\vec{V}_B\right) = \vec{V}_A + \vec{V}_B$$

Where, $-\vec{V}_B$ indicates that B moves in the direction opposite to A.

(c) A Jet airplane traveling at the speed of 500km/hr ejects its products of combustion at the speed of 1500km/h relative to the Jet plane. What is the speed of the latter with respect to an observer on the ground?

Ans: We have,

Velocity of the Jet plane, $V_J = 500\text{km / hr}$

Velocity of gases w.r.t. Jet plane $V_{gJ} = -1500\text{km / hr}$ (direction is opposite)

$$\vec{V}_{gJ} = \vec{V}_g - \left(-\vec{V}_J\right) = \vec{V}_g + \vec{V}_J$$

Now, we know that hot gases also come out in opposite direction of the Jet plane,

Velocity of the gas, $V_g = -1500 + 500 = -1000\text{km / hr}$.

8. Define (i) $v = u + at$ (ii) $v^2 - u^2 = 2as$ by calculus method

Ans:

(i) $v = u + at$

Ans: Acceleration is given by:

$$a = \frac{dv}{dt}$$

$$\Rightarrow dv = adt$$

Integrating on both sides,

$$\int dv = \int adt$$

Hence, $v = at + k$ …… (1)

Where, k is the constant.

When, $t = 0$ and $\theta = u$

We get $k = u$

Hence, $v = u + at$.

(ii) $v^2 - u^2 = 2as$

Ans: We have, $a = \frac{dv}{dt}$

Multiplying and dividing by dx,

$$a = \frac{dv}{dt} \times \frac{dx}{dx}$$

$$a = \frac{dv}{dt} \times \theta$$

$$adx = vdv$$

As $\frac{dx}{dt} = v$

On integrating,

$$a\int_{x_0}^{x} dx = \int_{v}^{\theta} vdv$$

Hence, $a\left(x - x_0\right) = \frac{u^2}{2} - \frac{v^2}{2}$

As $\left(x - x_0 = s\right)$

$$as = \frac{u^2 - v^2}{2}$$

$$v^2 - u^2 = 2as$$

9. Explain:

1) Elastic Body

Ans: Elastic Body: An elastic body is defined as the body which completely regains its original configuration immediately after the removal of deforming force on it. For example, Quartz and phosphor Bronze.

2) Plastic Body

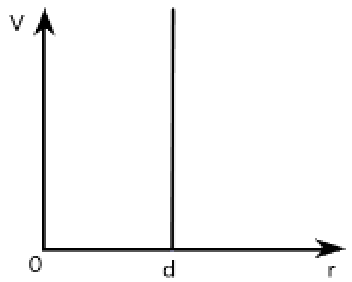
Ans: Plastic Body: A plastic body is defined as the body which does not regain its original configuration at all on the removal of deforming force, however the deforming force may be. For example, Paraffin wax.

3) Elasticity.

Ans: Elasticity: Elasticity is defined as the property of the body to regain its original configuration, when the deforming forces are removed.

10. Why is the force of repulsion responsible for the formation of a solid and not the forces of attraction?

Ans: When the motion of a large number of spheres, it is observed that two hard spheres do not attract each other, but rebound immediately on collision. That is, they do not come closer than their diameter 'd'. The interaction potential 'V' for a pair of hard spheres is

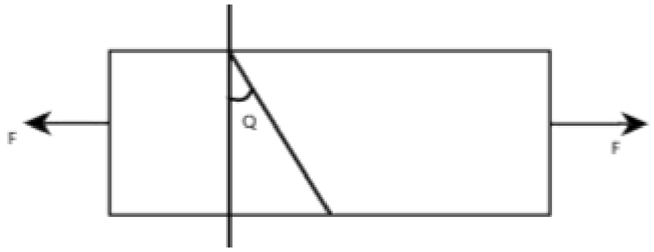


Where,

d is the diameter and r is the distance of interaction of 2 spheres.

It indicates that there is infinite repulsion when $r = d$ and no potential when $r > d$ and thus repulsive forces bind them together.

11. A bar of cross section A is subjected to equal and opposite tensile force F at its ends. If there is a plane through the bar making an angle Q, with the plane at right angles to the bar in the figure



(a) Find the tensile stress at this plane in terms of F, A and Q

Ans: We know that:

$$\text{Tensile stress} = \frac{\text{Normal force}}{\text{Area}}$$

And Normal force $= F\cos\theta$

$$\text{Tensile stress} = \frac{F\cos\theta}{\frac{A}{\cos\theta}} = \frac{F\cos^2\theta}{A}$$

(b) What is the shearing stress at the plane in terms of F,A and Q?

Ans: We know that:

$$\text{Shearing stress} = \frac{\text{Tangential stress}}{\text{Area}}$$

And Tangential stress $= F\sin\theta$,

$$\text{Area} = \frac{\text{Area}}{\cos\theta}$$

Hence,

Shearing stress$=\dfrac{F\sin\theta}{\dfrac{\text{Area}}{\cos\theta}}=\dfrac{F}{A}\sin\theta\cos\theta$

$\Rightarrow$ Shearing stress$= =\dfrac{2F\sin\theta\cos\theta}{2A}=\dfrac{F\sin^2\theta}{2A}$

(c) For what value of Q is tensile stress a maximum.

Ans: Calculating the value of Q for which tensile stress a maximum,

We know that $\text{Tensile stress}=\dfrac{F\cos^2\theta}{2A}$

Hence $\text{Tensile stress}\ \propto\cos^2\theta$,

When F and A are constant.

Now maximum value of $\cos^2\theta$ is 1

Hence calculating θ,

$\cos^2\theta=1$

$\therefore\cos\theta=1$

Hence, $\theta=0^\circ$

Thus, value of Q is 0° when the tensile stress is maximum.

12. The Young's modulus of steel is $2.0\times10^{11}\ \text{w/m}^2$. If the interatomic spacing for the metal is 2.8×10^{-10}m, find the increase in the interatomic spacing for a force of $10^9\ \text{N/m}^2$ and the force constant?

Ans: In the above question it given that:

Young's modulus of steel is $2.0\times10^{11}\ \text{w/m}^2$.

Force is $2.8\times10^{-10}\,\text{m}$.

Now, we know that:

$$Y = \text{Modulus of elasticity} = \frac{F \times l}{A \times \Delta l}$$

$$\Rightarrow \Delta l = \frac{Fl}{AY}$$

$$\Rightarrow \Delta l = \frac{10^9 \times 2.8 \times 10^{-10}}{2 \times 10^{11}} = 1.4 \times 10^{-12}\ m$$

Hence, $\Delta l = 0.014\mathring{A}$.

We know that the distance between 2 atoms is l.

Therefore, area of chain of atoms will be: $A = l \times l = l^2$

$$\Rightarrow Y = \frac{F \times l}{l^2 \times \Delta l} = \frac{F}{l \times \Delta l}$$

Now force constant, $K = \frac{F}{\Delta l}$.

$$\Rightarrow Y = \frac{K}{l}$$

$$K = Yl = 2.0 \times 10^{11} \times 2.8 \times 10^{-10}$$

Hence,

$$K = 56N / m$$

13. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

Ans: In the above question it given that:

Mass of the big structure is $M = 50,000\ kg$.

Inner radius of the column is $r = 30\ cm = 0.3\ m$.

Outer radius of the column is $R = 60\text{ cm} = 0.6\text{ m}$.

Young's modulus of steel is $Y = 2\times10^{11}\text{Pa}$.

Hence, total force exerted will be:

$F = Mg = 50000\times9.8\text{ N}$.

Now,

Stress = Force exerted on a single column = $\frac{50000\times9.8}{4}$=122500N

$$Y = \frac{\text{Stress}}{\text{Strain}},$$

$$\text{Strain} = \frac{\frac{F}{A}}{Y}$$

Where, $A = \pi\left(R^2 - r^2\right) = \pi\left((0.6)^2 - (0.3)^2\right)$

$$\text{Strain} = \frac{122500}{\pi\left((0.6)^2 - (0.3)^2\right)\times2\times10^{11}} = 7.22\times10^{-7}$$

Thus, the compressional strain of each column is 7.22×10^{-7}.

14. A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain.

Ans: In the above question it is given that:

Length of the piece of copper is $l = 19.1\text{mm} = 19.1\times10^{-3}\text{m}$.

Breadth of the piece of copper is $b = 15.2\text{mm} = 15.2\times10^{-3}\text{m}$

Area of the copper piece will be:

$A = l\times b$

$\Rightarrow$ A $= 19.1\times10^{-3}\times15.2\times10^{-3} = 2.9\times10^{-9}\,\text{m}^2$

Tension force applied on the piece of copper is $F = 44500\text{N}$.

Modulus of elasticity of copper is $\eta = 42\times10^{9}\,\text{N}/\text{m}^2$.

We know that:

$$\text{Modulus of elasticity}(\eta) = \frac{\text{Strerss}}{\text{Strain}}$$

$$\eta = \frac{\left(\frac{F}{A}\right)}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{F}{A\eta}$$

$$\Rightarrow \text{Strain} = \frac{44500}{2.9\times10^{-4}\times42\times10^{9}} = 3.65\times10^{-3}$$

Hence, the resulting strain is 3.65×10^{-3}.

15. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Ans: In the above question it is given that:

Edge of the aluminium cube is $L = 10\text{cm} = 0.1\text{m}$.

The mass attached to the cube is $m = 100\text{kg}$.

Shear modulus η of aluminium is $25\text{GPa} = 25\times10^{10}\,\text{Pa}$.

We know that:

$$\text{Shear modulus}(\eta) = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$n = \frac{\left(\frac{F}{A}\right)}{\left(\frac{L}{\Delta L}\right)}$$

Where,

F is the applied force.

$F = mg = 100 \times 9.8 = 980N$

Area of one of the faces of the cube is $A = 0.1 \times 0.1 = 0.01m^2$.

Vertical deflection of the cube is ΔL.

$$\Delta L = \frac{FL}{A\eta}$$

$$\Rightarrow \Delta L = \frac{980 \times 0.1}{0.01 \times 25 \times 10^9} = 3.92 \times 10^{-7} m$$

Clearly, the vertical deflection of this face of the cube is $3.92 \times 10^{-7} m$.

Long Answer Questions **4 Mark**

1. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm $(1atm = 1.013 \times 10^5 Pa)$. Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Ans: In the above question it is given that:

Initial volume is $V_1 = 100.0 l = 100 \times 10^{-3} m^3$.

Final volume is $V_2 = 100.5 l = 100.5 \times 10^{-3} m^3$.

Thus, the increase in volume is $V_2 - V_1 = 0.5 \times 10^{-3} m^3$.

Increase in pressure is $\Delta p = 100 atm = 100 \times 1.013 \times 10^5 Pa$.

The formula for bulk modulus is

$$\text{Bulk Modulus} = \frac{\Delta p}{\left(\frac{\Delta V}{V_1}\right)} = \frac{\Delta p V_1}{\Delta V}$$

$$\text{Bulk Modulus} = \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3} m^3} = 2.026 \times 10^9 \text{ Pa}$$

We know that Bulk modulus of air is $1 \times 10^5 Pa$.

$$\frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1 \times 10^5} = 2.026 \times 10^4$$

This ratio is very high because air is more compressible than water.

2. The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about $1.1 \times 10^8 Pa$. A steel ball of initial volume is $0.32 m^3$ dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches the bottom?

Ans: In the above question it is given that:

Water pressure at the bottom of the trench is about $1.1 \times 10^8 Pa$.

Initial volume of the steel ball is $V = 0.32 m^3$.

Bulk modulus of steel is $1.6 \times 10^{11} N/m^2$.

The ball is said to fall at the bottom of the Pacific Ocean, which is 11 km beneath the surface.

Consider the change in the volume of the ball on reaching the bottom of the trench to be ΔV.

Bulk modulus is given by the relation:

$$B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

$$\Delta V = \frac{pV}{B}$$

$$\Rightarrow \Delta V = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}} = 2.2 \times 10^{-4} \, m^3$$

Hence, the change in volume of the ball on reaching the bottom of the trench is $2.2 \times 10^{-4} m^3$.

3. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratio of their diameters if each is to have the same tension.

Ans: In the above question it is given that:

Tension force acting on each wire is the same.

Therefore, the extension produced in each wire is the same.

As the length of both wires is the same, the strain in both wires is also the same.

Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{\left(\frac{F}{A}\right)}{\text{Strain}} = \frac{\frac{4F}{\pi d^2}}{\text{Strain}} \qquad \ldots\ldots (1)$$

Where,

F is the Tension force,

A is the area of cross-section and

d is the diameter of the wire

From equation (1), it is clear that $Y \propto \frac{1}{d^2}$.

Young's modulus for iron is $Y_1 = 190 \times 10^9 \text{ Pa}$.

Let diameter of the iron wire be d_1.

Young's modulus for copper is $Y_2 = 100 \times 10^9 \text{ Pa}$.

Let diameter of the copper wire be d_2.

Therefore, the ratio of their diameters is given as:

$$\frac{d_2}{d_1} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{100 \times 10^9}} = \frac{1.31}{1}$$

Therefore, the ratio of diameters of copper wire to iron wire is 1.31:1.

Long Answer Questions **5 Marks**

1. A steel wire of length 4.7 m and cross-sectional area $3.0 \times 10^{-5} m^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area $4.0 \times 10^{-5} m^2$ of under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Ans: In the above question it is given that:

Length of the steel wire is $L_1 = 4.7\text{m}$.

Area of cross-section of the steel wire is $A_1 = 3.0 \times 10^{-5} \text{m}^2$.

Length of the copper wire is $L_2 = 3.5\text{m}$.

Area of cross-section of the copper wire is $A_2 = 4.0 \times 10^{-5} \text{m}^2$.

Now,

The change in length is given by:

$$\Delta L = L_1 - L_2 = 4.7 - 3.5 = 1.2\text{m}$$

Let the force applied in both the cases be F .

Therefore, Young's modulus of the steel wire is given by:

$$Y_1 = \frac{F}{A_1} \times \frac{L_1}{\Delta L} = \frac{F \times 4.7}{3.0 \times 10^{-5} \times 1.2} \quad \ldots\ldots (1)$$

And Young's modulus of the copper wire is given by:

$$Y_2 = \frac{F}{A_2} \times \frac{L_2}{\Delta L} = \frac{F \times 3.5}{4.0 \times 10^{-5} \times 1.2} \quad \ldots\ldots (2)$$

Therefore,

$$\frac{Y_1}{Y_2} = \frac{F \times 4.7 \times 4.0 \times 10^{-5} \times 1.2}{3.0 \times 10^{-5} \times 1.2 \times F \times 3.5} = \frac{1.79}{1}$$

Hence the ratio of Young's modulus of steel to that of copper is $1.79:1$.

2. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.

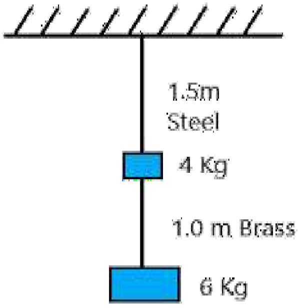


Fig 9.13

Ans: In the above question it is given that:

Diameter of the wires is $d = 0.25\text{m}$.

Hence $r = 0.125\text{m}$.

Length of the steel wire is $L_1 = 1.5\text{m}$.

Length of the brass wire is $L_2 = 1.0m$.

Total force exerted on the steel wire is $F_1 = (4+6)g = 10g$

$\Rightarrow F_1 = 10 \times 9.8 = 98N$.

Young's modulus for steel is given by

$$Y_1 = \frac{F_1}{A_1} \times \frac{L_1}{\Delta L_1}$$

Where,

ΔL_1 is the change in the length of the steel wire.

And A_1 is the area of cross-section of the steel wire.

$$\Rightarrow A_1 = \pi r_1^2$$

We have,

Young's modulus of steel is $Y_1 = 2.0 \times 10^{11} Pa$.

$$\Rightarrow \Delta L_1 = \frac{F_1 \times L_1}{A_1 \times Y_1}$$

$$\Rightarrow \Delta L_1 = \frac{98 \times 1.5}{\pi (0.125)^2 \times 2.0 \times 10^{11}} = 1.49 \times 10^{-4} m.$$

Total force on the brass wire is $F_2 = 6 \times 9.8 = 58.8N$.

Young's modulus for brass is given by

$$Y_2 = \frac{F_2}{A_2} \times \frac{L_2}{\Delta L_2}$$

Where,

ΔL_2 is the change in the length of the brass wire.

And A_2 is the area of cross-section of the brass wire.

$$\Rightarrow A_2 = \pi r_2^2$$

We have,

Young's modulus of brass is $Y_2 = 0.91\times10^{11}Pa$.

$$\Rightarrow \Delta L_2 = \frac{F_2 \times L_2}{A_2 \times Y_2}$$

$$\Rightarrow L_2 = \frac{58.8\times1}{\pi(0.125)^2\times0.91\times10^{11}} = 1.3\times10^{-4}m.$$

Clearly, the elongation of the steel wire is $1.49\times10^{-4}m$ and that of the brass wire is $1.3\times10^{-4}m$.

3. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is $0.65cm^2$. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Ans: In the above question it is given that:

Mass is $m = 14.5kg$.

Length of the steel wire is $l = 1.0m$.

Angular velocity is $\omega = 2rev/s$.

Cross-sectional area of the wire is $a = 0.065cm^2 = 0.065\times10^{-4}m^2$.

Consider the elongation of the wire when the mass is at the lowest point of its path to be Δl.

The total force on the mass when the mass is placed at the position of the vertical circle is given by:

$$F = mg + ml\omega^2$$

$$\Rightarrow F = 14.5\times9.8 + 14.5\times1\times2^4 = 200.1N$$

Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

$$\Rightarrow \Delta l = \frac{Fl}{AY}$$

We know that Young's modulus for steel is $2 \times 10^{11} \text{Pa}$.

Therefore,

$$\Rightarrow \Delta l = \frac{220.1 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}} = 1.53 \times 10^{-4} \text{m}$$

Thus, the elongation of the wire is $1.53 \times 10^{-4} \text{m}$.

4. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is $1.03 \times 10^3 \text{kg/m}^3$?

Ans: In the above question it is given that:

Pressure at the given depth is $p = 80.0\text{atm} = 80 \times 1.01 \times 10^5 \text{Pa}$.

Consider the given depth to be h.

Density of water at the surface is $\rho_1 = 1.03 \times 10^3 \text{kg/m}^3$

Consider ρ_2 to be the density of water at the depth h.

Consider V_1 to be the volume of water of mass m at the surface.

Consider V_2 to be the volume of water of mass m at the depth h.

Consider ΔV to be the change in volume.

$$\Delta V = V_1 - V_2$$

$$\Rightarrow \Delta V = m\left[\left(\frac{1}{\rho_1}\right) - \left(\frac{1}{\rho_2}\right)\right]$$

Now,

$$\text{Volumetric strain} = m\left[\left(\frac{1}{\rho_1}\right) - \left(\frac{1}{\rho_2}\right)\right] \times \left(\frac{\rho_1}{m}\right)$$

$$\frac{\Delta V}{V_1} = 1 - \left(\frac{\rho_1}{\rho_2}\right) \qquad \ldots\ldots (1)$$

Bulk modulus is given by:

$$\text{Bulk modulus} = \frac{pV_1}{\Delta V}$$

$$\frac{\Delta V}{V_1} = \frac{p}{B}$$

Compressibility of water is given by:

$$\frac{1}{B} = 45.8 \times 10^{-11} \text{Pa}^{-1}$$

$$\frac{\Delta V}{V_1} = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.71 \times 10^{-3} \qquad \ldots\ldots (2)$$

From equations (1) and (2) we get:

$$1 - \left(\frac{\rho_1}{\rho_2}\right) = 3.71 \times 10^{-3}$$

$$\rho_2 = \frac{1.03 \times 10^3}{\left[1 - \left(3.71 \times 10^{-3}\right)\right]} = 1.034 \times 10^3 \text{ kg / m}^3$$

Clearly, the density of water at the given depth (h) is $1.034 \times 10^3 \text{kg / m}^3$.

5. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of $7\times10^{6}\,\text{Pa}$.

Ans: In the above question it is given that:

The length of an edge of the solid copper cube is $l=10\text{cm}=0.1\text{m}$.

Hydraulic pressure is $p=7\times10^{6}\,\text{Pa}$.

Bulk modulus of copper is $B=140\times10^{9}\,\text{Pa}$.

Bulk modulus is given by the relation:

$$B=\frac{p}{\left(\frac{\Delta V}{V}\right)}$$

Where,

$\frac{\Delta V}{V}$ is the volumetric strain

ΔV is the change in volume.

V is the original volume.

$$\Delta V=\frac{pV}{B}$$

The original volume of the cube is $V=l^{3}$.

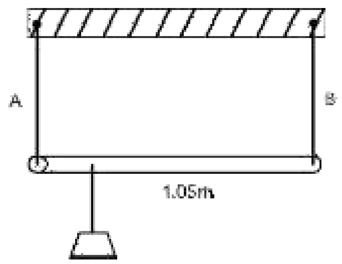
$$\Delta V=\frac{pl^{3}}{B}$$

$$\Rightarrow \Delta V=\frac{7\times10^{6}\times(0.1)^{3}}{140\times10^{9}}=5\times10^{-8}\,\text{m}^{3}=5\times10^{-2}\text{cm}^{3}$$

Clearly, the volume contraction of the solid copper cube is $5\times10^{-2}\text{cm}^{3}$.

6. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. 9.15. The cross-sectional areas of wires A and B are $1mm^2$ and $2mm^2$, respectively. At what point along the rod should a mass m be suspended in order to produce

(a) equal stresses



Ans: In the above question it is given that:

The cross-sectional area of wire A is $1.00mm^2 = 1.0 \times 10^{-6} m^2$.

Cross-sectional area of wire B is $2.0mm^2 = 2.0 \times 10^{-6} m^2$.

Young's modulus for steel is $Y_1 = 2 \times 10^{11} N/m^2$.

Young's modulus for aluminium is $Y_2 = 7 \times 10^{11} N/m^2$.

Consider a small mass m to be suspended to the rod at a distance y from the end where wire A is attached.

$$\text{Stress in the wire} = \frac{F}{a}$$

If the two wires have equal stresses,

$$\frac{F_1}{a_1} = \frac{F_2}{a_2}$$

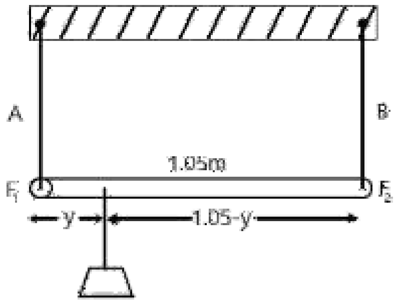
Where,

F_1 is the force exerted on the steel wire.

F_2 is the force exerted on the aluminium wire.

$$\frac{F_1}{a_1}=\frac{F_2}{a_2}=\frac{1}{2} \qquad \ldots\ldots (1)$$

Consider the figure given below:



Taking torque about the point of suspension, we get:

$$F_1 y = F_2\left(1.05-y\right) \quad \ldots\ldots (2)$$

From equations (1) and (2), we get:

$$\left(1.05-y\right)=\frac{1}{2}$$

$$\Rightarrow 2\left(1.05-y\right)=y$$

$$\Rightarrow y=0.7\text{m}$$

Clearly, to produce an equal stress in the two wires, the mass must be suspended at a distance of 0.7m from the end where wire A is attached.

(b) equal strains in both steel and aluminium wires.

Ans: Young's modulus is given by:

$$Y=\frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain}=\frac{\text{Stress}}{Y}=\frac{\left(\frac{F}{a}\right)}{Y}$$

When the strain in the two wires is equal,

$$\frac{\left(\frac{F_1}{a_1}\right)}{Y_1}=\frac{\left(\frac{F_2}{a_2}\right)}{Y_2}$$

$$\Rightarrow \frac{F_1}{F_2}=\frac{a_1Y_1}{a_2Y_2}$$

We have

$$\frac{a_1}{a_2}=\frac{1}{2}$$

$$\Rightarrow \frac{F_1}{F_2}=\frac{1\times 2\times 10^{11}}{2\times 7\times 10^{10}}=\frac{10}{7} \quad \ldots\ldots (3)$$

Consider the torques about the point where mass m, to be suspended at a distance y_1 from the side where wire A attached;

$$F_1y_1=F_2(1.05-y_1)$$

$$\Rightarrow \frac{F_1}{F_2}=\frac{(1.05-y_1)}{y_1} \quad \ldots\ldots (4)$$

From equations (3) and (4), we get:

$$(1.05-y_1)=\frac{10}{7}$$

$$\Rightarrow 7(1.05-y_1)=10y_1$$

$$\Rightarrow y_1=0.432\text{m}$$

Clearly, to produce an equal strain in the two wires, the mass must be suspended at a distance of 0.432m from the end where wire A is attached.

7. A mild steel wire of length 1.0 m and cross-sectional area $0.50\times 10^{-2}\text{cm}^2$ is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the

depression at the midpoint.

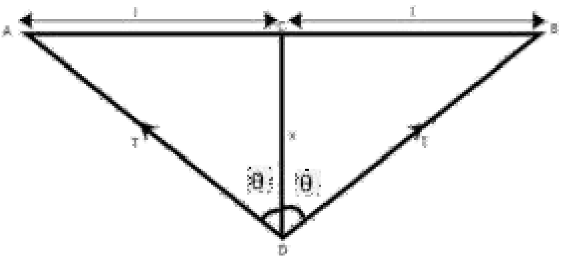
Ans: In the above question it is given that:

Mass is $m = 100g = 0.100kg$

Length of the mild steel wire is $1.0m$.

Cross-sectional area is $0.50 \times 10^{-2} cm^2$.

Now consider the figure given below:



If x is the depression at the mid point i.e., $CD = x$.

$$AB = BD = l = 0.5m$$

$$AD = BD = \sqrt{l^2 + x^2}$$

The increase in length will be:

$$\Delta l = AD + BD - AB = 2AD - AB$$

$$\Delta l = 2\sqrt{l^2 + x^2} - 2l$$

$$\Delta l = 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l$$

$$\Delta l = 2l\left(1 + \frac{x^2}{2l^2} - 1\right) = \frac{x^2}{1}$$

Now, we know that

$$\text{Strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{x^2}{2l^2}$$

If T is the tension in the wire, then

$2T\cos\theta = mg$

$$\Rightarrow T = \frac{mg}{2\cos\theta} \qquad \ldots\ldots (1)$$

Here,

$$\cos\theta = \frac{x}{\left(l^2 + x^2\right)^{\frac{1}{2}}} = \frac{x}{l\left(1 + \frac{x^2}{l^2}\right)^{\frac{1}{2}}} = \frac{x}{l\left(1 + \frac{x^2}{2l^2}\right)}$$

As $x << 1$;

$$1 + \frac{x^2}{2l^2} \approx 1$$

$$\Rightarrow \cos\theta = \frac{x}{l} \ldots\ldots (2)$$

From (1) and (2),

$$T = \frac{mg}{2x}$$

Also, stress is given by:

$$\text{Stress} = \frac{T}{A} = \frac{mgl}{2Ax}$$

And Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow Y = \frac{mgl}{2Ax} \times \frac{2l^2}{x^2} = \frac{mgl^3}{Ax^3}$$

$$\Rightarrow x = l\left(\frac{mg}{AY}\right)^{\frac{1}{3}}$$

$$\Rightarrow x = 0.5\left[\frac{0.1\times 10}{20\times 10^{11}\times 0.5\times 10^{-6}}\right]^{\frac{1}{3}} = 0.01074\text{m}$$

Clearly, the depression at the midpoint is 0.01074m.